assigment

1) i) 4, 7, 9, 9, 11, 15, 18, 18, 17, 25

$M = \text{mean} = \frac{\sum x_i}{n} = \frac{\cancel{116}\ 134}{10} = \cancel{11.6}\ 13.4$

median = 13 mode = 18

ii)

x_i	f_i	$x_i f_i$	CF
0	5	0	5
1	11	11	16
2	26	52	42
3	15	45	57
4	3	12	60
	60	120	

$\text{mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{120}{60} = 2$

$\text{median} = \frac{n+1}{2} = 30$ $\therefore$ median = 2

mode = 2

2) $\lambda = 0.5$

i) $P(X=0) = \frac{e^{-0.5}(0.5)^0}{0!} = e^{-0.5} = 0.607$

$P(X=1) = \frac{e^{-0.5}(0.5)^1}{1}$

$= e^{-0.5}(0.5)$

ii) $\left(P(X\geq 1) + P(X=0) + P(X=0)\right)6$

$= \left(\cancel{1 - P(X<1) + P(X=0)\times 2}\right)6$

$= \left(\left(1 - 0.90980\right) + 2(0.607)\right)6 =$

$= \left(1 - P(X<1) + e^{-0.5}0.5 + 2(0.607)\right)3$

3) $\alpha = 2 \quad \lambda = 1/2 \quad 0 < x < \alpha$

i) $\mu = \left(\frac{\alpha}{\lambda}\right) = \frac{2}{1/2} = 4$ $\quad Var(x) = \frac{\alpha}{\lambda^2} = \frac{2}{(1/2)^2} = 8$

$SD(x) = \sqrt{8}$

ii) $CDF = \int_0^\alpha \frac{\left(\frac{1}{2}\right)^2}{1} x^1 e^{-\frac{1}{2}x} = \frac{1}{4}\int x e^{-\frac{1}{2}x} dx$

$= \frac{1}{4}\left(x\left(e^{-1/2x}\right)\cdot(-2) - \int e^{-1/2x}(-2)\, dx\right.$

$= \frac{1}{4}\left(-2xe^{-\frac{1}{2}x} - (-2)\left[e^{-1/2x}\cdot(-2)\right]\right)$

$= \frac{1}{4}\left(-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right)_0^\alpha = \frac{1}{4}\left(\cancel{(-)}\ 0 - (0-4)\right) = \frac{1}{4}\times 4 = 1$

$P(Late \cap L2) = 40\% \times 50\% = 0.2$

5) $P(L1) = 40\%$ $P(L2) = 50\%$ $P(L3) = 10\%$

$P(Late/L1) = 20\%$ $P(Late/L2) = 40\%$ $P(Late/L3) = 10\%$

$\cancel{P(L2/Late) = P(Late/L2)}$ $\dfrac{P(L2 \cap Late)}{P(Late)} = \cancel{0.2}$

$P(L2/Late) = \dfrac{P(Late/L2)}{Total} = \dfrac{40\% \times 50\%}{40\% \times 20\% + 50\% \times 40\% + 10\% \times 10\%}$

$= \dfrac{2000\%}{800\% + 2000\% + 100\%} = \dfrac{20}{8+20+10} = \dfrac{20}{38} = \dfrac{10}{19}$

6) $X \sim U(1,2)$ $Y = \dfrac{3}{6-X}$ PDF of Y

$Y \sim U\left(\dfrac{3}{5}, \dfrac{3}{4}\right)$

$f(x) = \dfrac{1}{2-1} = \dfrac{1}{1} = 1$

$f(y) = f\left(\dfrac{3}{6-X}\right) = \dfrac{1}{\frac{3}{4} - \frac{3}{5}} = \dfrac{1}{\frac{15-12}{20}} = \dfrac{20}{3}$

9

$7(iv)$

7) i) $F(3) = 0.35 + 0.15 + 0.05 = 0.55$

ii) $E(x) = 0.05 + 0.3 + 1.05 + 1.6 + 0.25 = 3.25$

iii) $E(x^2) = \frac{11.45}{1} = \cancel{2.49}\ \cancel{2.29}\ 11.45.$ $\quad E(x) = 3.25$

$Var(x) = E(x^2) - (E(x))^2 = 0.8825$

~~iv~~ 8) $P(\text{exceeding } 1000) = 0.2$

$P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= (0.8)^{15} + 150.13194139S + '' = 0.3980$

9) P(winning) = 0.01 $\qquad k = 3$

$$P(X=7) = {}^{7-1}C_{3-1}(0.01)^3(0.99)^4 = 0.0000144092$$

10) P(in a week) = 2 = λ $\qquad$ P(in a day) = 0.4

$$P(X>2) = \cancel{P(X \geq 2) - P(X=2)} \quad 1 - P(X \leq 2)$$

$$P(X=2)\ \frac{e^{-0.5}(0.5)^2}{2} = 0.151632665$$

$$P(X>2) = 1 - 0.98561 = 0.01439$$

11) $P(M) = P(F) = 0.5 = \frac{1}{2}$

$P(\text{3 next boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0.125$

12) $\lambda = 10$ $\quad \frac{1}{0.1} \quad \frac{10}{1} \quad \lambda = 1$ (Per 0.1 days)

$P(x < 0.1) = \cancel{\frac{e^{-0.1}(0.1)^{0}}{}} \quad \cancel{\frac{e^{-10}(10)^{0.1}}{0.1}}$

13) $f_x(x) = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$

$P(80 < X < 100) = \int_{80}^{100} \frac{1}{45} dx = \frac{1}{45}(20) = \frac{20}{45} = \frac{4}{9}$

14) $X \sim \text{gama}(\overset{\alpha}{5}, \overset{\lambda}{0.4})$ $\quad P(X < 22.89)$

$2\lambda X \sim \chi^2_{10}$

$P(X < 22.89) = P(2\lambda X < 2\lambda(22.89))$

$= P(\chi^2_{10} < 18.132)$

$P(X < 18) = 0.9450 \quad P(X < 18.5) = .9529$

$0.5 = 0.0079 \quad 1 \quad 0.0158 \quad 0.132 \quad 0.0020856$

$P(\chi^2_{10} < 18.132) = \cancel{P(\chi} \; 0.9470856$

15) $P(x) = \frac{k}{(x+5)^4}$

i) Find k $\quad k\int_0^{\alpha} (x+5)^{-4} dx = 1 = -\frac{k}{3}\left[(x+5)^{-3}\right]_0^{\alpha} = \frac{k}{3}\left(\frac{1}{5^3}\right) = 1$

$k = 5^3 \times 3 = 375$

ii) $F10 = -\frac{375}{3}\left[\int_0^{10} (x+5)^{-3}\right] = -\frac{375}{3}\left(15^{-3} - 5^{-3}\right) = 0.96296296$

iii) $E(x) = \int_0^{\alpha} \frac{kx}{(x+5)^4} dx = k\int \frac{x\,dx}{(x+5)^4} = k\int \frac{t-5}{t^4} dt$ $\quad x+5 = t, \; dx = dt$

$= k\int_0^{\alpha}\left(\frac{1}{t^3} - \frac{5}{t^4}\right) = k\left[\frac{-t^{-2}}{2} + \frac{5t^{-3}}{3}\right]_0^{\alpha} =$