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B-69.

23/11/21

Calculus Assignment

1. Evaluate :

$$\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$$

Soln.

$$= \frac{2[9 + 2(-3)(h) + h^2] - 18}{h}$$

$$= \frac{2[9 - 6h + h^2] - 18}{h}$$

$$= \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$$

$$= \frac{\cancel{h}(-12 + 2h)}{\cancel{h}}$$

$$= -12 + 2(0)$$

$$= \boxed{-12}$$

2. Determine if the func. is cont. or disc. at a) $x = 4$ b) $x = 6$

$$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

Soln. a) $\lim_{x\to 4^-} g(x)$ $\lim_{x\to 4^+} g(x)$ $g(4) = 8$

$\lim_{x\to 4^-} (2x)$ $x < 6$ $x - 1 = 3 \neq 8$

$2(4) = 8$

It does. exist, ∵ LH lim = RH Lim,

It is continuous at $x = 4$.

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b) $x = 6$

$2x = 2(6) = 12$

$x - 1 = 6 - 1 = 5$

$12 \neq 5$ (LH lim $\neq$ RH lim)

$\therefore$ It is <u>discontinuous</u>.

3.) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

<u>Soln</u> Rationalizing the deno.

$= \dfrac{x-9}{3-\sqrt{x}} \times \dfrac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \dfrac{x-9(3+\sqrt{x})}{3^2 - x}$

$= \dfrac{x-9\ (3+\sqrt{x})}{9-x}$

$= \dfrac{\cancel{x-9}\ (3+\sqrt{x})}{-\cancel{(x-9)}}$

$= -(3+\sqrt{9})$

$= \boxed{-6}$

4.) Det. if the func. is cont. or disc. at:
a) $x = -1$ b) $x = 0$ $x = 3$

$f(x) = \dfrac{4x+5}{9-3x}$

Soln
* At $x = -1$ $f(-1) = \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{-4+5}{9+3} = \dfrac{1}{12}$

$\therefore$ continuous.

b) $x = 0$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{0+5}{9} = \frac{5}{9}$$ ∴ continuous

c) $x = 3$

$$f(3) = \frac{4(3)+5}{9-3(3)} = 0$$, not defined

∴ infinite discontinuous

5. $$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$ dividing each term by e^4

$$\frac{6e^{x} - e^{-6x}}{8e^{x} - e^{-2x} + 3e^{-3x}}$$

since $e^{-\infty} = 0$ $$\frac{6e^{\infty} - e^{-6\cdot\infty}}{8e^{\infty} - e^{-2\cdot\infty} + 3e^{-3\cdot\infty}}$$

$$\frac{6e^{\infty} - 0}{8e^{\infty} - 0 + 0}$$

$$\frac{6\cancel{e^{\infty}}}{8\cancel{e^{\infty}}} = \frac{3}{4}$$

6. $$g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y\to 6} g(y)$

b) $\lim_{y\to -2} g(y)$

Soln :

∵ $y < -2$

a) $g(6) = 1 - 3(6) = -17$

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b) $y \to -2$ $\quad y \geq 2$, $\quad 1-3y$

LHL

$$\lim_{y\to 2^-} g(y) = \lim_{y\to 2^-}(y^2+5) = 9$$

RHL : $\lim_{y\to 2^+} g(y) = \lim_{y\to 2^+}(1-3y) = -5$

7. Product Rule :

$$f(t) = (4t^2-t)(t^3-8t^2+12)$$

$$= 4t^2-t \cdot \frac{d}{dt}(t^3-8t^2+12) + t^3-8t^2+12 \cdot \frac{d}{dt}(4t^2-t)$$

$$\to 4t^2-t \cdot (3t^2-8(2t)+0) + t^3-8t^2+12\,(8t-1)$$

$$\to (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)\cdot(8t-1)$$

$$\to 4t^2(3t^2-16t) - t(3t^2-16t) + 8t^4-64t^3+96t-t^3+8t^2-12$$

$$\to 12t^4-64t^3-3t^3+16t^2+8t^4-64t^3+96t-t^3+8t^2-12$$

$$\to \boxed{20t^4-132t^3+24t^2+96t-12}$$

$$\boxed{4(5t^4-33t^3+6t^2+24t-3)}$$

8. Quotient Rule:

$$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

Soln

$$\frac{2w^2+1 \cdot \frac{d}{dw}(3w + w^4) - 3w + w^4 \frac{d}{dw}(2w^2+1)}{(2w^2+1)^2}$$

$$= \frac{2w^2+1\left[3(1) + 4w^3\right] - 3w + w^4\left[4w + 0\right]}{(2w^2+1)^2}$$

$$= \frac{(2w^2+1)(3+4w^3) - 3w + w^4 (4w)}{(2w^2+1)^2}$$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \frac{-6w^2 + 4w^5 + 3 + 4w^3}{(2w^2+1)^2}$$

$$= \frac{w^2 (4w^3 + 4w - 6) + 3}{2w^2 + 4w^4 + 1}$$

$$= \boxed{\frac{2w^2 (2w^3 + 2w - 3) + 3}{2w^2 + 4w^4 + 1}}$$

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9. Diff: $f(x) = 2e^x - 8^x$

Soln:

$\frac{d}{dx} \Rightarrow 2e^x - 8^x \cdot \log 8$

10 $y = z^5 - e^z \ln(z)$

$\frac{dy}{dz} = 5z^4 - e^z \frac{\ln d(z)}{dz} - e^z \cdot \frac{d \ln(z)}{dz}$

Product rule:

$= 5z^4 - \left(e^z \frac{1}{z} + \ln z \cdot e^z\right)$

$= 5z^4 - \left(\frac{e^z}{z} + \ln z \cdot e^z\right)$

$\frac{dy}{dz} = \boxed{5z^4 - e^z \cdot \ln(z) - \frac{e^z}{z}}$

11. Chain rule:

a) $f(x) = (6x^2 + 7x)^4$

let $6x^2 + 7x = u$

$\therefore \frac{du}{dx} = 12x + 7$

$\therefore f(x) = u^4$

$\frac{d}{du} = 4u^3$

here $\frac{du}{dx} =$

$= \boxed{4(6x^2 + 7x)^3 \cdot 12x + 7}$

b) $f(t) = 5 + e^{4t + t^7}$

$$\text{let } u = 4t + t^7$$
$$\frac{du}{dt} = 4(1) + 7t^6$$
$$= 4 + 7t^6$$

$$\therefore \text{let } t/y = 5 + e^{4t+t^7}$$
$$t/y = 5 + e^{u}$$

$$\frac{dy}{du} = e^{u}$$

$$\frac{dy}{dx} = \left(e^{4t+t^7}\right) . \left(4 + 7t^6\right)$$

— by chain rule

12. a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7 - x^3}$$

$$\frac{d^2z}{dx^2} = \frac{7 - x^3(0) - 1\,(0 - 3x^2)}{(7 - x^3)^2}$$

$$= \frac{+\,3x^2}{(7 - x^3)^2}$$

12.) a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \left(\frac{-\,3x^2}{7 - x^3}\right)$$

$$\frac{d^2z}{dx^2} = \frac{7 - x^3(-6x) + 3x^2\,(-3x^2)}{(7 - x^3)^2}$$

$$= \frac{-\,42x + 6x^4 - 9x^4}{(7 - x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

$$\frac{d^2 z}{dx^2} = \boxed{-\frac{3x\,(14-x^3)}{(7-x^3)^2}}$$

b) $$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

let $6+2v-v^2 = u$

$$\frac{(6+2v-v)^4 \cdot \frac{d(2)}{dv} - 2 \cdot \frac{d(6+2v-v^2)^4}{dv}}{(6+2v-v^2)^{6}}$$

$$\frac{(6+2v-v)^4 \cdot 0 - 2\cdot 4(6+2v-v^2)\cdot(2-2v)}{(6+2v-v^2)^{5}}$$

$$= \frac{-8\,(6+2v-v^2)\cdot(2-2v)}{(6+2v-v^2)^{5}}$$

$$= \frac{-8\,(2-2v)}{(6+2v-v^2)^{5}}$$

$$Q(v) = \boxed{\frac{-16\,(1-v)}{(6+2v-v^2)^5}} \rightarrow \frac{du}{dv}$$

$$\frac{(6+2v-v^2)^5\cdot(0+16) - [-16+16v\cdot(5\cancel{(6+2v-v^2)^4}\cdot(2-2v)]}{5\cancel{(6+2v-v^2)^4}}$$

$\cancel{16\,(6+2v-v^2)^5 + 16 + 16v\cdot(30+10v-5v}$

$$= \frac{16\,(6+2v-v^2) + 16 + 16v\cdot(2-2v)}{5}$$

$$= \boxed{\frac{16\,[9x^2 + 16 - 18x]}{5}}$$

*

c) $f(t) = \ln(1+t^2)$

$= \left(\dfrac{0+2t}{1+t^2}\right)$

1st der $= \dfrac{2t}{1+t^2}$

X 2nd : $\dfrac{1+t^2(2) \cancel{+} - 1+t^2(2)}{(1+t^2)^2}$

$\dfrac{\cancel{2+2t^2 - 2 - 2t^2}}{(1+t^2)^2}$

$\cancel{= 0}$

2nd. $\dfrac{1+t^2(2) - 2t(2t)}{(1+t^2)^2}$

$= \dfrac{2+2t^2 - 4t}{(1+t^2)^2}$

$= \boxed{\dfrac{-(2t+2)^2}{(1+t^2)^2}}$

$\therefore \dfrac{d^2 t}{dt^2} = \dfrac{-(2t+2)^2}{(1+t^2)^2}$

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13. find the max. & min. val. of :

$y = x^3 - 3x^2 - 9x + 12$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

$\cancel{x^2 - x - 3x - 3 = 0}$

$\cancel{x(x-1) - 3(x+1)}$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

minimum val. → -15 maximum val. → 17

$f''(x) = 6x - 6$

$f''(-1) = -6 - 6 = -12$

$f''(3) = 18 - 6 = 12$

$f(-1) = -1 - 3 + 9 + 12$

$= 17$

maximum val.

$f(3) = 27 - 27 - 27 + 12$

$= -15$

minimum val.

∴ Min val of the func. is -1 & max. val. is 3.

14. $y = 4x^3 - 18x^2 + 24x - 7$

$\frac{dy}{dx} = 12x^2 - 36x + 24$

$12(x^2 - 3x + 2) = 0$

$(x^2 - x - 2x + 2) = 0$

$x(x-1) - 2(x-1) = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

$f'(x) = 12x^2 - 36x + 24$

$f''(x) = 24x - 36$

$f''(2) = 48 - 36 = 12$

$f''(1) = 24 - 36 = -12$

$f(1) = 4 - 18 + 24 - 7 = 3$

max. val.

$f(2) = 32 - 72 + 48 - 7$

$= 80 - 119 \cancel{+ 48 - 7}$

$= 1$ min. val.

15. $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, \; x = -2$

$f''(x) = 6x + 6$

$f''(0) = 6 \quad f''(-2) = -6$

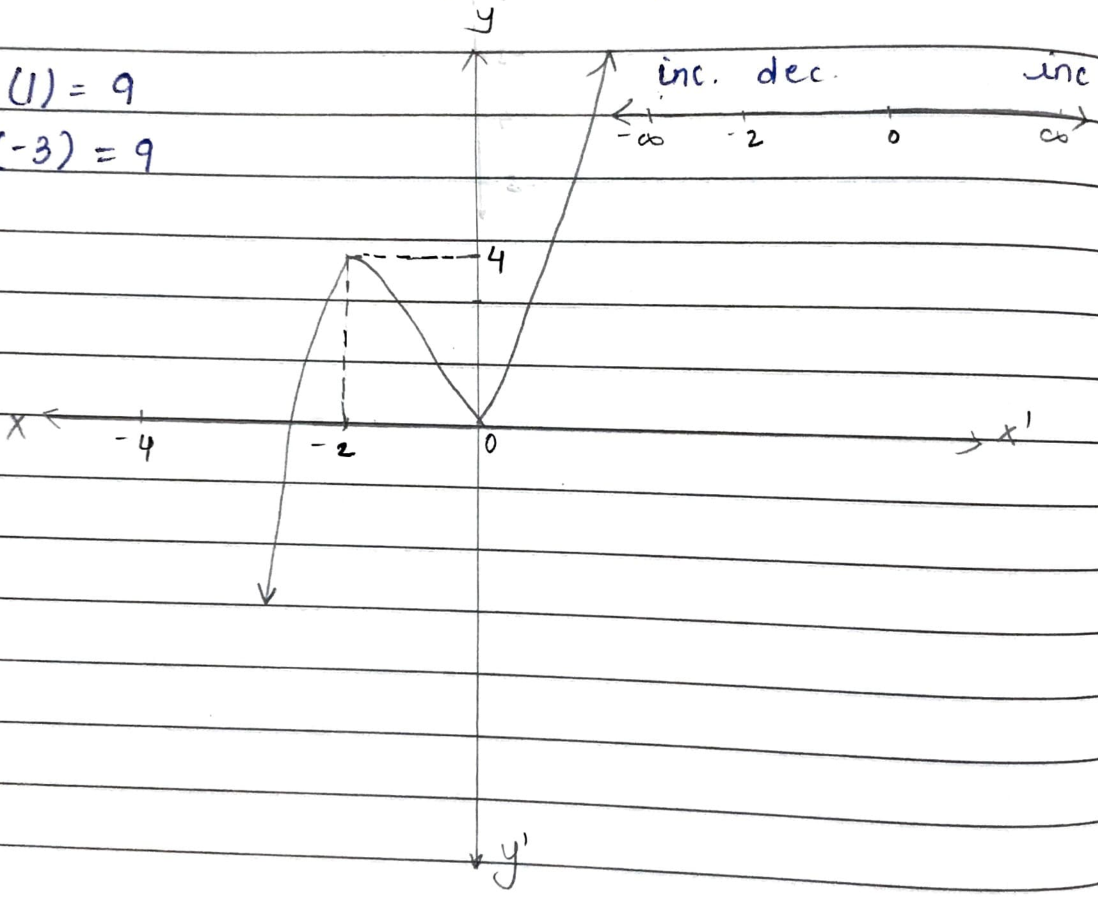
$\therefore$ At 0, $f(x)$ lies minimum val. & At -2 $f(x)$ has maximum val.

$f(0) = 0$ (min val.)

$f(-2) = -8 + 12 = 4$ (max. val.)

$f'(1) = 9$

$f(-3) = 9$



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16. $f(x) = 3x^2 - 6x$ $f'(x) = 6x - 6$ $f''(x) = 6$

$(x = 1)$ $\therefore$ At $x = 1$, $f(x)$ is min.

$f'(0) = -6$, $f'(2) = 6$

dec. inc