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Probablity and Stats

Assignment - 1

Q1

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25 = x

n = 10

$$\text{mean} = \frac{\Sigma x_i}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \frac{134}{10} = 13.40$$

$$\text{median} = \frac{\left(\frac{n}{2}\right)^{th\ term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{5^{th} \text{ term} + 6^{th} \text{ term}}{2}$$

$$= \frac{15+18}{2}$$

$$= 16.5$$

mode = 18 [repeated 3 times]

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{term} = 2.5^{th} \text{ term}$$

$$= \frac{7+9}{2} = 8$$

$$Q_3 = 3\left(\frac{n}{4}\right) = 7.5^{th} \text{ term}$$

$$= 18$$

$$IQR = Q_3 - Q_1 = 18 - 8 = 10$$

$$x^2 = 16 + 49 + 81 + 121 + 225 + 324 + 324 + 625$$

$$SD = \sqrt{\frac{\Sigma x^2}{n}} = \sqrt{\frac{2170}{10}} = \sqrt{217} = 14.7309$$

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(ii)

Number of Households (x)	Number of People (f)	CF	fx	fx^2
0	5	5	0	0
1	11	16	11	11
2	26	42	52	104
3	15	57	45	135
4	3	60	12	48
	60		120	298

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$

median $= \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$

$= \frac{30^{th} \text{term} + 31^{st} \text{term}}{2} = \frac{2+2}{2} = 2$

mode $= 2$ [$f = 26$]

$Q_1 = \frac{n}{4}^{th}$ term $= \frac{60}{4}^{th}$ term $= 15^{th}$ term $= 1$

$Q_3 = \frac{3n}{4}^{th}$ term $= \frac{180}{4}^{th}$ term $= 45^{th}$ term $= 3$

IQR $= Q_3 - Q_1 = 3 - 1 = 2$

SD $= \sqrt{\frac{\Sigma fx^2}{n}} = \sqrt{\frac{298}{60}} = \sqrt{4.96} = 2.2286$

Q2] X = No. of claims arising on ~~into~~ policies

$X \sim Poi(0.5)$

$\therefore \lambda = 0.5$

i) $P(X = 0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!}$

$= \frac{e^{-0.5} \cdot 1}{1}$

$= e^{-0.5} = 0.60653066$

ii) $P(X \geq 1) = 1 - P(X < 0)$

$= 1 - 0.60653066$

$= 0.39346934$

iii) Didn't understand.

Q3] $X \sim Gamma(2, 1/2)$

$\therefore \alpha = 2, \lambda = 1/2$

i) $E(X) = \frac{\alpha}{\lambda} = \frac{2}{\left(\frac{1}{2}\right)} = 2 \times 2 = 4$

$V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{\left(\frac{1}{2}\right)^2} = 4 \times 2 = 8$

iii) $\lambda = 1/2$

$2\lambda = 1$

$X \sim Gamma(2, 1/2)$

$\therefore 2\lambda x \sim \chi^2$

$= 1x \sim \chi_4$

$\therefore F_x(x) = P(0 < \chi^2_4 < \infty)$

Q4 male staff : 10

female staff = 8

6 males = fully qualified

7 females = fully qualified.

P To find = P(2 qualied female | atleast 1 female)

P(atleast 1 female) = $\frac{8}{18}$

P(2 qualied female ∩ atleast 1 female) = $\frac{6}{18}$

P(2 qualified female | atleast female) = $\frac{P(A \cap B)}{P(B)}$

$= \frac{\left(\frac{6}{18}\right)}{\frac{8}{18}} = \frac{6}{\cancel{18}} \times \frac{\cancel{18}}{8}$

$= \frac{\cancel{6}^{3}}{8_{4}} = \frac{3}{4}$

Q5 $P(\text{level }1) = 40\%$
$P(\text{level }2) = 50\%$
$P(\text{level }3) = 10\%$
~~P(late)~~
$P(\text{late}/\text{level }1) = 20\%$
$P(\text{late}/\text{level }2) = 40\%$
$P(\text{late}/\text{level }3) = 10\%$

$$P(\text{level }2/\text{late}) = \frac{P(\text{level }2).P(\text{late}/\text{level }2)}{\sum_{i=1}^{3} P(\text{level }i).P(\text{late}/\text{level }i)}$$

$$= \frac{(0.5)(0.4)}{(0.4)(0.2)+(0.5)(0.4)+(0.1)(0.1)}$$

$$= \frac{0.2}{(0.08+0.2+0.01)}$$

$$= \frac{0.2}{0.29}$$

$= 0.689655172$

$= 68.9655\%$

Q6 $X \sim U(1,2)$

$y = \frac{3}{6-x}$

$6-x = \frac{3}{y}$

$x = 6 - \frac{3}{y} = \frac{6y-3}{y}$

$$f_x(x) = \frac{1}{2-1} = 1$$

$$f_{\frac{64-3}{4}}\left(\frac{64-3}{4}\right) = 1$$

$$f_y(y) = \frac{64-3}{4}$$

Q7

x	$P(X=x)$	$xP(X=x)$	$x^2 . P(X=x)$	$x^3 . P(X=x)$
1	0.05	0.05	0.05	0.05
2	0.15	0.30	0.6	1.2
3	0.35	1.05	3.15	9.45
4	0.4	1.6	6.4	25.6
5	0.05	0.25	1.25	6.25
	1	3.25	11.45	42.55

i) $F(3) = P(X \leq 3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = \sum_x x . P(X=x) = 3.25$

iii) $V(X) = E(x^2) - [E(X)]^2$

$= 11.45 - (3.25)^2$

$= 0.8875$

iv) Skewness $= E(X^3) - [E(X)]^3$

$= 42.55 - (3.25)^3$

$= 8.221875$

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Q8] P(claim > 1000) = 0.2

~~P(claims)~~ P(2 claims) = $\frac{2}{15}$

P(2 claims | exceed 1000) = $\frac{\left(\frac{2}{15}\right)}{0.2}$

$= \frac{2}{15 \times 0.2}$

$= \frac{1}{1.5}$

$= 0.6667$

Q9] P(winning) = 0.01
P(losing) = 1 - P(winning) = 1 - 0.01 = 0.99 = q

X ~ winning raffle
X ~ Bin (7, 0.01)

$P(X=3) = {}^7C_3 \cdot p^3 \cdot q^4$
$= 35(0.01)^3 \cdot (0.99)^4$
$= 0.000035 \cdot (0.99)^4$
$= 0.000033621$

Q10] $E(x) = \frac{2}{5}$

P(2 claims in a day) =

Q11) P(male) = 0.5
P(female) = 0.5

Probablity (~~one~~ Next 3 child = boys)
$= 0.5 \times 0.5 \times 0.5$
$= 0.125$

Q12] Claim ~ Poi(10)
$\therefore \lambda = 10$

$P(X = 2 / 0.1 \text{ days}) = \frac{e^{-10} \cdot 10^2}{2!} \times \frac{1}{0.1}$

$= \frac{0.004539993}{0.2}$

$= 0.022699965$

13) $X \sim U(75, 120)$

$$F_x(100) = \frac{100-75}{120-75} = \frac{25}{45} = \frac{5}{9}$$

$$F_x(80) = \frac{80-75}{120-75} = \frac{5}{45} = \frac{1}{9}$$

$$P(80 \le x \le 100) = F_x(100) - F_x(80)$$
$$= \frac{5}{9} - \frac{1}{9}$$
$$= \frac{4}{9}$$

14) $X \sim \text{Gamma}(5, 0.4)$

$\therefore \alpha = 5$

$\lambda = 0.4$

$$2\lambda x \sim \chi^2_{10}$$

$$\therefore 0.8x \sim \chi^2_{10}$$

$$P(X < 22.89) = P(0.8x < 18.312)$$
$$= P(\chi^2_{10} < 18.312)$$
$$= 0.9541728$$

$$f_x(x) = \frac{k}{(x+5)^4}, \quad x > 0$$

$$\int_0^\infty \frac{k}{(x+5)^4}\, dx = 1$$

$$k \int_0^\infty (x+5)^{-4}\, dx = 1$$

$$k \left[\frac{(x+5)^{-5}}{-5}\right]_0^\infty = 1$$

$$k \left[-\frac{1}{5(x+5)^5}\right]_0^\infty = 1$$

$$k \left[\frac{-1}{5(\infty+5)^5}\right] - \frac{1}{5(5)^5} = 1$$

$$k \left[\frac{-1}{\infty} + \frac{1}{5^6}\right] = 1$$