SRM Assignment-1

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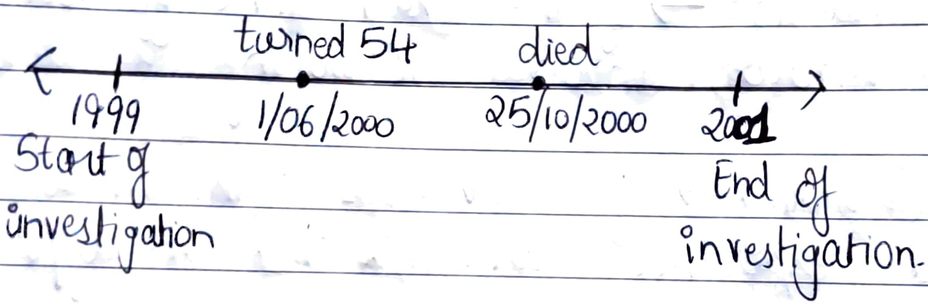
Q1 $\quad {}_u q_x = P[T_x \le u] = P[K_x = 0 \text{ and } S_x \le u]$

$\therefore \quad {}_u q_x = P[K_x = 0] \times P[S_x \le u]$

$P[S_x \le u] = \int_0^u 1 \cdot dx = u$ (due to uniform distributio)

$\therefore \quad {}_u q_x = u \times q_x .$

Q2



(a) Central Exposure to risk-

(25/10/2000) - (1/06/2000) = 147 days

Or 21 weeks.

(b) Initial exposed to risk-

52 weeks.

Q3 • Left censoring → If data regarding joining dates is not present. Left censoring is not present.

• Right censoring → If data regarding the policy surrender is not present. OR if the policy is terminated before maturity.

• Interval Censoring → If the exact time of surrender is not known. Interval Censoring is present.

• Informative Censoring → If individuals with similar subsequent experience are seperated in different groups.

Q4 (i) Complete expectation of life →

$$\mathring{e}_x = \int_0^{\omega-x} {}_tP_x\,dt = E[T_x]$$

(ii) Curtate expectation of life →

$$e_x = \sum_{k=1}^{\infty} {}_kP_0 = \sum_{k=1}^{\infty} e^{-0.0325k} = \frac{e^{-0.0325}}{1-e^{-0.0325}} = 30.27$$

(iii)

$${}_9P_{36} = e^{-\int_0^9 0.0325\,dt} = e^{-[0.0325t]_0^9}$$

$$\Rightarrow e^{-0.2925} = 0.7464 \text{ or } 74.6\%$$

(iv) median of lifetime means $_xP_0 = 0.5$

$\therefore \; _xP_0 = 0.5 \Rightarrow$ [illegible]

$\Rightarrow \cancel{\log 0.5}$

$$e^{(-0.0325x)} = 0.5$$

$$x = \frac{-(\log 0.0325)}{5} = 21.33$$

Q5 (i) Gompertz law is viable for ages above 35 and below 90.

(ii) We know that, $_tP_x = e^{-\int_0^t \mu_{x+s}\, ds}$

putting $\mu_x = Bc^x$,

$$_tP_x = e^{-\int_0^t Bc^{x+s}\, ds} \qquad \left(c^{x+s} = c^x \cdot e^{s\log c}\right)$$

$$\int_0^t Bc^{x+s}\, ds = \int_0^t Bc^x e^{s\log c}\, ds = \frac{Bc^x}{\log c} \times \left[e^{s\log c}\right]_0^t$$

$$\Rightarrow \frac{Bc^x}{\log c}\left[c^t - 1\right] \qquad \left(g = \frac{-B}{\log c}\right)$$

$$\Rightarrow (\log g) \times c^x(c^t - 1)$$

$$\Rightarrow \; _tP_x = \left(e^{\log(g)}\right)^{c^x(c^t+1)}$$

$$\Rightarrow g^{c^x(c^t+1)}$$

Q6. (i) Female smoker of age 30 at entry.

(ii) $\dfrac{h_j(t)}{h_i(t)} = \dfrac{e^{-0.05}}{e^{0.1}} = 0.86070$

($j \to$ male smoker wt age 30 at entry.
$i \to$ female smoker wt age 40 at entry.)

$$S(t) = \exp\left(-\int_0^t h(s)\,ds\right)$$

$$\therefore S_j(t) = \left(S_i(t)\right)^{0.86070}$$

Hence, $S_j(t) > S_i(t)$ for all $t > 0$.

(iii) $\dfrac{h_j(t)}{h_i(t)} = \dfrac{e^{0.2}}{e^{0.05}} = 1.161$

$$S(t) = \exp\left(-\int_0^t h(s)\,ds\right)$$

$$\therefore S_j(t) = \left(S_i(t)\right)^{1.161}$$

Hence, $S_j(t) < S_i(t)$ for all $t > 0$.

Q7 (i) ~~Polig~~ Policy rate interval would be the most appropriate because it ~~not~~ corresponds to the definition of the deaths.

If birthdays are uniformly distributed, avg. age at the start of interval $= \left(x - \frac{1}{2}\right)$

(ii) ~~Let~~ All notations are age x next birthday.

$$P_x(t) = \frac{1}{2}\left(P_{x,t} + P_{x+1,t}\right)$$

assuming policies to be uniformly distributed over the calender year,

$$E_x^c = \int_0^{10} P_x(t)\,dt = \frac{1}{2}\sum_{t=0}^{9}\left(P_x(t) + P_x(t+1)\right)$$

assuming in force population varies linearly b/w dates of investigation.

$$E_x = E_x^c + \frac{1}{2}\sum_{t=0}^{10} \theta_{x,t}$$

assuming that in aggregate the deaths occur on average halfway through the policy year.

Q8. (i) Censoring Present →
- type 1
- type 2
- random censoring
- right censoring.

(ii) The censoring is likely to be informative because the patients who died were probably ~~not~~ recovering less well that patient who discharged from the hospital.

(iii) Kaplan-Meier estimate →

T	n	d	c	d/n	(1-d/n)	S(t)
0	13					
5	13	1	0	0.0769	0.9231	0.92
7	12	1	0	0.0833	0.9167	0.85
14	11	1	2	0.0909	0.9091	0.77
28	8	1	2	0.1250	0.8750	0.67
35	5	1		0.2000	0.8000	0.54

∴ value survival function at end of investigation = 0.54.

(iv) The survival of a patient from the infection given treatment is approx 50%.

Q9 (i) (a) Under UDD:

$$\int_0^1 {}_tp_x\,dt = \int_0^1 (1-tq_x)\,dt = \left[t - \tfrac{t^2}{2}q_x\right]_0^1$$

~~given~~ $\boxed{q_x = 0.3}$

$$\therefore m_x = \frac{0.3}{1-0.15} = 0.35294$$

(b) Under constant force of mortality:

$$q_x = 1-e^{-\mu}$$

$$\int_0^1 {}_tp_x\,dt = \int_0^1 e^{-\mu}\,dt = \frac{1}{\mu}\times(1-e^{-\mu}) = \frac{q_x}{\mu}$$

$$\therefore m_x = \mu = -\ln(1-q_x) \Rightarrow -\ln 0.7$$

$$= 0.35668.$$

Q10 (i) Under Cox model, each individual's hazard is proportional to the baseline hazard.

(iii) The baseline hazard refers to annual policy taken through the online chanel & premiums are paid by direct debit.

(iv) B_F, B_D, B_M

$$\frac{\exp[(B_D \times 1)]\,\cancel{\exp}}{\exp[B_D \times 1 + B_F \times 1 + B_M \times 1]} = 0.75$$

$$\Rightarrow \quad \frac{1}{e^{B_F + B_M}} = \frac{3}{4}$$

$$\Rightarrow \quad e^{B_F + B_M} = \frac{4}{3} \quad -(i)$$

$$\frac{e^{B_D}}{e^{B_F}} = 1 \quad -(ii)$$

$$\frac{e^{B_M}}{e^{B_D \times 2}} = 0.75 \quad -(iii)$$

from eq (i) & (ii),

$$e^{(B_D + B_M)} = \frac{4}{3} = e^{B_D} \times e^{B_M} \quad -(iv)$$

from eq (iii),

$$\cancel{\exp}\ (e^{B_D})^2 \times 0.75 = e^{B_M}$$

$\therefore$ Substituting in eq (iv),

$$(e^{B_D}) \times (e^{B_D})^2 \times 0.75 = \frac{4}{3}$$

$$\left(e^{\beta_D}\right)^3 = \frac{16}{9}$$

$$e^{\beta_D} = 1.2114$$

$\beta_D = 0.19179$
$\beta_F = 0.19179$
$\beta_M = 0.0959.$

Q11 (ii)

T	S(t)	N(t)	nt	dt	ct
0	1	0	12	0	
1	0.9167	0.0833	12	1	2
3	0.7130	0.22	9	2	2
6	0.4278	0.4	5	2	3

(iii) total deaths = 1+2+2 = 5. We started with 12 insects. ∴ 12-5 = 7 insects were right Censored.

Q12 (i) Gompertz is an ~~exponentialgial~~ exponential function used for ages over 35 and below 90.

$\lambda_x = Bc^x$; λ_x = force of mortality of a person aged x.

(ii) Substituting $BB = e^{B_0 + B_1 x1 + B_2 x2}$,

$\lambda_x = e^{B_0 + B_1 x1 + B_2 x2} \times c^x$ defining x as duration since 50^{th} birthday.

The first part depends of covariates and second part on duration.

$\therefore$ duration is not a factor and it is a proportional hazards model.

(iii) baseline hazard is a non-smoker female.

(iv) Given, $x_1 = 0$; $x_2 = 1$; $x = 4$

$$\therefore \lambda_{54} = e^{BB0 + BB_1 \times 0 + B.B_2 \times 1} \times c^4$$

$$= e^{-4 \times 0.65} \times 1.05^4$$

$$= 0.04266.$$

Q15 (i) Advantages of central exposed to risk:

- Easier to calculate and has an intuitive appeal since the total observed waiting time.
- Easier to comprend.
- More versatile and easy to extend the concept of central exposed to risk to cover more elaborate situations.

(ii)

- Rita - Central exposed to risk = 3 months
  Initial exposed to risk = 1 year

- Sita - Central exposed to risk = 1 year
  Initial exposed to risk = 1 year.

- Nita - Central exposed to risk = 9 months
  Initial exposed to risk = 9 months

- Gita - Central exposed to risk = 0
  Initial exposed to risk = 0

(iii) Total exposed to risk →

- Central exposed to risk = 0.25 + 0.75 + 1 = 2 years
- Initial exposed to risk = 1 + 1 + 0.75 = 2.75 years