Assignment - 01

Q1) Actual length = $r_1 = 12.5$ cm

Actual Area = πr^2

$= \pi (12.5)^2$

$= 490.87385 \text{ cm}^2$

Approx. radius = 12.65

" area = πr^2

$= \pi (12.65)^2$

$= 502.72551 \text{ cm}^2$.

1) Absolute Error = Approx Area − Actual Area

$= 502.72551 - 490.87355$

$= 11.58166 \text{ cm}^2$.

2) Relative Error = $\dfrac{\text{Absolute Error}}{\text{Actual area}}$

$= \dfrac{11.58166}{490.87385} = 0.02414$

3) Percentage Error = Relative Error × 100

$= 0.02414 \times 100$

$= 2.414\%$

Q2) 1) Using linear interpolation

$x_1 = 4$ $\quad x_2 = 6$ $\quad y_1 = 0.60206$

$y_2 = 0.7781513$

$6 - 4 = y_2 - y_1$

$2 = 0.1760913$

1 cause a change of $\dfrac{0.1760913}{2} \times 1$

$\therefore = 0.08804565$.

$x = 5$

$y = 0.60206 + 0.08804565$

$= 0.69010565.$

2) $x_1 = 4.5$ $\quad x_2 = 5.5$ $\quad y_1 = 0.6532125$

$y_2 = 0.7403627.$

$(5.5 - 4.5) = 0.7403627 - 0.6532125$

$1 = 0.0871502.$

$\therefore$ change caused by $0.5 = 0.0435751$

$\therefore x = 5$

$y = 0.6532125 + 0.0435751 = 0.696787.$

Q3) $x^4 - x - 10 = 0$

$x_0 = 1.5$ $\quad y_1 = -6.4375$

$x_1 = 2$ $\quad y_2 = 4.$

$x_2 = \dfrac{x_0 + x_1}{2} = 1.75.$

$y_2 = -2.37109$

$x_3 = \dfrac{x_1 + x_2}{2} = 1.875$ $\quad y_3 = 0.48462$

$x_4 = \dfrac{x_2 + x_3}{2} = 1.8125.$ $\quad y_4 = -1.02025$

$x_5 = \dfrac{x_3 + x_4}{2} = 1.84375$ $\quad y_5 = -0.28773$

$x_6 = \dfrac{x_4 + x_5}{2} = 1.859375$ $\quad y_6 = 0.93378$

Q4) $x^3 - x - 1 = 0$

x	y
0	-1
1	-1
2	5

$f'(x) = 3x^2 - 1$

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= 1.5$

$y_1 = 0.875 \quad f'(y_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \left(\frac{0.875}{5.75}\right) = 1.34783$$

$f(x_2) = 0.100699 \quad f'(x_2) = 4.44994$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252$$

$y_3 = 0.00205 \quad f'(x_3) = 4.26846$

$\cancel{f'(x_3) = 4.26}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252$$

$y_3 = 0.00205 \quad f'(x_3) = 4.26846.$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.32472$$

$y_4 = 0.000008712$

Q5) $A^2 = A$

$$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$$
$$= 7A - I - 7A$$
$$= -I$$

Q6) $\begin{bmatrix} 1 & -2 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} \Rightarrow |A| = -6 - 4 + 8 = -2$

$$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= -\frac{1}{2}\begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Q7) $\vec{a} = 8\hat{i} - 9\hat{j} \quad \vec{b} = 7\hat{i} - 9\hat{j}$

$$\vec{a}\cdot\vec{b} = ab\cos\theta = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$$
$$= 56 + 81 = 137$$

$a = \sqrt{145} \quad b = \sqrt{130} \quad \vec{a}\cdot\vec{b} = ab\cos\theta \quad \cos\theta = \frac{137}{\sqrt{145}\sqrt{130}}$

$\theta = 3.75856°$

Q8) Displacement Vector $= \vec{R}$

$$\vec{R} = \vec{a} + \vec{b}$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta \qquad \theta = 30^\circ$$

$$R^2 = 75^2 + 25^2 + 2(75)(25)\cos 30^\circ$$

$$R^2 = 97.4556 \text{ metre}$$

Q9) $d = 3$ $\quad AP = 101, 104, 107 \ldots 998$

~~AP = 101, 104, 10~~ $\quad a = 101 \quad a_n = a + (n-1)d$

$$998 = 101 + (n-1)3$$

$$S_n = \frac{n}{2}(a + a_n) \qquad n = 300$$

$$= \frac{300}{2}(101 + 998) = 164850.$$

Q10) $a_5 = 32 \quad a_8 = 128 \quad ar^3 = 32 \quad ar^7 = 128$

$$\frac{ar^7}{ar^5} = \frac{128}{32} \rightarrow r^2 = 4 \qquad r = \pm 2$$

$$\therefore r = 2.$$

Q11) $(3x+4)^5 = 3x^5 + (5\times(3x)^4\times4) + (10\times(3x)^3\times4^2)$

$$+ (10\times(3x)^2\times4^3) + (5\times3x\times4^4) + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2$$

$$+ 3480x + 1024.$$

Q12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A - \lambda I|$

Expanding using 3^{rd} row

$$= (3-\lambda)[-10 - 2\lambda + 5\lambda + \lambda^2 + 6]$$

$$= -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

Comparing coefficient

$$a + b + c + d = 1 - 13 + 12 = 0.$$

$$\lambda^2 + \lambda - 12 = 0$$

$$(\lambda - 3)(\lambda + 4) = 0$$

$$\lambda = 3 \text{ or } -4$$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5 \quad f(5) = -13 \quad f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6.$$

$f(x_1) = 9 \quad f'(x_1) = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 5.71875 \quad f(x_2) = 0.84787$$

Q14) $(x^2-x+1)^4 = ((x+1)^2+x)^4$

$((x-1)^2+x)^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$

$((x-1)^2+x)^4 = (x-1)^8 + 4(x-1)^6 x + 6x^2(x-1)^4 + 4x^3(x-1) + x^4.$

Q15) $e^{-x} = 3\log(x)$

$3\log x + e^{-x} = 0$

x	y
0.1	-2.09516
0.2	-1.27818
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$x_0 = 0.6 \quad y_0 = -0.116$

$x_1 = 0.7 \quad y_1 = 0.03188$

$x_2 = \frac{x_0+x_1}{2} = 0.65 \quad y_2 = -0.03921$

$x_3 = \frac{x_2+x_1}{2} = 0.675 \quad y_3 = -0.00293$

$x_4 = \frac{x_3+x_2}{2} = 6.6875 \quad y_4 = 0.01465$

$x_5 = \frac{x_4+x_3}{2} = 0.68125 \quad y_5 = 0.0059.$