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FE Assignment 2 - Roll no 426

1. $S = 65$; $\sigma = 25\%$ p.a., $r = 2\%$ pa , $T = 0.5$

i) $C_t = S_0 \phi(d_1) - Ke^{-rt} \phi(d_1)$

$$d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{20\%^2}{2}\right) 0.5}{20\% \sqrt{0.5}} = 1.08996$$

$d_2 = d_1 - \sigma\sqrt{T} = 0.91348$

$C_P = 65\phi(0.09) - 5\, 5 e^{-0.0 \times 0.5} \phi(0.913948)$
$= 11.4646$

ii) Delta of call option = $\frac{\text{Change in price of call option}}{\text{change in price of underlying}}$

iii) $\Delta c = \phi(d_1)$, $\Delta p = -\phi(d_1)$

iv) $\Delta p = -0.13786$

2.

i) $\text{Delta} = \frac{\text{Change in Price of Derivative}}{\text{Change in price of underlying}}$

$\text{Vega} = \frac{\text{Change in price of option}}{\text{Change in volatility of underlying assets}}$

ii) Put call Parity $\boxed{C + K\exp(-rT) = p + S}$

$\rightarrow C - p = S - K - e^{-rT}$

$$\frac{d}{d\sigma}(C-p) = \frac{d}{d\sigma}(S - Ke^{-rT})$$

$V_C - V_p = 0$ $\therefore \boxed{V_C = V_p}$ (S & K are constant)

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iii) $S_0 = 55$, $x = 50$, $\sigma = 25\%$ p.a., $r = 5\%$, $T = 1$

$$d1 = \frac{\ln\left(\frac{55}{50}\right) + \left(5\% + \frac{1}{2} \times 25\%^2\right) \times 1}{25\%} = 0.70624$$

$d2 = 0.70624 - 25\% = 0.4562$

$C_t = 55(0.72115) - 50e^{-0.05}(0.67729)$
$= \$9.69$

$p = 2.211$ ← from put call parity

iv) Delta hedging → Portfolio for which overall delta = 0. The portfolio immune to change in the price of U.A

Vega hedging → Portfolio for which overall vega = 0, the portfolio is immune to small changes in the assumed volatility.

v)

	PV	Delta	Vega
Call	$c = 9.6525$	Δc	V_c
Put	$P = 2.2140$	Δp	V_p
forward		1	0

Vega hedging → $xV_c + yV_p + zV_F = 0$

$[V_c = V_p;\ V_F = 0]$

$V_c(x + y) = 0$

$x + y = 0$

$y = -x$

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$d2 = 0.1673$

$\phi(d1) = 0.6346$

$\phi(d2) = 0.5669$

$C_t = 50 \times 0.6346 - 49 e^{-0.05 \times 30} (0.5664)$

$= 4.66$

iii) 4.66 (same as European call)

iv) $S_t + p = c + Ke^{-rT}$

$p = 4.66 + 99 e^{-0.05 \times 28} - S_0$

$p = 2.75$

v) value of underlying asset will fall.

Q4. $\tilde{Z}_t = Z_t + \int_0^t Y_s \, ds$

$\tilde{Z}_t$ = SBM under Q

If Z_t is SBM under P.

$\tilde{Z}_t = Z_t + \int_0^t Y_s \, ds$

Q5. i) Delta $= \phi(d1)$

$= \frac{\text{chg. in price of U.A}}{\text{chg. in price of a call.}}$

ii) d_1 s.

$$0.3 = \frac{\ln\left(\frac{40}{198.91}\right) + (0.002 + 0.5 + \sigma^2) 5}{6\sqrt{5}}$$

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Delta hedging → $x\Delta c + y\Delta p + 2\Delta F = 0$

$[y = -x, \Delta p = \Delta c - 1, \Delta F = 1)$

$x\Delta c - x(\Delta c - 1) + 2 = 0$

$x + 2 = 0$

$x = -2$

$x = -y = -2$

Portfolio = 1000

$x \cdot C + y \cdot P + 2 \cdot F = 1000$

$(F = 0)$

$x \cdot c + y \cdot P = 1000$

$x \cdot 9.6525 + y \cdot 2.290 = 1000$

$x = 134.43$

$y = -134.43$

Portfolio → long posi. 134 call opt.
short posi 134 put opt.
short posi. 134 forward.

Q3. $\boxed{C_t = \Sigma(e^{-r(T-t)} C_T / F_t)}$

Price of Derivative in Black scholes

C_T = payoff under derivative at T

C_t = derivative value at t

F_t = filteration @ t > 0

ii) $S_0 = 50$, $K = 49$, $r = 5\%$, $\sigma = 25\%$, $T = 0.5$

$C_t = S_0 \phi(d_1) - Ke^{-rT} \phi(d_1)$

$$d_1 = \frac{\ln\left(\frac{50}{49} + (5\% + 12.5\%) 0.5\right)}{25\% \cdot \sqrt{0.5}}$$

$= 0.399$

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$\therefore 0.3\sqrt{S} = -0.1378 + (0.1 + 2.5\sigma^2)$

$2.5\sigma^2 = 0.670830 - 0.0375 = 0$

$\sigma = \dfrac{0.67083 + \sqrt{0.67083 - 4 \cdot 0.8\,0.6376)}}{0 \text{ or } 2.5}$

$\sigma = 32\%$

iii) $R_0 = 30$, $\sigma = \sqrt{15\%}$ pa.

$\dfrac{S_1}{S_0} = k_S$, $\dfrac{R_1}{R_0} < k_R$

Payoff $\rightarrow$ $ce^{-rT}\, g\left(\dfrac{S_1}{S_0} < k_S\right) g\left(\dfrac{R_1}{R_0} < k_R\right)$

v) $ce^{-rT}\, g(S_T/S_0 < k_S)\, g(R_T/R_0 < k_R)$

$= 50e^{-0.02}\, g(S_T/S_0 < 0.8)\, g(R_T/R_0 < 0.6)$

$= 50e^{-0.02}\left(1 - \phi\left(\dfrac{(\log(1/0.8) + 0.02 - 0.5)\times 0.3}{0.32}\right)\right)$

$= (1 - \phi(\log(1/0.6) + 0.02 - 0.5 \times 0.15)\,(\phi\,.18)$

$= 50e^{-0.02}\,(1 - 0.7257)(1 - 0.88039)$

$= 1.61$

6. i) $d_1 = \dfrac{\left(\ln\left(\dfrac{S_u}{K}\right) + (r + 1/2\,\sigma^2)T\right)}{\sigma\sqrt{T}}$

$1000000\,\Delta = -24830$

$\Delta = 0.75$

ii) $\Delta = -0.2483$, $d_1 = 0.68$

$\sigma = 0.68 + \sqrt{0.3709}$

$= 70\%$.

iii) $Ke^{-rT}\phi(-d_2) = e^{-r\phi}(-d_1 + \sigma\sqrt{T})$

$630e^{-0.03}\phi(-0.609)p = 630e^{-0.03} \cdot 0.2712$

$= 165.806p$

$165.806 - 24830 \times \frac{640}{100000} = 6.894p$

Q7. i) Delta $= \frac{\delta f}{ds}$

gamma $= \frac{d^2 f}{ds^2}$

vega $= \frac{\delta d}{\delta \sigma}$

ii) Delta $= 0.801$

iii) hedge delta $= 17.91 - 0.801 \times 60 = 20.15$

iv) $f(S_0\,6 + 8) \simeq f(8+\sigma) + 8df/d\sigma$

option price $= 17.91 + 29.00 \times 0.002$

$= 18.49$

8) i) Δ first partial derivation of option price wr.t uA.

ii) $6 = -0.0600 + \frac{1}{2}6^2$

9) i) BPE of Black scholes is

$\frac{1}{2}6^2 D_x{}^2 g_{xx} + (r-q)xg_x - \delta g + g_c = 0$

ii) $g(t,x) = (x^n / S_0{}^{n-1})\, e^{u(T-t)}$

$\boxed{PDE \rightarrow u = \frac{1}{2}\sigma^2 n(n-1) + (n-1)rnq}$

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Q10. i) Put call Parity for a stock paying no divident.

$$C_t + Ke^{-r(T-t)} - P_t = S_t$$

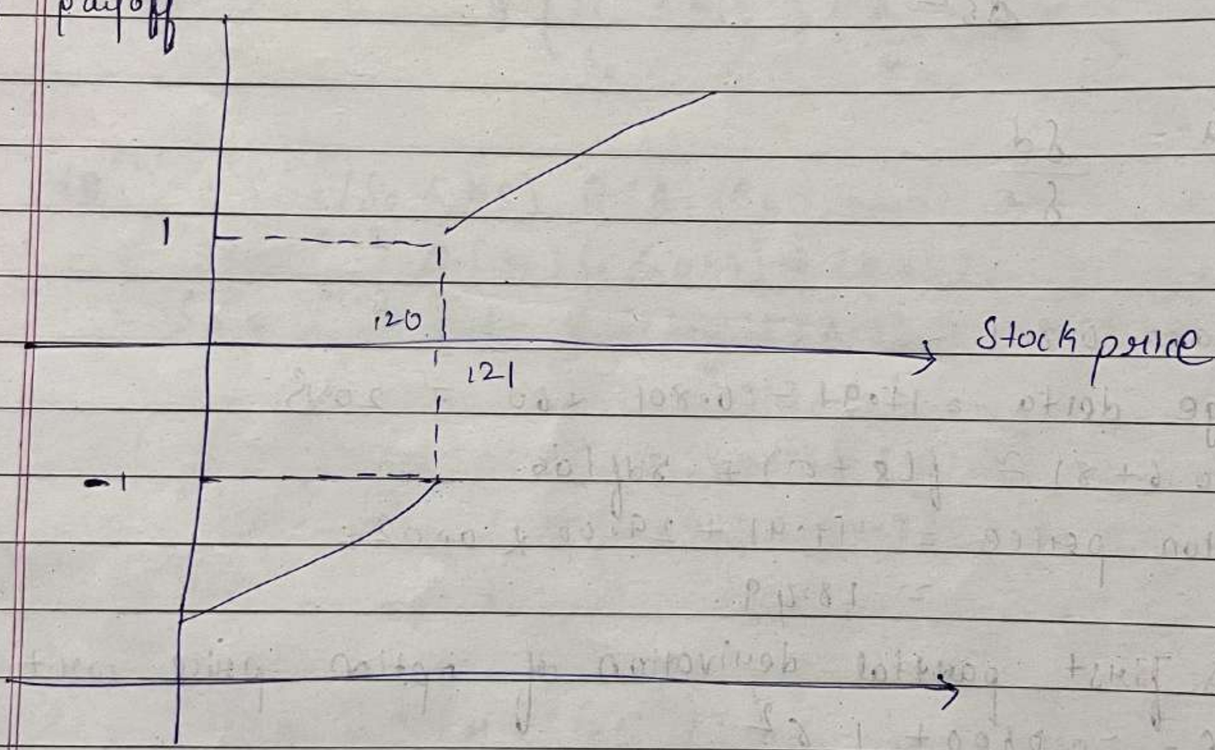
ii) $S\,\phi(d_1)\,Ke^{-rT}\,\phi\, d_2$

$S_0 = 110$, $K = 120$, $r = 0.02$, $T = 1$

$$d_1 = \frac{\left(\log\left(\frac{11}{12}\right) + 0.02 + \frac{1}{2}\sigma^2\right)}{\sigma}$$

$$d_2 = d_1 - \sigma$$

iii) payoff



iv) $S_1 = 121 \leq D \leq S_1 - 120$

$S_0 = 121e^{-r} \leq V \leq S_0 - 120e^{-r}$