Numerical Method and Algebra

1) $\sqrt{7\sqrt{7\sqrt{7}\ldots}}$

$= 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} \cdot 7^{1/16} \ldots$

$= 7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\ldots}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots$ is in form of infinite G.P

Sum of infinite G.P $= \frac{a}{1-r}$

$= \frac{1/2}{1-1/2}$

$= 1$

$7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots}$

7^1

$= 7$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$18x^2 = 2y^2$

$\frac{x^2}{y^2} = \frac{2}{18}$

$\frac{x}{y} = \sqrt{\frac{2}{18}}$

3) $x + y = 4 \quad \text{—} (1)$

$x^2 + y^2 - 4y = 12 \quad \text{—} (2)$

$x = 4 - y$ — from (1) in 2

$(x-y)^2 + y^2 - 4y = 12$

$16 + y^2 - 8y + y^2 - 4y = 12$

$16 + 2y^2 - 12y = 12$

$y^2 - 6y + 8 = 6$

$y^2 - 6y + 2 = 0$

$y = \frac{6 \pm \sqrt{36-8}}{2}$

$= \frac{6 \pm \sqrt{28}}{2}$

$= 3 \pm \sqrt{7}$

$x_1 = 4 - y_1$
$= 4 - 3 + \sqrt{7}$
$= 1 + \sqrt{7}$

$x_2 = 4 - y_2$
$= 4 - 3 - \sqrt{7}$
$= 1 - \sqrt{7}$

the ordered pairs are
$(1+\sqrt{7}, 3-\sqrt{7})$ and $(1-\sqrt{7}, 3+\sqrt{7})$

4.) $d = 2r^2 - 16r + 34$ → density

(i) $r = 3.5$
$d = 2.5$ mill particles/ft^3

(ii) $d = 2r^2 - 16r + 34 = 0$
differentiation
$4r - 16 = 0$ - (1)
$r = 4$ (400 RPM)
differentiation
4

minima
$d = 2(4)^2 - 16(4) + 34$
$d = 2$

∴ the no of revolution for minimum smoke = 4

(iii) $d = 100$

$100 = 2x^2 - 16x + 34$

$50 = x^2 - 8x + 17$

$0 = x^2 - 8x + (-33)$

$x = (x - 11)(x + 3)$

$x = 11$

the speed must be 11 revolution per minute.

5) $C_1 = 1500 + 25$ (km)

$C_2 = 2100 + 22$ (km)

(i) cost to rent car from Co 1

$C_1 = 1500 + 25n$

(ii) cost to rent car from Co 2

$C_2 = 2100 + 22n$

(iii) $C_2 < C_1$

$1500 + 25n > 2100 + 22n$

(iv) $8n > 600$

$n > 200$

it is less expensive to take C_2 when the no of km is more than 200

6) $x^3 - 7x^2 + 14x - 6 = 0$

(a) $a = 0$, $b = 1$

a	b	$a + b_2$	f(a)	f(b)	$F\left(\frac{a+b}{2}\right)$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.625	0.9844	0.2598

$x = 0.625$

(b) $a = 1$ $b = 3.2$ $f(x) = x^3 - 7x^2 + 16$ $x = 6$

a	b	a+b/2	f(a)	f(b)	f((a+b)/2)
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.08582

$x = 2.925$

(c) $[3.2, 4]$

a	b	a+b/2	f(a)	f(b)	f((a+b)/2)
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.0461	0.0180

$x = 3.425$

7.) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

(a)

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$
1	-1	156.1398	-1105	-0.60814
2	-0.6081	43.994	-418.898	-0.2354
3	-0.23541	1.537	-234.246	-0.04759
4	-0.04759	0.0004	-221.704	-0.04066
5	-0.04066	~ 0	-221.70443	-0.04066
	-0.04066	~ 0	-221.70443	-0.04066

$x = -0.04060$

(b)

i	x_i	$F(x)$	$F'(x_i)$	$x_{i+1} = x_i - \frac{F(x_i)}{F'(x_i)}$
1	0.9	-36.585	509.62	0.97179
2	0.97178	6.37668	691.80	0.96284
3	0.9625	0.11523	666.87	0.9624
	0.9624	0.3267	666.411	0.9624
	0.9624			

$x = 0.9624$

8.) (a) Average $= \frac{70+86+81+83+x}{5}$

$= \frac{320+x}{5}$

(b) Average = B

$80 < \frac{320+x}{5} < 90$

(c) $80 < \frac{320+x}{5} < 90$

$400 < 320 + x < 450$

$80 < x < 130$

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9.) $x_1 = 0.04 \quad x_2 = 0.21 \quad x = 0$

$F(x_1) = 8.54 \quad F(x_2) = -11.81 \quad F(x) = ?$

$\frac{x - x_1}{x_2 - x_1} = \frac{F(x) - F(x_1)}{F(x_2) - F(x_1)}$

$\frac{-0.04}{0.16} = \frac{F(x) - 8.54}{-20.35}$

$F(x) = 13.6275$

10.) $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

for profit $R(x) > C(x)$

$\therefore \quad 32x - 0.21x^2 > 195 + 12x$

$20x - 195 - 0.21x^2 > 0$

$0.21x^2 - 20x + 195 < 0$

$0.21x^2 - 20x + 195 = 0$

$x = \dfrac{20 \pm \sqrt{212.8}}{0.42}$

$x = 82.35, \ 12.88$

$(x - 82.38)(x - 12.88) < 0$

$x \in (12.88, 82.38$