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Unit 1 and 2
Assignment

Q1. (i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

$n = 10$

→ Mean, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{134}{10} = 13.4$

→ ~~Me~~ Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= (5.5^{th})$ term

$= \frac{11+15}{2}$

$= 13$

→ Mode $= 18$

→ Standard deviat^n, $\sigma = \cancel{\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}}$ $\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$

x_i	$(x_i-\bar{x})^2$
4	88.36
7	40.96
9	19.36
9	19.36
11	5.76
15	2.56
18	21.16
18	21.16
18	21.16
25	134.56

$\Sigma(x_i-\bar{x})^2 = 374.4$

$\sigma^2 = \frac{374.4}{10} = 37.44$

$\sigma = \sqrt{37.44} \doteq 6.12$

→ Last term of Q_1 ~~term~~ is the $\left(\frac{n}{4}\right)^{th}$ term $= 2.5^{th}$ term

Last term of $Q_1 = \frac{7+9}{2} = 8$

Last term of Q_3 is the $\left(\frac{3n}{4}\right)^{th}$ term $= 7.5^{th}$ term

Last term of $Q_3 = \frac{18+18}{2}$

$= 18$

Inter Quartile Range $= Q_3 - Q_1$

$= 18 - 8$

$= 10$

(ii)

x_i	f_i	cf
0	5	5
1	11	16
2	26	42
3	15	57
4	3	60

$n = 60$

→ Mean, $\bar{x} = \frac{\sum f_i x_i}{n}$

$= \frac{120}{60}$

$= 2$

→ Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= (30.5)^{th}$ term

$(30.5)^{th}$ term lies in the x_i, 2

Thus, Median $= 2$

→ Mode $= 2$

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x_i	f_i	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
0	5	4	20
1	11	1	11
2	26	0	0
3	15	1	15
4	3	4	12

$$\sum f_i (x_i-\bar{x})^2 = 58$$

$$\sigma^2 = \frac{58}{60} = 0.97$$

Stand. devn; $\sigma = \sqrt{0.97} = 0.98.$

→ Last term of Q_1 is $\left(\frac{n}{4}\right)^{th}$ term $= 15^{th}$ term

15^{th} term $= 1$

Las term of Q_3 is $\left(\frac{3n}{4}\right)^{th}$ term $= 45^{th}$ term

45^{th} term $= 3$

Interquartile Range $= Q_3 - Q_1$

$= 3-1$

$= 2.$

Q2. $\lambda = 0.5$

(i) Poisson distribut^n is used as Poi (0.5).

(ii) P(no claim arrising) $= P(X=0)$

$$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{(0.5)^0 e^{-0.5}}{0!}$$

$$= e^{-0.5}$$

$$= 0.60653066$$

(ii) This will be a binomial distribution

Let success be one or more than one claim arises.

Failure is no claim arising.

$$P(s) = P(X>0) = 1 - P(X=0)$$

$$= 1 - 0.60653066$$

$$= 0.39346934$$

$$P(f) = P(X=0) = 0.60653066$$

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Thus, $X \sim \text{Bin.}(3, 0.39346934)$

$P(X=1) = {}^3C_1 (0.39346934)^1 (0.60653066)^2$

$= 0.434247843$

$= 0.434$

~~(iii) Since~~

(iii) Since, $\alpha = 1$ {As we are waiting for 1 claim & and we need to calculate the waiting period for the next claim, we will use the exponential ~~funct^n~~ distribut^n of $Exp(0.5)$.
Let time be, T.

$P(X>2) = 1 - P(X \leq 2)$
$= 1 - F(2)$
$= 1 - (1 - e^{-\lambda x})$
$= e^{-\lambda x}$
$= e^{-0.5 \times 2}$
$= e^{-1}$

$P(X>2) = \cancel{0.368}\ 0.367879441$
$= 0.368$

Q3. (i) $\alpha = 2 \quad \lambda = 1/2$

$\rightarrow E(X) = \dfrac{\alpha}{\lambda}$

$= \dfrac{2}{1/2}$

$= 4$

$\rightarrow$ ~~[illegible]~~ $V(X) = \dfrac{\alpha}{\lambda^2}$

$= \dfrac{2}{1/4}$

$= 8$

Standard deviatn $= \sqrt{8}$

$= 2\sqrt{2}$

$= 2.83$

(ii) Coefficient of Skewness $= \dfrac{2}{\sqrt{\alpha}}$

$$= \frac{2}{\sqrt{2}}$$

$$= 1.414$$

Hence the data will be positivily skewed. Thus, the shape of the graph is rightly skewed.

(iii) $$f_x(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$$

$$= \frac{(1/2)^2}{(2-1)!} x^{2-1} e^{-x/2}$$

$$= \frac{1}{4}\left(x e^{-x/2}\right)$$

$$\cancel{F_x(x) = \int_{2}^{\infty}}$$

$$F_x(x) = \frac{1}{4}\int_0^x x e^{-x/2}$$

$$= \frac{1}{4}\left[x e^{-x/2}(-2) - \int e^{-x/2}\right]_0^x$$

$$= \frac{1}{4}\left[-2x e^{-x/2} + 2e^{-x/2}\right]_0^x$$

$$= \frac{1}{4}\left[2e^{-x/2} - 2x e^{-x/2}\right]_0^x$$

$$= \frac{1}{4}\left[2e^{-x/2} - 2x e^{-x/2} - 2\right]$$

$$= \frac{1}{2}\left(e^{-x/2} - x e^{-x/2} - 1\right)$$

~~0-625~~

~~0-705882353~~

Q4. P(choosing 2 members from the group) = $^{18}C_2 = P(A)$

~~P(atleast one~~

P(atleast one female) = 1 − P(both are male

$P(F) = 1 - \frac{^{10}C_2}{^{18}C_2}$

$= 0.705882353$

P(2 Qualified Females) = $\frac{^{7}C_2}{^{18}C_2}$

$P(F \cap Q) = 0.137254902$

$P(Q|F) = \frac{P(F \cap Q)}{P(F)}$

$= \frac{0.137254902}{0.705882353}$

$= 0.1945$.

~~Q5.~~

~~$P(L_1) = 0.4$~~

~~$P(L_2) = 0.5$~~

~~$P(L_3) = 0.1$~~

~~$P(L|L_1) = 0.2$~~

~~$P(L|L_2) = 0.4$~~

~~$P(L|L_3) = 0.1$~~

Q5. P(Delivering via Level 1) = $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

P(Arriving late using level 1) = $0.2 = P(L|L_1)$

$P(L|L_2) = 0.4$

$P(L|L_3) = 0.1$

~~$P(L_3|L) = P(L$~~

$$P(L_2 | L) = \frac{P(L_2)\, P(L | L_2)}{P(L_1)\, P(L|L_1) + P(L_2)\, P(L|L_2) + P(L_3)\, P(L|L_3)}$$

$$\cancel{= 0.5 \times 0.2}$$

$$= \frac{0.5 \times 0.4}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29}$$

$$= 0.69$$

Q6. $X \sim U(1,2)$

$$f_X x = \frac{1}{\beta - \alpha} = \frac{1}{2-1}$$

$$= 1$$

$$\cancel{f_X} \quad Y = \frac{3}{6-x}$$

$$Y(6-x) = 3$$

$$6 - x = \frac{3}{Y}$$

$$x = 6 - \frac{3}{Y}$$

$$g^{-1}(y) = 6 - \frac{3}{Y}$$

$$\frac{d}{dy} g^{-1}(y) = -3\left(-\frac{1}{Y^2}\right)$$

$$= \frac{3}{Y^2}$$

$f_X\, g^{-1}(y) = 1$ {As R.V X can still only take values b/w 0 and 1}

$$\cancel{f_Y} \quad \cancel{f_Y(y) =} \quad \cancel{\Rightarrow f_Y(y) = f_X\, g^{-1}(y) + \left|\frac{d}{dy}\right.}$$

$$f_Y(y) = f_X\, g^{-1}(y) \left| \frac{d}{dy} g^{-1}(y) \right| = 1 \times \frac{3}{Y^2} = 3y^{-2}.$$

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Q7. (i) $F(3) = P(X \leq 3)$
$= 0.05 + 0.15 + 0.35$
$= 0.55$

(ii) $E(X) = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_5p_5$
$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$
$= 3.25$

(iii) $V(X) = x_1^2p_1 + x_2^2p_2 + x_3^2p_3 + x_4^2p_4 + x_5^2p_5$
$= 1^2(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$
$= 11.45$

(iv) Coefficient of Skewness $= \frac{\text{Skewness}}{\sigma^3}$

$= \frac{\mu_3}{\sigma^3}$

$= \frac{E(X-\mu)^3}{\sigma^3}$

$\cancel{= E(X^3) - E(X)^3 - 3E}$

$\cancel{= \frac{E\{X^3 - 3X^2\mu + 3\mu^2X - (E(X))^3\}}{\sigma}}$

$= \frac{E[X^3 - 3X^2\mu + 3\mu^2X - \mu^3]}{\sigma^3}$

$= \frac{E(X^3) - 3E(X^2)\mu + 3\mu^2(E(X)) - \mu^3}{\sigma^3}$

$= \frac{E(X^3) - 3\mu[E(X^2) - \mu E(X)] - \mu^3}{\sigma^3}$

$\mu E(X) = [E(X)]^2$

$= \frac{E(X^3) - 3\mu\sigma^2 - \mu^3}{\sigma^3}$

$$= \frac{E(X^3) - 3E(x)\sigma^2 - [E(x)]^3}{\sigma^3}$$

$E(X^3) = 1(0.05) + 8(0.15) + 27(0.35) + 64(0.4) + 125(0.05)$

$= 42.55$

Coefficient of skewness $= \frac{42.55 - 3(3.25)(11.45) - (3.25)^3}{(\sigma^2)^{3/2}}$

$= -2.67$

Q8. Let success be defined as claim amount exceeding 1000.

$P(s) = 0.2$

$P(f) = 0.8$

It is a binomial distributⁿ.

$P(X < 3) = P(X \le 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= 0.035184372 + 0.131941395 + 0.230897442$

$= 0.398.$

Q9. Let success be defined as winning the prize.

$P(s) = 0.01$

$P(f) = 0.99$

~~P(X=3~~ It is a binomial distributⁿ.

$P(X=3) = {}^7C_3 (0.01)^3 (0.99)^4$

$= 0.000033621$

Q10. It will follow a Poisson distributⁿ.

$$\lambda = \frac{2}{5} = 0.4$$

$$P(X>2) = 1 - P(X \leq 2)$$

$$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^0 e^{-0.4}}{0!}$$

$$= e^{-0.4}$$

$$= 0.670320046$$

$$P(X=1) = \frac{(0.4)^1 e^{-0.4}}{1!}$$

$$= 0.268128018$$

$$P(X=2) = \frac{(0.4)^2 e^{-0.4}}{2!} = 0.053625604$$

$$P(X \leq 2) = 0.992073668$$

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 0.007926332.$$

Q.11. $P(b) = P(g)$

$$P(b) + P(g) = 1$$

$$2P(b) = 1$$

$$P(b) = \frac{1}{2}$$

$$P(g) = \frac{1}{2}$$

~~P(E)~~

Let E be the event of having 3 boys.

$$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$P(E) = \frac{1}{8}$$

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Q12. It is following exponential dist^n as Exp(10)

$\lambda = 10$

$\alpha = 1$

~~$P(X \leq 0.1) = 1 - e^{-\lambda x}$~~

~~$= 1 - e^{-1}$~~

~~$P(X$~~

$t = 0.1$ days

$P(T < 0.1) = F(0.1)$

$= 1 - e^{-\lambda t}$

$= 1 - e^{-1}$

$= 0.632120559$

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Q13. $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75}$$

$$= \frac{1}{45}$$

$$P(80 < X < 100) = \frac{100-80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444.$$

Q14. $X \sim \text{Gamma}(5, 0.4)$

Thus, $2\lambda X \sim \chi^2_{2\alpha}$

$0.8X \sim \chi^2_{10}$

We need to calculate $P(X < 22.89)$

$P(X < 22.89) = P(0.8X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$P(\chi^2_{10} < 18) = 0.9450$

$P(\chi^2_{10} < 18.5) = 0.9529$

0.5 causes a change of 0.0079 in probability

$0.5 = 0.0079$

$0.312 = \frac{0.0079}{0.5} \times 0.312$

$= 0.0049296$

Thus, $P(\chi^2_{10} < 18.312) = 0.9450 + 0.0049296$

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$= 0.9499296$

Thus, $P(X < 22.89) = P(\chi^2_{10} < 18.312)$

$= 0.9499296.$

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Q15. $f_X(x) = \frac{k}{(x+5)^4}, x > 0$

(i) $\int_0^\infty f_X(x) = 1$

$k \int_0^\infty \frac{1}{(x+5)^4} dx = 1$

Let $(x+5) = t$
$dx = dt$

$k \int_5^\infty t^{-4} dt = 1$

$k \left[\frac{t^{-3}}{-3} \right]_5^\infty = 1$

$k \left[-\frac{1}{3t^3} \right]_5^\infty = 1$

$k \left[0 - \left(-\frac{1}{3(125)} \right) \right] = 1$

$\frac{k}{375} = 1$

$k = 375$

(ii) $F(10) = P(X \leq 10)$
$= \int_0^{10} \frac{k}{(x+5)^4}$
$= 375 \int_0^{10} \frac{1}{(x+5)^4}$

Let, $(x+5) = t$
$dx = dt$
$= 375 \int_5^{15} t^{(-4)}$

$$\cancel{= 375 \left[\frac{-1}{3t^3} \right]_5^{15}}$$

$$\cancel{= 375 \int_5^{15} \frac{1}{3t^3}}$$

$$= 375 \left[-\frac{1}{3t^3} \right]_5^{15}$$

$$= 375 \left[\frac{1}{3t^3} \right]_{15}^{5}$$

$$= 375 \left[\frac{1}{375} - \frac{1}{10125} \right]$$

$$= 1 - \frac{375}{10125}$$

$$= 0.963.$$

(iii) $E(X) = \int_0^\infty x \cdot f_x(x)\, dx$

$$= \int_0^\infty \frac{x.k}{(x+5)^4}$$

$$= 375 \int_0^\infty \frac{x}{(x+5)^4}$$

Let, $(x+5) = t$

$dx = dt$

$$= 375 \int_5^\infty \frac{t-5}{t^4}$$

$$= 375 \int_5^\infty \frac{1}{t^3} - \frac{5}{t^4}$$

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$$= 375\left[-\frac{1}{2t^2} + \frac{5}{3t^3}\right]_5^\infty$$

$$= 375\left[\frac{5}{3t^3} - \frac{1}{2t^2}\right]_5^\infty$$

$$= 375\left[0 - 0 - \frac{5}{375} + \frac{1}{50}\right]$$

$$= 375\left[\frac{1}{50} - \frac{5}{375}\right]$$

$$= \frac{375}{50} - 5$$

$$\cancel{= 70.}$$

$$= 7.5 - 5$$

$$E(x) = 2.5.$$