FINANCIAL MATHEMATICS - ASSIGNMENT

(1)

$$20000 = 427.90 \, a_{\overline{60|}} @ \, {}^{(12)}/_{12}$$

$$\frac{20000}{427.90} = \left[\frac{1 - (1+i)^{-60}}{9}\right]$$

$$46.73989 = \frac{1 - (1+9)^{-60}}{9}$$

$$9 = 10.79\%$$

$$1+9 = \left[1 + \frac{9^{(12)}}{12}\right]^{12}$$

$$\frac{9^{(12)}}{12} = 0.008583$$

$$= 0.8583\%$$

$$9^{(12)} = 10.2996\%$$

(2) a. Discounted Payback Period:

In Many practical problems, the net cash flow changes sign only once this change being from negative to positive. In these circumstances the balance in the investor's account with change from negative to positive at a unique time t, or it will always be negative. in which case the project is not viable. If this time t, exists it is reffered to as the discounted payback period.

b. Payback Period:

In Many practical problems the net cash flow change, sign only once, this change being from negative to positive In these circumstances the balance in the investor's account will change from negative to positive at a unique time t, or it will always be negative, in which case the project is not viable. It is the smallest value of t such that $A(t) \geq 0$, where $A(t) = \sum_{s \leq t} C_s(1+j_1)^{(t-s)} + \int_0^t p(s)(1+j_1)^{(t-s)}\,ds$ where, j_1 = rate of interest applicable to investors borrowings. The payback period of a project is found by counting the numbers of years it takes before the cumulative forecasted cash flow equals the initial investment.

(iii) Project A:

$$170000 = 20000(1+i)^{-1} + 20000(1+i)^{-2} + 20000(1+i)^{-3}$$
$$- 20000(1+i)^{-1} - 10000(1+i)^{-2}$$

$$170000 = 10000(1+i)^{-2} + 20000(1+i)^{-3}$$

$$i = 0.07424$$

$$i = 7.424\%$$

project B:

$$20000 = 14000\, a_{\overline{6}|} \; @ \, i\,\%$$
$$20000 = 14000 \left[\frac{1-(1+i)^{-6}}{i} \right]$$
$$\frac{100}{7} = \left[\frac{1-(1+i)^{-6}}{i} \right]$$

$$7 - 7(1+i)^{-6} - 100\, i = 0$$

$$20000 = 14000\, a_{\overline{6}|} \; @ \, i\,\% + 200000 (1+i)^{-6}$$
$$200 = 14 \left[\frac{1-(1+i)^{-6}}{i} \right] + 200 (1+i)^{-6}$$
$$200\,i = 14 - 14(1+i)^{-6} + 200\, i\, [1+i]^{-6}$$
$$i = 7\%$$

b)
$$\text{Project A} = -170000 - 20000\,(1.06)^{-1} - 10000\,(1.06)^{-2}$$
$$+ 20000\,(1.06)^{-1} + 20000\,(1.06)^{-2} + 20000\,(1.06)^{-3}$$
$$= -170000 + 10000\,(1.06)^{-2} + 20000\,(1.06)^{-3}$$
$$= 6823.82$$

$$\text{Project B} = -20000 + 14000\, a_{\overline{6}|} \; @ \, 6\% + 20000\,(1.06)^{-6}$$
$$= 9834.65$$

3)
$$\text{Annuity } 1 = 200\,(v)^{1/4}\,(a_{\overline{21}|} \; @ \, 8\%) = 2161.363$$
$$\text{Annuity } 2 = 320\,(1+i)^{1/12}\,[a^{(4)}_{\overline{6.25}|} \; @ \, 8\%] = 2997.872$$
$$\text{Annuity } 3 = 180\,(a^{(12)}_{\overline{18.75}|} \; @ \, 8\%) = 1780.665$$
$$\text{Total} = 6899.90$$
$$6899.90 = A\,(1+p)^{1/4}\,(a^{(2)}_{\overline{21.5}|})$$
$$A = 656.56$$

(5) i TWRR $= \frac{16.4}{12.4} - 1 = 0.3226 = 32.26\%$

TWRR of equity fund $= \frac{15.5}{12.1} - 1 = 28.10\%$

ii a $1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4} = 4(1640)$

$i = 29.32\%$

b $400(\ddot{s}_{n}^{(4)}) = 100(16.4)\left[\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.8}\right]$

~~$i = 29.8\%$~~ $i = 29.8\%$

(iii) a $121(1+i) + 920(1+i)^{3/4} + 1030(1+i)^{1/2} + 1310(1+i)^{1/4} = 4(15.50)$

$i = 67.5\%$

b. $400\left[\ddot{S}_{\overline{1}|}^{(4)}\right] = 100(15.5)\left[\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right]$

$400(1.4135) = 100(15.5)\left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right)$

$i = 7.088\%$

(a) The notation $(I\ddot{a})_{\overline{n}|}$ is used to denote the present value of an increasing annuity due payable for n time units the n^{th} payment (of amount n) being made at time (n-1)

present value

$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + \cdots + nv^{n-1} \rightarrow 1$

Multiplying both sides by v.

$v(I\ddot{a})_{\overline{n}|} = v + 2v^2 + 3v^3 + \cdots (n-1)v^{n-1} + nv^n \rightarrow 2$

Subtracting (1) from (2)

$(I\ddot{a})_{\overline{n}|}(1-v) = 1 + v + v^2 + v^3 + \cdots + v^{n-1} - nv^n$

$(I\ddot{a})_{\overline{n}|}(1-v) = \ddot{a}_{\overline{n}|} - nv^n$

$$(I\ddot{a})_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{d}$$

(6) (i) $160000 = Xa_{\overline{10|}} @ 8\%$

$X = 23844.65$

(ii) Loan outstanding After 4^{th} payment.

$23844.65\,[a_{\overline{6|}} @ 8\%] = 110231.44$

$\therefore\ y a_{\overline{6|}} = 110231.44$

$\therefore\ y = 25309.72$

(iii) Loan outstanding after 7^{th} payment

$25309.72\,[a_{\overline{3|}} @ 10\%] = 62942.79$

$z a_{\overline{3|}} @ 9\% = 62942.74$

$\therefore\ z = 24869.77$

$\therefore$ yield $= 160000 - 146.5(a_{\overline{4|}}) - 443.95(a_{\overline{7|}})$

$-24865.72(a_{\overline{10|}}) = 0$

$\therefore\ i = 8.4\%$

(8) i $PV = 2.7 + 0.2\, a_{\overline{3|}} @ i\%$ — ①

$0.1v^3 + a_{\overline{10|}}(0.025v + 0.2v^2 + 0.1v^3) @ 9\%$

① = ② $\therefore\ i = 9.829$.

ii $0.1v^3 + 0.85\, \bar{a}_{\overline{10|}}\, v^3$

$= 0.1v^3 + 0.85\, a_{\overline{10|}}\, v^3 - 2.7 - 0.2\, a_{\overline{3|}} @ 10\%$

$= 1.0089$

as NPV is positive, it's good to invest.

(11) i TWRR $\Rightarrow (1+i)^2 = \frac{500}{460}\left(\frac{584+50}{500+40}\right)\left(\frac{x}{550}\right) = 1.44$

$\therefore\ X = 655.78$

ii MWRR $= 400(1+i)^2 + 40(1+i)^{7/4} - 50(1+i)^3 = 655.78$

MWRR $= 19.85\%$

(iii) $460(1+i)+40(1+i)^{3/4}=600$

$i=20.445\%$

$(1+i)=\frac{655.70}{56}=1.1923$

$LIRR=(1+i)^n=1.9923\,(1.2044)$

(12) ii (a) $DPP=-800(1+i)^t-50(1+i)^{t-1}+100\,(\bar{s}_{\overline{t-2|}}\ @\ 7\%)=0$

$t=17.8$ yr.

(b) $AV\rightarrow 100\,(\bar{s}_{\overline{7.2|}}\ @\ 6\%)=480$

$AV=480000$

(iii) $P=6\%$

$100\,\bar{s}_{\overline{1|}}=102.971$

$DPP=\$-800(1+i)^t-50(1+i)^{t-1}+102.97\,\bar{s}_{\overline{t-2|}}\ @\ 7\%=0$

$t=18$ yr

$-800(1+i)^{10}-50(1+i)^{17}+102.97+\bar{s}_{\overline{16|}}\ @\ 7\%=9.7714$

$AV=9.7714(1+i)^9+100\,\bar{s}_{\overline{7|}}\ @\ 6\%=462.79$

$AV=462.790$.

(13) i $X\,a^{(12)}_{\overline{25|}}\ @\ 7\%=9880$

$X=821.76$

ii Loan outstanding after 10/03/29

$X\,a^{(16)}_{\overline{34/12|}}\ @\ 7\%=648600$

iii Loan outstanding after 10/9/26 $\Rightarrow X\,a^{(12)}_{\overline{83/6|}}\ @\ 7\%$

Loan outstanding After 10/9/26 $\Rightarrow X\,a^{(12)}_{\overline{13.75|}}\ @\ 7\%$

Capital repaid $=X\left[\frac{v^{13.75}-v^{83/6}}{i^{2}}\right]\ @\ 7\%=26.86$

iv Capital repayment continue in 12 installments is

$X\left[a^{(12)}_{\overline{22/3|}}-a^{(12)}_{\overline{19/3|}}\right]\ @\ 7\%=516.2$

Total Interest $=X-516.20=303.56$.