Uddhav Jadhav 91 Probability and statistics

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Q1.i.) ans) Givendata :-

4 , 7, 9, 9 , 11 , 15 , 18, 18, 18 , 25

mean = $\frac{\Sigma x_i}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$

$\therefore$ ~~mean~~ $\bar{x} = 13.4$

$\boxed{\text{mean of the given data is } 13.4}$

Mode is the observation with maximum frequency

$\therefore$ mode = 18

$\boxed{\text{mode of the given data is } 18}$

Standard deviation $= \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}}$

$\therefore \Sigma (x_i - \bar{x})^2 = (4 - 13.4)^2 = (-9.4)^2 = 88.36$

$= (7 - 13.4)^2 = (-6.4)^2 = 40.96$

$= (9 - 13.4)^2 = (-4.4)^2 = 19.36$

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$= (11 - 13.4)^2 = (-2.4)^2 = 5.76$

$= (15 - 13.4)^2 = (1.6)^2 = 2.56$

$= (18 - 13.4)^2 = (4.6)^2 = 21.16$

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$= (25 - 13.4)^2 = (11.6)^2 = 134.56$

$\Sigma (x_i - \bar{x})^2 = 374.4$

~~stanca~~ $s = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}}$

$s = \sqrt{\frac{374.4}{\cancel{10}\ 9}}$

$s = 6.4498061986$

Standard deviation of the given data is 6.4498061986

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$Q_1 =$ value of $\left(\frac{n+1}{4}\right)^{th}$ observation

$Q_1 =$ value of $(2.75)^{th}$ observation

$Q_1 =$ value of 2^{nd} observation $+ 0.75$ (value of 3^{rd} observation - value of 2^{nd} observation)

$Q_1 = 7 + 0.75(9-7)$

$Q_1 = 7 + 1.5$

$Q_1 = 8.5$

$Q_3 =$ value of $3\left(\frac{n+1}{4}\right)^{th}$ observation

$Q_3 =$ value of $(8.25)^{th}$ observation

$Q_3 =$ value of 8^{th} observation $+ 0.25$ (value of 9^{th} observation - value of 8^{th} observation)

$Q_3 = 18 + 0.25(18-18)$

$Q_3 = 18 + 0.25 \times 0$

$Q_3 = 18 + 0$

$Q_3 = 18$

Inter Qualtile Range $= \cancel{\frac{Q_3 - Q_1}{2}}$ ~~or~~ $Q_3 - Q_1$

Inter Quartile Range $= 18 - 8.5$

Inter Quartile Range $= 9.5$

Q2.i ans) $P(\text{no claims}) = P(X=0)$ where $X \sim \text{Poisson}(0.5)$
$= 0.60653$ .... {from tables}

Q2.ii ans) Let Y = number of years with a claim
the $Y -$ binomial $(3, 0.03935)$
$\cancel{P(Y=1)}$ $P(Y=1) = 3(0.03935)(0.60653)^2 =$

Q2.iii ans) P(~

~~Q2.i ans) P(no~~

~~Q2.i ans) P(no claims)~~

Q2.i ans) $P(\text{no claims}) = P(X=0)$ where $X \sim \text{Poisson}(0.5)$
$= 0.60653$ .... {from tables}

Q2.ii ans) Let Y = number of years with claim
the $Y \sim \text{binomial}(3, 0.3935)$
$P(Y=1) = 3(0.3935)(0.60653)^2 = 0.4342807356$

Q2.iii ans) Let T = time until next claim
then $T \sim e(0.5)$ .... {e is exponential}
$P(T>2) = e^{-0.5(2)}$ .... {or by integration}
$= e^{-1}$
$= 0.3678794412$

Q3.i ans) $\alpha = 2 \quad \lambda = \frac{1}{2}$

$$E(X) = \frac{\alpha}{\lambda} = \frac{2}{\frac{1}{2}} = \frac{2}{0.5} = 4$$

$E(X) = 4$

$$Var(X) = \frac{\alpha}{\lambda^2} = \frac{2}{(\frac{1}{2})^2} = \frac{2}{\frac{1}{4}} = \frac{2}{0.25} = 8$$

$Var(X) = 8$

Standard deviation $= (8)^{\frac{1}{2}} = 2.8284271247$

Q3.ii ans) Shape of distribution is positively skewed and ~~#platykur~~ Platykurtic.

Q3.iii ans) Cummulative distribution function for $G(\alpha = 2, \lambda = \frac{1}{2})$

$$F_X(x) = \int_0^x \frac{(\frac{1}{2})^2 x^{2-1} e^{-\frac{1}{2}x}}{\Gamma 2} dx$$

$$= \frac{\frac{1}{2}}{2!} \int_0^x x e^{-\frac{1}{2}x} dx$$

$$= \frac{1}{4} \left\{ x \left[\frac{e^{-\frac{1}{2}x}}{(-\frac{1}{2})} \right]_0^x - \int_0^x \frac{e^{-\frac{1}{2}x}}{(-\frac{1}{2})} dx \right\}$$

$$= \frac{1}{4} \left\{ -2x \left[e^{-\frac{1}{2}x} - 1 \right] + 2 \int_0^x e^{-\frac{1}{2}x} dx \right\}$$

$$= \frac{1}{4} \left\{ -2x \left[e^{-\frac{1}{2}x} - 1 \right] + 2 \left[\frac{e^{-\frac{1}{2}x}}{\cancel{(-\frac{3}{2})}(-\frac{1}{2})} \right]_0^x \right.$$

$$= \frac{1}{4} \left\{ 2x - 2x e^{-\frac{1}{2}x} - 4 \left[e^{-\frac{1}{2}x} - 1 \right] \right\}$$

$$= \frac{1}{4} \left\{ 2x - 2x e^{-\frac{1}{2}x} - 4 e^{-\frac{1}{2}x} + 4 \right\} = \frac{1}{2} \left\{ x(1 - e^{-\frac{1}{2}x}) + 2(1 - e^{-\frac{1}{2}x}) \right\}$$

$$F_X(x) = \frac{1}{2} \left\{ (1 - e^{-\frac{1}{2}x})(x+2) \right\}$$

Q4 ans) Given:-
10 males and 8 females, total = 18
of which 6 males and 7 females are qualified actuaries
To find:-
Probability that a team of two workers randomly selected of two qualified females given that it contains at least on female

$$= \frac{\text{No. of ways to select two qualified females}}{\text{No. of ways to select at least one female}}$$

Now number of ways to select 2 qualified females out of 7 are ${}^{7}C_2 = 21$ ways and
Number of ways to select at least one ~~qualified~~ female =
(Total number of ways to select 2 workers) - (Number of ways to select both male workers)

$= {}^{18}C_2 - {}^{10}C_2$

$= 153 - 45$

$= 108$

$\therefore$ Required probability $= \frac{21}{108}$

$= 0.19444444$

Q5 ans) Lets consider the following events :-

L_1 : Package delivered via level 1 service

L_2 : Package delivered via level 2 service

L_3 : Package delivered via level 3 service

L : Package was delivered late

Given that :-

$P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

Also, $P(L/L_1) = 0.2$

$P(L/L_2) = 0.4$

$P(L/L_3) = 0.1$

Now, we can calculate the probability that the package was sent late using law of total probability

$\therefore P(L) = P(L_1) \times P(L/L_1) + P(L_2) \times P(L/L_2) + P(L_3) \times P(L/L_3)$

$P(L) = (0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)$

$P(L) = 0.08 + 0.2 + 0.01$

$P(L) = 0.29$

Now the probability that a late package was sent via level 2 service can be calculated ~~as~~ ~~usi~~ using Bayes' theorem

$$\therefore P(L_2/L) = \frac{P(L_2) \times P(L/L_2)}{P(L)}$$

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$$= \frac{P(L2) \times P(L/L2)}{P(L1) \times P(L/L1) + P(L2) \times P(L/L2) + P(L3) \times P(L/L3)}$$

$$= \frac{(0.5 \times 0.4)}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.29}$$

$$= 0.6896551724$$

The probability that a late package was sent via level 2 service is 0.6896551724

Q6ans) $X \sim U(1,2)$

$$Y = \frac{3}{6-x}$$

p.d.f ~~(propar~~ (probability distribution function)
of ~~[illegible]~~ X

$$= f(x) = \frac{1}{2-1} = 1 \qquad 1 \le x \le 2$$

find X in terms of Y

$$Y = \frac{3}{6-\cancel{x}\, X}$$

$$6 - X = \frac{3}{Y}$$

$$X = 6 - \frac{3}{Y}$$

$$|J| = \frac{dx}{dy} = 0 - \frac{3}{(-y^2)} = \frac{3}{y^2}$$

~~p.d.f of y of y of y = p.d.f of~~
~~pdf of y = pdf o~~
pdf of y = pdf of x . |J|

$$h(y) = f(x) \cdot |J|$$

$$= 1 \times \frac{3}{y^2} \quad \ldots\ldots\ldots\ldots \{1 \le x \le 2\}$$

$$h(y) = \frac{3}{y^2}$$

Hence pdf of y is $\frac{3}{y^2}$

Q7.i ans) $F(3) = P(X \leq 3)$

$F(3) = P(X=1) + P(X=2) + P(X=3)$

$F(3) = 0.05 + 0.15 + 0.35$

$F(3) = 0.55$

Q7.ii ans) $E(X) = \Sigma x P(X)$

$E(X) = 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.05$

$E(X) = 3.25$

Q7.iii ans) $V(X) = \Sigma (x-\mu)^2 P(X) = \Sigma (\mu - x)^2 P(X)$

$= (3.25-1)^2 \times 0.05 + \cancel{(3.25-2)^2 \times 0.15}$

$\cancel{(3.25}$

~~Q7 iii) X(J)~~

Q7.iii ans) $V(X) = \Sigma (x-\mu)^2 P(X) = \Sigma (\mu - x)^2 P(X)$

$V(X) = (3.25-1)^2 \times 0.05 + (3.25-2)^2 \times 0.15 +$
$(3.25-3)^2 \times 0.35 + (3.25-4)^2 \times 0.4 +$
$(3.25-5)^2 \times 0.05$

$V(X) = 0.8875$

Q7. iv ans) Coefficient of skewness = $\frac{3.(\text{mean} - \text{median})}{S.D}$

Now, mean $(\bar{x})$ = 3.25

median = 3

$SD = \sqrt{V(X)}$

$SD = \sqrt{0.8875}$

$\therefore$ Coefficient of skewness

$= \frac{3(3.25 - 3)}{\sqrt{0.8875}}$

$= \cancel{3 \times} \frac{0.75}{0.9420721841}$

$= 0.7961173387$

Coffecient of skewness = 0.7961173387

Q8ans) Let X = Number ~~*~~ of insurance claims exceed 100

~~who~~ we have,

$p = 0.2$ ~~n=15~~

$n = 15$

Here, X is independent

Therefore, $X \sim \text{Binomial}(n = 15, p = 0.2)$

The probability mass function ~~of~~ of X is

$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad : x = 0, 1, 2, \ldots\ldots n, q = 1-p$

$= 0.8$

Consider,

$P(\text{Fewer than } 3) = P(X < 3)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \binom{15}{0} (0.2)^0 (0.8)^{15-0} + \binom{15}{1} (0.2)^1 (0.8)^{15-1} + \binom{15}{2} (0.2)^2 (0.8)^{15-2}$

Use probability mass function (PMF)

$= \cancel{(0.8)^{15}} \cancel{(0.8)^{14}} \cancel{(0.8)^{14}} \cancel{(0.8)^{15}} + \cancel{15 \times (0.2) (0.8)^{14}} +$

$\cancel{105 \times (0.2)^2 (0.8)^{15-2}}$

~~Use PMF probabil mass fun~~ P

~~= 0.8~~

~~Use probability mass function (PMF)~~

$= \cancel{(0.8)^{15} + 15 \times (0.2)(0.8)^{14} + 105 \times (0.2)^2 (0.8)^{13}}$

$= \cancel{0.0352 + 0.1319 + 0.2309}$

$= \cancel{0.3980}$

~~Hence, probability is 0.3980~~, ~~He~~ ~~...~~

~~Use Prob~~

Use Probability mass function (PMF),

$= (0.8)^{15} + 15 \times (0.2)(0.8)^{14} + 105 \times (0.2)^2$

$(0.8)^{13}$

$= 0.0351843721 + 0.1319413953 + 0.2308974419$

$= 0.3980232093$

$\therefore$ probability is 0.3.980232093

Q9ans) Given :-

Probability of winning a prize $= 0.01$

$\Rightarrow P = 0.01$

$q = 1 - p$

$q = 1 - 0.01$

$q = 0.99$

~~Since~~ $\because$ he takes ~~upto~~ seven weeks upto ~~to~~ win third prize that means in first seven weeks he won 2 prizes, then probability of winning third prize in 18^{th} week = ?

Use negative binomial for $(r-1)$ success in $(n-1)$ trial

7 — 2 prizes

→ 3rd prize

$\cancel{P(x) = {}^{(n-1)}C_{(r-1)} \, p^r q^{n-r}}$

$\cancel{P(x) =}$

$\cancel{P(x) = {}^{(n-1)}C_{(r-1)}} \, p^r q^{(n-r)}$

$= \cancel{{}^{(8-1)}C_{(3-1)} \, (0.01)^3 \, (0.99)\, (8-3)}$

$$P(x) = {}^{(n-1)}C_{(r-1)} \, p^r q^{(n-r)}$$

$$= {}^{(8-1)}C_{(3-1)} \, (0.01)^3 (0.99)^{(8-3)}$$

$$= {}^7C_2 \times 0.000001 \times (0.99)^5$$

$$= 21 \times 0.000001 \times 0.9509900499$$

$$= 0.0000199708$$

Hence, probability that he takes seven weeks upto and including the week where he wins his third price is 0.0000199708

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Q10 ans) Two claims in 5 days on average $\Rightarrow \frac{2}{5} = 0.4$ claims per 1 day on average.

P(more than 2 claims in one day) $= P(X > 2 \mid X \sim poisson(0.4))$

$= 1 - P(X \leq 2 \mid X \sim poisson(0.4))$

$= 1 - \sum_{i=0}^{2} \frac{e^{-0.4} \times 0.4^{2}}{i!}$

$= 1 - e^{-0.4}\left(1 + 0.4 + \frac{(0.4)^2}{2}\right.$

$= 1 - 1.48e^{-0.4}$

$= 0.0079263319$

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Q11 ans) n (boys) = 5
n (girls) = 2
P (next three children are boys)

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

because both boys and girls have equally like probability

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Q12ans) Let $x(t)$ be the poisson process with rate $\lambda = 10/\text{day}$.

We have to calculate probability that the time between two claims is less than 0.1 days

Let Y denote the time between two ~~cla~~ claims

We know that the inter arrival time follows an ~~an~~ exponential ~~distribution~~ distribution with parameter $\lambda = 10$

$\therefore P[Y < 0.1] = \int_0^{0.1} \frac{1}{10} e^{-\frac{y}{10}} dy$ .... { ∵ the PDF of Y is ~~$f_Y(y) =$~~ $f_Y(y) = \frac{1}{10} e^{-y/10}, \; y>0$
$= 0 \quad \text{o.w}$ }

$= \frac{1}{10} \int_0^{0.1} e^{-y/10} dy$

$= \frac{1}{10} \left[\frac{e^{-y/10}}{-\frac{1}{10}} \right]_0^{0.1}$

~~$= 1 - e^{-\frac{0.1}{10}}$~~ $= 1 - e^{-0.1/10}$

$= 1 - e^{-0.01}$

$= 0.0099501663$

Probability that the time between ~~2 claims~~ two claims is ~~the~~ less than 0.1 days is 0.0099501663

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Q13ans) Here, in the given problem,
$X \sim$ Uniform $(a = 75, b = 120)$
F : ~~PDF~~ ~~Prop~~ Probability ~~of~~ distribution function of X is

$$\cancel{f_X(x)} \quad f_X(x) = \begin{cases} \frac{1}{120-75} = \frac{1}{45}; & \text{if } 75 \leq x \leq 120 \\ 0; & \text{o.w} \end{cases}$$

$\therefore$ Probability that X lies between 80 and 100 is

$$P(80 \leq X \leq 100)$$

$$= \int_{80}^{100} f_X(x)\,dx$$

$$= \int_{80}^{100} \frac{1}{45}\,dx$$

$$\cancel{= \frac{1}{45}(x)_{80}^{100}}$$

$$\cancel{= \frac{100-80}{45}}$$

$$= \frac{1}{45}(x)_{80}^{100}$$

$$= \frac{100-80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444444444$$

Probability that X lies between 80 and 100 is 0.444444444

Q14 ans) $P(X < 22.89) = \frac{(0.4)^5}{\Gamma(5)} \int_0^{22.89} x^4 e(-0.4x)\,dx$

$$= \frac{(0.4)^5}{\Gamma(5)} \left[-\frac{x^4}{0.4} e(-0.4x) + \frac{4}{(0.4)} \int x^3 e(-0.4x)\,dx \right]_0^{22.89}$$

{using integration by parts}

$$= \frac{(0.4)^5}{\Gamma(5)} \left[-\frac{x^4}{0.4} e(-0.4x) - \frac{4x^3}{(0.4)^2} e(-0.4x) + \frac{4\times 3}{(0.4)^2} \int x^2 e(-0.4x)\,dx \right]_0^{22.89}$$

$$= \frac{(0.4)^5}{\Gamma(5)} \left[-\frac{x^4}{0.4} e(-0.4x) - \frac{4x^3}{(0.4)^2} e(-0.4x) - \frac{4\times 3x^2}{(0.4)^3} e(-0.4x) + \frac{4\times 3\times 2}{(0.4)^3} \int x\, e(-0.4x)\,dx \right]_0^{22.89}$$

$$= \frac{(0.4)^5}{\Gamma(5)} \left[-\frac{x^4}{0.4} e(-0.4x) - \frac{4x^3}{(0.4)^2} e(-0.4x) - \frac{4\times 3x^2}{(0.4)^3} e(-0.4x) - \frac{4\times 3\times 2\times x}{(0.4)^4} e(-0.4x) - \frac{4\times 3\times 2\times 1}{(0.4)^5} e(-0.4x) \right]_0^{22.89}$$

$= 0.9501$

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Q1s ans) $f(x) = \frac{k}{(x+5)^4}$, if $x > 0$

$= 0$ o.w

we know, $\int_x f(x) = 1$

$\Rightarrow k \int_0^\infty \frac{1}{(x+5)^4} dx = 1$

$\Rightarrow k . \left[-4 (x+5)^{-5} \right]_0^\infty = 1$

$\Rightarrow k . -4 \left[0 - \frac{1}{5^5} \right] = 1$

$\Rightarrow k = \frac{5^5}{4} = \frac{\cancel{3125}}{4}$

$\cancel{\Rightarrow k = 781.25}$ $\cancel{k}$ $\Rightarrow k = \frac{3125}{4}$

Q2. ii ans) c.d.f $= F(x) = \int_0^x f(t) dt$

$= \int_0^x \frac{k}{(t+5)^4} dx$

$= k \left[-4 \times (t+5)^{-5} \right]_0^x$

$= k . -4 \left[\frac{1}{(x+5)^{-5}} - \frac{1}{5^5} \right]$

$F(10) = 4k \left[\frac{1}{5^5} - \frac{1}{15^5} \right]$

$F(10) = 4k \times \frac{242}{759375} = \frac{968}{243}$ $\cancel{\frac{968}{243}}$

$F(10) = \cancel{3.98353909447}$ $\frac{968}{243}$

Q15. iii ans) $E(X) = \int_0^\infty x \cdot \frac{k}{(x+5)^4} dx$

$$E(X) = k \int_0^\infty \frac{x+5-5}{(x+5)^4} dx$$

$$E(X) = k \left[\int_0^\infty \frac{1}{(x+5)^3} dx - 5 \int_0^\infty \frac{1}{(x+5)^4} dx \right].$$

$$E(X) = k \left[1 - 3 \cdot (x+5)^{-4} \right]_0^\infty - 5 \left[\because k \int_0^\infty \frac{1}{(x+5)^4} dx \right.$$

$$E(X) = k \left[-3 \left(0 - \frac{1}{5^4} \right) \right. - 5$$

$$E(X) = k \times \frac{3}{5^4} - 5$$

$$E(X) = 15 - 5$$

$$E(X) = 10$$