Calculus No. 1

1) $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$

$\rightarrow \therefore \lim_{h\to 0} \frac{2(9-6h+h^2)-18}{h}$

$= \lim_{h\to 0} \frac{\cancel{18}-12h+2h^2-\cancel{18}}{h}$

$= \lim_{h\to 0} \frac{-12h+2h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(-12+2h)}{\cancel{h}}$

$\therefore -12+2h = -12+2(0) = \underline{\underline{-12}}$

2) a) $x=4$; b) $x=6$

$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$

$\rightarrow$ a) $x=4$

$\therefore$ LHL $\Rightarrow \lim_{x\to 4^-} g(x) = 2(4) = 8$

$\therefore$ RHL $\Rightarrow \lim_{x\to 4^+} g(x) = 4-1 = 3$

$\because 8 \neq 3$

$\therefore$ LHL $\neq$ RHL

Hence discontinuous at $x=4$

b) $x=6$

$\therefore$ LHL $\Rightarrow \lim_{x\to 6^-} g(x) = 2(6) = 12$

$\therefore$ RHL $\Rightarrow \lim_{x\to 6^+} g(x) = 6-1 = 5$

$\therefore 12 \neq 5$ i.e LHL $\neq$ RHL

Hence discontinuous at $x=6$

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3) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

→ Multiply & divide by conjugate

$\therefore \lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x \to 9} \frac{(\cancel{x-9})^{-1}(3+\sqrt{x})}{\cancel{9-x}}$ $\quad \because -(9-x) = x-9$

$= -1(3+\sqrt{9})$

$= -1(6)$

$= \underline{\underline{-6}}$

4) a) $x=-1$; b) $x=0$; c) $x=3$

$f(x) = \frac{4x+5}{9-3x}$

→ a) $x=-1$

$f(-1) = \frac{4(-1)+5}{9-3(-1)}$

$= \frac{-4+5}{9+3} = \frac{1}{12}$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} \left(\frac{4x+5}{9-3x} \right) = \frac{1}{12}$

Here $\lim_{x \to -1} f(x) = f(-1)$

Hence function is <u>continuous</u> at $\underline{\underline{x=-1}}$.

b) $x=0$

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$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left[\frac{4x+5}{9-3x}\right] = \frac{5}{9}$$

Here $\lim_{x \to 0} f(x) = f(0)$

Hence, func. is continuous at $x=0$.

c) $x = 3$

$$\therefore f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \text{Not defined}$$

Hence, func. is discontinuous at $x=3$.

5) $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\longrightarrow \lim_{x \to \infty} \frac{\dfrac{6e^{4x} - e^{-2x}}{e^{4x}}}{\dfrac{8e^{4x} - e^{2x} + 3e^{-x}}{e^{4x}}}$$

$$= \lim_{x \to \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{\lim_{x \to \infty} 6 - e^{-6x}}{\lim_{x \to \infty} 8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{6 - e^{-6(\infty)}}{8 - e^{-2(\infty)} + 3e^{-5(\infty)}}$$

$$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$$

$$= \frac{6 - 0}{8 - 0 + 0} \quad \because e^{-\infty} = 0$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

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6) $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{y \to 6} g(y)$ (b) $\lim_{y \to -2} g(y)$

→ a) $\lim_{y \to 6} g(y)$

$\because 6 > -2$

$\therefore \lim_{y \to 6} 1-3y$

$= 1-3(6) = 1-18 = \underline{\underline{-17}}$

b) $\lim_{y \to -2} g(y)$

$\because -2 = -2$

$\therefore \lim_{y \to -2^-} g(y) = y^2+5$ | $\lim_{y \to -2^+} g(y) = 1-3y$

$= 4+5 = 9$ | $= 1-(-6) = 7$

$\therefore \lim_{y \to -2} g(y) = \underline{\underline{9}}$ or $\underline{\underline{7}}$

7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

→ Using Product Rule

$\therefore f'(t) = (4t^2-t)\frac{d}{dx}(t^3-8t^2+12) +$

$(t^3-8t^2+12)\cdot\frac{d}{dx}(4t^2-t)$

$= (4t^2-t)\cdot(3t^2-16t) +$

$(t^3-8t^2+12)\cdot(8t-1)$

$\therefore f'(t) = [12t^4-64t^3-3t^3+16t^2] +$

$[8t^4-64t^3+96t-t^3+8t^2-12]$

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$\therefore f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8) $R(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2 + 1}$

$\rightarrow \therefore R'(\omega) = \dfrac{(2\omega^2+1)\dfrac{d}{dx}(3\omega+\omega^4) - (3\omega+\omega^4)\cdot\dfrac{d}{dx}(2\omega^2+1)}{(2\omega^2+1)^2}$

$= \dfrac{(2\omega^2+1)(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}$

$= \dfrac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}$

$\therefore R'(\omega) = \dfrac{\omega^5 + \omega^3 - 3/2\,\omega^2 + 3/4}{\omega^4 + \omega^2 + 1/4}$

... taking out 4 common.

9) $f(x) = 2e^x - 8^x$

$\rightarrow \therefore f'(x) = 2e^x - \dfrac{d}{dx} 8^x$

$= 2e^x + (-8^x \log 8)$ .....

$\dfrac{d}{dx}[a^x] = a^x \log a$

$\therefore f'(x) = 2e^x - 8^x \log 8$

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10) $y = z^5 - e^z \ln(z)$

$\rightarrow \therefore \frac{dy}{dx} = \frac{d}{dx} z^5 - \frac{d}{dx} e^z . \ln(z)$

$= 5z^4 - \left[e^z . \frac{d}{dx} \ln(z) + \ln(z) . \frac{d}{dx} e^z\right]$

$= 5z^4 - \left[e^z . \frac{d}{dx} \ln(z) + \ln(z) . e^z\right]$

$= 5z^4 - \left[e^z . \frac{1}{z} + e^z \ln(z)\right]$

$= 5z^4 - \left[\frac{e^z}{z} + e^z \ln z\right]$

$\therefore y' = 5z^4 - \frac{e^z}{z} - e^z \ln(z)$

11) a) $f(x) = (6x^2 + 7x)^4$ b) $f(t) = 5 + e^{4t + t^7}$

$\rightarrow$ a) $f(x) = (6x^2 + 7x)^4$

$\therefore f'(x) = 4(6x^2 + 7x)^3 (12x + 7)$

$\therefore f'(x) = 4(6x^2 + 7x)^3 \times (12x + 7)$

b) $f(t) = 5 + e^{4t + t^7}$

$\therefore f'(t) = 0 + e^{4t + t^7} \left(\frac{d}{dx} 4t + t^7\right)$

$= e^{4t + t^7} (4 + 7t^6)$

$\therefore f'(t) = e^{4t + t^7} (4 + 7t^6)$

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12> a> $z = \ln(7-x^3)$; b> $g(v) = \frac{2}{(6+2v-v^2)^4}$

c> $f(t) = \ln(1+t^2)$

→ a> $z = \ln(7-x^3)$

$\therefore \frac{d}{dx} z = \frac{d}{dx} \ln(7-x^3)$

$\therefore z' = \frac{1}{(7-x^3)} \times (0-3x^2)$

$= \frac{-3x^2}{7-x^3}$

$\therefore z'' = \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$

$= \frac{(7-x^3)(-6x) - (-3x^2)(0-3x^2)}{(7-x^3)^2}$

$= \frac{-42x + 6x^4 - [+9x^4]}{(7-x^3)^2}$

$= \frac{-42x - 3x^4}{(7-x^3)^2}$

$= \frac{3x(-14-x^3)}{(7-x^3)^2}$

$\therefore z'' = \frac{-3x(14+x^3)}{(7-x^3)^2}$

b) $g(v) = \dfrac{2}{(6+2v-v^2)^4}$

$\therefore g'(v) = \dfrac{(6+2v-v^2)^4 \cdot 0 - 2\left[4(6+2v-v^2)^3 \times (0+2-2v)\right]}{\left[(6+2v-v^2)^4\right]^2}$

$= \dfrac{-2\left[4\cancel{(6+2v-v^2)^3} \times (2-2v)\right]}{(6+2v-v^2)^{\cancel{8}\,5}}$

$= \dfrac{-2\left[4(2-2v)\right]}{(6+2v-v^2)^5}$

$= \dfrac{-16+16v}{(6+2v-v^2)^5}$

$= \dfrac{-16(1-v)}{(6+2v-v^2)^5}$

$\therefore g''(v) = \dfrac{d}{dx} \dfrac{-16+16v}{(6+2v-v^2)^5}$

$\rightarrow = \dfrac{\left[(6+2v-v^2)^5 \times 16\right] - \left[-16+16v(2-2v) \times (5(6+2v-v^2)^4)\right]}{(6+2v-v^2)^{10}}$

$= \dfrac{16(6+2v-v^2)^5 - \left[-32+64v-32v^2 \times 5(6+2v-v^2)^4\right]}{(6+2v-v^2)^{10}}$

taking out $(6+2v-v^2)^4$ common

$= \dfrac{\cancel{(6+2v-v^2)^4}\left[16(6+2v-v^2) - (-160+320v-160v^2)\right]}{(6+2v-v^2)^{\cancel{10}\,6}}$

$= \dfrac{96+32v-16v^2+160-320v+160v^2}{(6+2v-v^2)^6}$

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$$= \frac{256 + (-288V) + 144V^2}{(6+2V-V^2)6}$$

$$\therefore g''(V) = \frac{16(16 - 18V + 9V^2)}{(6+2V-V^2)6}$$

c) $f(t) = \ln(1+t^2)$

$$\therefore f'(t) = \frac{1}{1+t^2} \times 2t$$

$$= \frac{2t}{1+t^2}$$

$$\therefore f''(t) = \frac{(1+t^2)\frac{d}{dx}2t - (2t)\frac{d}{dx}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)\cdot 2 - 2t(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{}$$

$$\therefore f''(t) = \frac{2-2t^2}{(1+t^2)^2}$$

13) $x^3 - 3x^2 - 9x + 12$

$\longrightarrow$ $\therefore f(x) = x^3 - 3x^2 - 9x + 12$

$\therefore f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$\therefore f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x+1)(x-3)$

At $x = -1$

$f''(-1) = 6(-1) - 6 = -12 < 0$

$\therefore f$ is maximum at $x = -1$.

$\therefore f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$\underline{y_{max} = 17}$

At $x = 3$

$f''(3) = 6(3) - 6 = 18 - 6 = 12 > 0$

$\therefore f$ is minimum at $x = 3$.

$\therefore f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= \cancel{27} - \cancel{27} - 27 + 12$

$\underline{y_{min} = -15}$

14) $4x^3 - 18x^2 + 24x - 7$

$\rightarrow \therefore f(x) = 4x^3 - 18x^2 + 24x - 7$

$\therefore f'(x) = 12x^2 - 36x + 24$

$\therefore f''(x) = 24x - 36$

$\therefore f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x-1)(x-2)$

At $x = 1$

$f''(1) = 24(1) - 36 = -12 < 0$

$\therefore f$ is maximum at $x = 1$

$\therefore f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$\therefore \underline{y_{max} = 3}$

At $x=2$

$f''(2) = 24(2) - 36 = 12 > 0$

$\therefore f$ is minimum at $x=2$

$\therefore f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$\therefore y_{min} = 1$

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15) $f(x) = x^2(x+3)$

$\rightarrow f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$\therefore f'(x) = 3x^2 + 6x$

$= 3(x^2 + 2x)$

Let $x^2 + 2x = 0$

$\therefore x = 0$ or $x = -2$

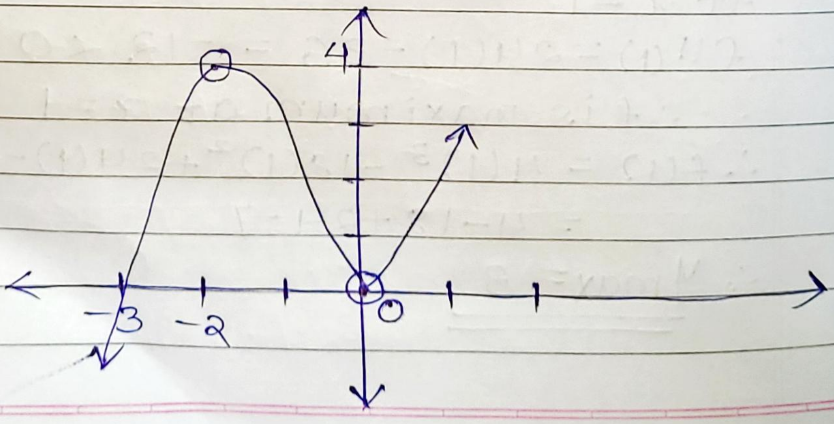
$f''(x) = 6x + 6$

$\therefore f''(0) = 6 > 0 \quad \therefore$ Minimum

$f''(-2) = -6 < 0 \quad \therefore$ Maximum

$\therefore f(0) = 0$

$f(-2) = 4$



16) $f(x) = 3x^2 - 6x$

$\rightarrow$ $f(x) = 3x^2 - 6x$

$\therefore f'(x) = 6x - 6$

$= 6(x-1)$

Let $x - 1 = 0$

$\therefore x = 1$

$\therefore f''(x) = 6 > 0$ $\therefore$ Minimum

$\therefore f(1) = -3$