

PRLI 2 - Assignment 2

Q1

- ↳ sensitivity analysis may be carried out at an individual policy level and at a portfolio level.
- ↳ At an individual policy level, sensitivity analysis allows the company to understand the impact of misestimating parameter values in the model.
- ↳ It can help show what the reductions would be in profits emerging, return on capital or other metrics targeted.
- ↳ This may help assess what margins may be included in the parameter values, for the risk that are not borne out in reality.
- ↳ At a portfolio level, sensitivity analysis can be used to assess the impact of shifts in mix of business.
- ↳ Some parts of the portfolio may be more profitable than others and this analysis will highlight the possible impact on overall profitability of the product.
- ↳ sensitivity analysis on the volume of business can also be used to assess the overall profits emerging.

Q2

Annual prem $\rightarrow 100000$; ~~rate~~ risk discount rate $\rightarrow 8\%$
 Allocation % (1st yr) $\rightarrow 90\%$; Allocation % (2nd yr +) $\rightarrow 101.5\%$
 Interest [Unit] $\rightarrow 6\%$; Interest [Non-unit] $\rightarrow 4\%$
 Max charge $\rightarrow 6.5\%$; B/o spread $\rightarrow 5\%$
 Min death benefit $\rightarrow 500000$

	INR	% prem	Total
Initial exp/ commission	2500	20%	12500
Renewal exp/ commission	500	4.5%	5000

Multiple decrement table:

x	q_x^d	q_x^s	$(aq)_x^d$	$(aq)_x^s$	(ap)	$t-1(ap)$
40	0.000788	0.1	0.000749	0.09996	0.899291	1.00
41	0.000962	0.05	0.000938	0.04998	0.949086	0.899291
42	0.001104	0.05	0.001076	0.04997	0.948951	0.853504
43	0.001208	0.05	0.001178	0.04997	0.948852	0.809934

(i)		Yr 1	Yr 2	Yr 3	Yr 4
	value of units at start of year	0	90177	196809	309274
	allocation	90000	101500	101500	101500
	B/O	4500	5075.00	5075	5075
	Interest	5130	50051196	17594	24342
	management charge	453	4989	1554	2150
	val of units at end of year	90177	196809	309274	421891
	Answer opt a.				

(ii)		Yr 1	Yr 2	Yr 3	Yr 4
	unallocated man	10000	-1500	-1500	-1500
	B/O spread	4500	5075	5075	5075
	Emp & commission	12500	5000	5000	5000
	Interest	80	-57	-57	-57
	man charge	453	989	1554	2150.2
	extra death benefit	307	284	205	85
	end of yr cashflow	2226	-777	-133	583

Answer - opt a

(iii)

prob in force	1	0.899291	0.853504	0.809934
disc factor	0.925925	0.851338	0.793832	0.735029
E.PV of profit	1718.96			
Premium signature	100000	0.326769	13174.25	6429517
E.PV of premiums	320737.09			
Profit margin	0.54%			

(iv)

To calculate the expected provisions at the end of each year we have (utilising the end of yr cashflow figures and decrement tables in (i) above) :

$${}_2V = 133 / 1.04 = 127.88$$

$$\text{Cashflow in yr 2} = -777 - (ap)_{41} \times {}_2V = -898.37$$

$${}_1V = 898.37 / 1.04 = 863.82$$

we want it in the beginning of the year

$$\therefore \text{Year 2} \rightarrow 127.88 (ap)_{41} = 121.37$$

$$\text{Year 1} \rightarrow 863.82 (ap)_{40} = 776.83$$

$\therefore$ Answer (a)

(v) Based on the expected provisions in (a) above, the cash flow for 2 yrs will be zeroised whilst year 1 will become $2226 - 776.83 = 1449.17$

	1449.17	0	0	583
prob in force	1	0.899291	0.853504	0.809934
discount fact	0.925925	0.851338	0.793832	0.735029
EPV (profit)	1688.9			

Q3

(i) The vector of balancing items in the projected revenue account for each policy year is called the profit vector. Profit vector gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.

(ii) The objective specified for expected level of profit is termed as "profit criterion"

Eg : NPV = 40% of initial sales commission
Profit margin = 3% of the EPV of premium income.

(iii) Risk discount rate = risk free rate + margin for risk
= 10% + 3% = 13%.

$$EPV \text{ profit (@13\%)} = -250v + 150v^2 + 200v^3 + 225v^4 = 172.84$$

Let the first year premium be P .

$$\begin{aligned} EPV \text{ premiums (@13\%)} &= P(1 + 1.04v + 1.04^2v^2 + 1.04^3v^3) \\ &= P(1 + 0.919338 + 0.845094 + 0.776754) \\ &= 3.541186P \end{aligned}$$

$$\text{Profit margin} = EPV \text{ profit} / EPV \text{ premiums}$$

$$5\% = 172.84 / 3.541186P$$

$$\therefore P = 976.17$$

Q4

- (i) Experience basis is used to calculate the expected future profits of a contract for comparison with a stated profit criterion. It represents the best estimate of expected future experience.

Pricing basis is used while setting premium. Assuming that the risk discount rate reflects fully the uncertainties in the assumptions, the pricing basis represents the insurer's realistic expected outlook.

Valuation basis is used to calculate the reserves to be held by an insurer. It represents prudent assumptions of expected future experience.

- (ii) While the student actuary is correct that reserves are held such that there is an acceptably low probability of insolvency occurring in the future, it is incorrect to assume that everyone will die on the day after the valuation date. This is because:

↳ A company whose reserving basis is extremely pessimistic will be holding extremely high reserves leading to large capital requirements. This will require higher profits resulting in higher premiums/charges for customer.

↳ The reserving basis chosen has to satisfy any local legislation and professional guidance which exists to protect the interests of the policyholders.

(iii) We need the profit vector of the policy in order to calculate the non unit reserves. It can be derived by dividing the given profit signature with the probability of surviving to the start of that year = $(-20, -35.2/P_{70}, 50/2P_{70}, -28.7/3P_{70}, -20/4P_{70}, 90.21/5P_{70})$
 $= (-20, -35.79, 52.12, -30.65, -22.27, 104.4)$

Let the reserves required at the end of policy year n be denoted as nV . No reserve is need at end of 5th year $\therefore 5V = 0$

$$4V = 22.27/1.06 = 21.01$$

$$3V = (30.65 + 21.01, P_{78})/1.06 = 48.06$$

The cashflow of year 3 is sufficient to set up this reserve.

Hence $2V = 0$. Allowing for this, revised profit in 3rd year will be $\rightarrow 52.12 - 48.06, P_{72} = 5.54$

$$1V = 35.79/1.06 = 33.76$$

$$\text{Revised profit in year 1} = -20 - 33.76, P_{70} = -53.2$$

$$\therefore \text{Revised profit vector is } (-53.2, 0, 5.54, 0, 0, 104.4)$$

Q5

(i) Unit fund represents the value of unit of the policyholder at any time. It does not necessarily represent the amount that the policyholder is entitled to any point. For eg, on surrender, policyholder will get the surrender value which is the proportion of unit fund. On death, maturity or surrender, the units held will be used to pay the benefit.

Non-unit fund will arise from the following sources:

- (a) premium less cost of allocation i.e. the difference between the premium paid and the amount invested in the unit fund
- (b) expenses incurred by the insurer.
- (c) interest earned on non-unit fund.
- (d) FMC taken from unit fund.
- (e) cost of outgoes if outgo amount is greater than the value of units.
- (f) Profit on surrender.

(ii) The life company sets up non unit reserves if the overall cash flow in any month other than month one is negative without reserves. The policy should be self-financing after month one. Reserves are set up early so that money can be ~~and~~ released as and when required to eliminate the negative cash flow. setting up reserves will result in delay of profits but gives the comfort of the company that capital is sufficient to support the policy throughout the term.

Q6

(i)	Year	Age	Profit vector	q_n	P_n	$P(n-1)$	Profit signature	PV
	1	30	-250	0.00059	0.99941	1	(250)	(235.85)
	2	31	-400	0.00062	0.999398	0.99941	(399.76)	(355.79)
	3	32	-600	0.000617	0.999383	0.998808	(599.29)	(503.17)
	4	33	1500	0.000626	0.999364	0.998192	(497.29)	1185.99

NPV @ 6% = 91.18

(ii) $2V = 600 / 1.03 = 582.52$

$1V = (400 + 582.52 \times 0.99941) / 1.03 = 953.57$

Revised 1 Year cashflow = $-250 - 953.57 \times 0.99941 = -1203$

NPV = $-1203 / 1.06 + 1185.99 = 51.08$

(iii) NPV after zeroising negative cashflows to achieve a single financing phase is smaller. This is because the negative cashflows have been accelerated hence being discounted less.

Q7

Decrement table

Years	L_x	q_x	w_x	No. of deaths	No. of surrenders	No. of maturity	No. of survivors
1	1	0.003358	10%	0.063358	0.0996642		0.8969718
2	0.896978	0.004903	5%	0.00439788	0.044629		0.847951
3	0.847951	0.00565	5%	0.0047992	0.042158		0.801002
4	0.801002	0.006352	5%	0.00508796		0.79591404	0

Unit fund									
Year	Premium	Allocated prem	Bid offer spread	Fund before deduction	Pol Admin charge	Fund after deduction	growth rate	FmC	Fund after growth & FmC
1	5500	5362.5	5094.38	5094.38	60	5034.38	4%	52.36	5183.9
2	5500	5362.5	5094.38	10277.78	60	10217.78	3.5%	105.75	10461.65
3	5500	5362.5	5094.38	15564.03	60	15504.03	3.5%	160.47	15886.2
4	5500	5362.5	5094.38	20980.58	60	20920.58	3.5%	216.53	21436.27

Non Unit fund.

Year	Prem - Cost of Allocation	Admin charge	Expenses	Commission	Death benefit cost	Surrender benefit cost	Maturity benefit cost	FmC	Non-unit Surv. In.	Profit
1	405.62	60	265	275	4.89	-89.7	0	52.36	-1.86	61.13
2	405.62	60	75	137.5	13.32	-44.8	0	105.75	6.33	346.68
3	405.62	60	0	137.5	23	-44.7	0	160.47	8.2	518.49
4	405.62	60	0	137.5	34.68	-	59.62	216.53	8.2	458.56

(i)

Prem - allocation	Admin charge	Exp.	Commission	Death benefit cost	Surrender benefit cost	Mat. Benefit cost	FmC
405.62	60	265	275	1395.85	0	0	52.36

Non unit investment income	Profit
-1.86	-1419.73

(b)	Prem-allocation	Admin charge	Exp	Com	Death benefit cost	Surrender	Maturity	FMC	Non unit	Profit
	405.62	60	265	275	0	-900	0	52.34	-1.86	876.12

(ii)	Prem-allocation	Admin charge	Exp	Com	Death benefit	Surrender	Maturity	FMC	Non unit	Profit
Year (a) 2	405.62	60	75	137.5	0	-	0	105.75	6.33	365.2

(b)	Prem-allocation	Admin charge	Exp	Comm.	Death benefit cost	Surrender benefit cost	Maturity B.C.	FMC	Non unit IT	Profit
Year 2	405.62	60	75	137.5	0	0	0	105.75	6.33	365.2

(iii)	Profit vector per policy	Profit signature	NPV @ 5%
			1062.99
	61.13	61.14	1054.95
	396.68	355.82	751.88
	518.49	439.65	349.82
	458.56	367.31	-

Q8 Let 'P' be the annual premium payable

$$P \ddot{a}_{61:\overline{30}|} = 500000 A_{61:\overline{30}|} + (100 + 0.025P) (\ddot{a}_{61:\overline{30}|} - 1) + 800$$

$$P \times 3.730 = 500000 \times 0.95654 + (100 + 0.025P) (3.730 - 1) + 800$$

$$P = 429343 / 3.662$$

$$P = 117250$$

Reserve required on the policy per unit sum assured:

$$1V_{61:\overline{30}|} = \frac{1 - \ddot{a}_{62:\overline{29}|}}{\ddot{a}_{61:\overline{30}|}} = \frac{1 - 2.857}{3.722} = 0.23240$$

$$2V_{61:\overline{30}|} = 1 - \frac{1.951}{3.722} = 0.47582$$

$$3V_{61:\overline{30}|} = 1 - \frac{1}{3.722} = 0.73133$$

Multiple decrement table.

x	$q_x^{(d)}$	$q_x^{(s)}$	$(aq)_x^{(s)}$	T	$(op)_{[61]+t-1}$	${}_{t-1}(op)_{[61]}$
61	0.006433	0.05	0.04968	1	0.943887	1
62	0.009696	0.05	0.04952	2	0.940784	0.943889
63	0.011344	0.05	0.04943	3	0.939226	0.93709
64	0.012716			4	0.937284	0.93403

Year	Prem.	Exp.	opening reserve	Interest	Death claim	surrender claim	maturity claim	Closing reserve	Profit
1	117250.7681	806	0	5892.5389	3216.5	5825.02		10679.67	3552.12
2	117250.7681	3031.27	116200	11520.9749	4848	11612.52		223821.92	1658.04
3	117250.7681	3031.27	237910	17606.4749	5672	17387.12		343442.08	3234.75
4	117250.7681	3031.27	365665	23994.2249	6358		493642		3878.72

Year	Profit signature	Discount factor	NPV of profit signature
1	3552.12	0.9259259	3289
2	1565	0.8573388	1341.79
3	2872.45	0.7938322	2280.25
4	3234.97	0.7350299	2377.8
			9288.79

Year	Premium	${}_{t-1}V_{[61]}$	Discount factor	NPV of premium
1	117250.7681	1	1	117250.77
2	117250.7681	0.94389	0.925926	102473.91
3	117250.7681	0.88799	0.857339	89263.98
4	117250.7681	0.83403	0.793832	77629.38
				386618.04

Profit margin = 2.4% [i.e. $\frac{\text{NPV of profit signature}}{\text{NPV of premium}}$]

Q9

(i) Calculating the dependent probabilities:

x	$(aq)_x^A$	$(aq)_x^S$	$(ap)_x$	$n - 42 (ap)_n$
42	0.000122	0.099908	0.899170	1
43	0.001150	0.099855	0.898965	0.899170
44	0.001327	0.0000	0.998673	0.808323

Calculating non-unit cashflow:

Year	unallocated premium	B/o spread	Exp.	Comm.	Interest	Annual management charges	Death cost	Maturity cost
1	7000	75	2000	1000	285.25	39.85	15.54	0
2	2000	200	750	300	80.5	149.17	9.45	0
3	750	231.25	750	300	-4.81	283.45	0	3354.37

Year	End of Year cashflow	Probability in force	Discount factor	EPV of profit	Premium signature	EPV of premiums	Profit margin
1	4384.56	1	0.884956	3083.46	1000		
2	1370.22	0.899170	0.783147	3083.46	8991.7	24287.61	12.7%
3	-3144.48	0.808323	0.693050	3083.46	8083.23		

(ii) Reserves do not reduce profits; they only defer the release of profits. If the interest earned on reserves is equal to the risk discount rate then there is no effect of deferring the profit as the opportunity cost on money held as reserves is zero. $\therefore$ the profit margin will be same as calculated earlier.

(iii) Since the mortality rates and surrender rate is the same in the valuation basis and pricing basis, we will be able to use the dependent probabilities derived in (i)

$$2V = 4532.25/1.07 = 4235.75$$

$$1V \times 1.07 - (OP)_{43} \times \frac{1}{2} = -(1760.10) \quad \therefore 1V = 4269.06$$

These need to be adjusted as the question asks for the values in respect of beginning of the year.

$$\therefore \text{Year 2} \quad 4235.75 \times (ap)_{43} = 3807.79$$

$$\text{Year 1} \quad 4269.06 \times (ap)_{42} = 3838.61$$

Q 10

(i)

Policy year	Premium allocated	Cost of allocation	Plus fund bld	Fund before charge	Fund management charge	Policy admin. fees	Fund cld
1	17500	15750	15750	16852.50	84.26	500	16268.24
2	27500	24750	41018.24	43889.52	219.45	500	43170.07
3	27500	24750	67920.07	72674.47	363.37	500	71811.10
4	27500	24750	96561	103320.38	516.6	500	102303.78

Expected ~~pro~~ profit by projecting the non unit cashflows are:

Policy year	Profit on allocation	expenses	Non unit interest	Fund mang. charge	Policy admin. fees	Non unit death cost	Profit in each yr.
1	9250	3500	402.5	84.26	500	730.98	6005.78
2	250	1100	-59.5	219.45	500	327.45	-517.5
3	250	1100	-59.5	363.37	500	0	-46.13
4	250	1100	-59.5	516.6	500	0	107.1

(ii) The non-unit reserves are:

$$\text{start of year 3} : 46.13 / 1.07 = 43.11$$

$$\text{End of year 2} : 43.11 \times 0.985 + 517.5 = 559.96$$

$$\text{Start of year 2} : 559.96 / 1.07 = 523.33$$

$$\text{End of year 1} : 523.33 \times 0.985 = 515.48$$

$\therefore$ Revised profits are:

Policy year	Non unit reserve bld	Int. on reserves	Increase in reserves	Profit ignoring reserves	Profit allowing for reserves
1	0	0	515.48	6005.78	5490.3
2	523.33	36.63	-480.87	-517.5	0
3	43.11	3.02	-43.11	-46.13	0
4	0	0	0	107.1	107.1