Financial Mathematics

Speed PAGE NO.: DATE: / /

Assignment 2

Q1. $20{,}000 = 12 \times 427.96\, a_{\overline{5}|}^{(12)}$ @ i.

$\cancel{\frac{20{,}000}{12 \times 427}}$ $\frac{20{,}000}{12 \times 427} = \frac{1-(1+i)^{-5}}{i^{(12)}}$

$3.894991042 = \frac{1-(1+i)^{-5}}{12\left((1+i)^{1/12}-1\right)}$

$i = 10.8\%$

Flat rate $= \frac{\text{Total interest}}{\text{Original loan} \times \text{Term of loan}}$

$= \frac{\text{Total repayment} - 20{,}000}{20{,}000 \times 5}$

$= \frac{25{,}674 - 20{,}000}{1{,}00{,}000}$

$= \frac{5674 \times 100}{1{,}00{,}000}$

Flat rate $= 5.674\%$

APR $= 2 \times$ Flat rate

$= 2 \times 5.674$

$= 11.3\%$

New loan

$20{,}000\,(1.108) = 12 \times \cancel{427}\ 274.49\, a_{\overline{x}|}^{(12)}$ @ $i = 10.8\%$

$\frac{20{,}000\,(1.108)}{274.49 \times 12} = \frac{1-(1.108)^{-x}}{12\left((1.108)^{1/12}-1\right)}$

$6.727628207 = \frac{1-(1.108)^{-x}}{0.102996083}$

$0 = 1 - (1.108)^{-x} - 0.692919356$

$x = 11.513$

Time for loan = 11.5 years

Interest on original loan = 5674.

Restructured loan = $11.5 \times 274.49 \times 12 - 20{,}000(1.08)$

$= 15719.62$

More interest = ₹1,00,45-62.

Q2. (i) Discounted Payback Period

The discounted payback period is a capital budgeting procedure used to determine the profitability of a project. A DPP gives the no. of years it takes to break even from undertaking the initial expenditure by discounting future cashflows and recognising the time value of money.

Payback Period

The payback period is the time you need to recover the cost of your investment ~~in simple terms~~. In simple terms, it is the time an investment takes to reach the break even point.

(ii) The net present value method evaluates a capital project in terms of its financial returns over a specific ~~p~~ time period, whereas the payback method is concerned with the time that will elapse a project repays the complete initial investment, unlike the NPV.

(iii) Project A

0	1	2	3
−1,70,000	+20,000 −20,000	+20,000 −10,000	+2,00,000

→ NPV @ $i = 6\%$

$$NPV_A = -1,70,000 + 10,000\, v^2 + 2,00,000\, v^3$$
$$= ₹\ 6823.821$$

→ For IRR, $NPV_i = 0$

$$-1{,}70{,}000 + 10{,}000\,v^2 + 2{,}00{,}000\,v^3 = 0$$

$$-17 + (1+i)^{-2} + 20(1+i)^{-3} = 0$$

$$i = 7.4\%$$

$$\Rightarrow IRR_A = 7.4\%$$

<u>Project B</u>

Time	0	1	2	3	4	5	6
Cash flow	−2,00,000	14,000	14,000	14,000	14,000	14,000	14,000+2,00,000

→ NPV

$$NPV_B = -2{,}00{,}000 + 14000\,a_{\overline{6|}} + 2{,}00{,}000\,v^6$$
$$= 9834.308.$$

→ For IRR_B

NPV=0

$$-2{,}00{,}000 + 14{,}000\left(\frac{1-v^6}{i}\right) + 2{,}00{,}000\,v^6 = 0$$

$$-200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200\,v^6 = 0$$

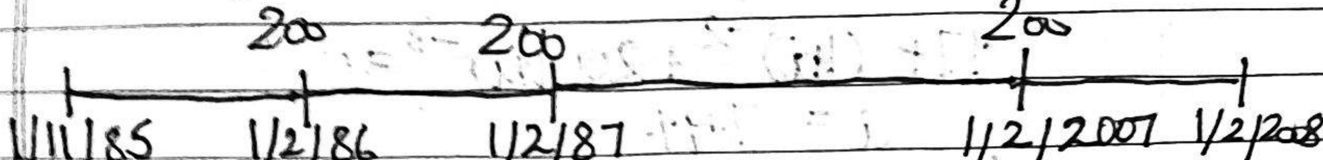
$$-200i + 14 - 14(1+i)^{-6} + 200(1+i)^{-6} = 0$$

$$i = 7\%$$

$$IRR_B = 7\%$$

⇒ Maximum interest rate, the bank can charge is upto the IRR of both the projects.

Q3. **Annuity 1**



$$PV = 200\, \ddot{a}_{\overline{22|}} @ 6\% \times v^{0.25}$$

$$= 200\, a_{\overline{22|}} \times \frac{i}{d} \times (1.06)^{0.25}$$

$$= 2515.8893$$

Annuity 2

$$PV = \left(320\, \ddot{a}^{(4)}_{\overline{16|}} + 80(1.06)^{-16} \right) v^{0.25}$$

$$= 320 \left(a_{\overline{16|}} \times \frac{i}{d^{(4)}} + 80(1.06)^{-16} \right) (1.06)^{-0.25}$$

$$PV = 3336.8$$

Annuity 3

$$PV = \frac{180}{12}\, \ddot{a}_{\overline{225|}} @ \frac{i^{(12)}}{12}$$

$$= \frac{180}{12} \left(\frac{1-(1.004867583)^{-225}}{0.004844005} \right)$$

$$= 2058.130188$$

Let X be the new installment

$$X\,\ddot{a}^{(2)}_{\overline{22}|}\ @\ 6\% = 2515.8893 + 3336.8 + 2058.130188$$

$$X a_{\overline{22}|} \times \frac{i}{d^{(2)}} = 7910.81948$$

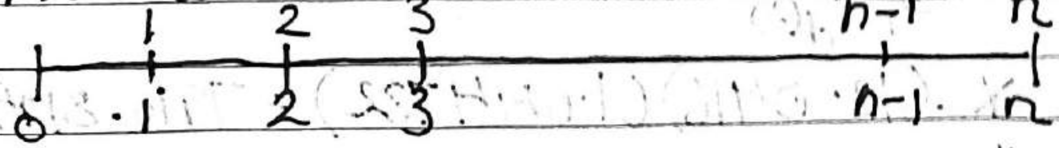
$$X\,(12.0416)(1.044782) = 7910.81948$$

$$\Rightarrow X = 628.7986431$$

Q4. Derive from 1^{st} Principle

$$I\ddot{a}_{\overline{n}|} = ?$$

Arrears



$$PV_0 = v + 2v^2 + 3v^3 + \cdots + nv^n \quad -①$$

$$(1+i)(Ia_{\overline{n}|}) = 1 + 2v + 3v^2 + \cdots + nv^{n-1} \quad -②$$

Subtracting ① from ②

$$i(Ia_{\overline{n}|}) = 1 + v + v^2 + v^3 + \cdots + v^{n-1} - nv^n$$

$$= \ddot{a}_{\overline{n}|} - nv^n$$

$$Ia_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$(1+i)(Ia_{\overline{n}|}) = (1+i) \times \frac{(\ddot{a}_{\overline{n}|} - nv^n)}{i}$$

$$I\ddot{a}_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

PAGE NO.:
DATE: / /

Q5. (i) TWRR: Property fund

$$(1+i) = \frac{13.1}{12.4} \times \frac{14.8}{13.1} \times \frac{15.8}{14.8} \times \frac{16.4}{15.8}$$

$$i = \frac{41}{31} - 1$$

$$i = 32.26\%$$

→ TWRR: Equity fund.

$$(1+i) = \frac{\cancel{9.2}}{12.1} \times \frac{\cancel{10.3}}{\cancel{9.2}} \times \frac{\cancel{13.1}}{\cancel{10.3}} \times \frac{15.5}{\cancel{13.1}}$$

$$= \frac{15.5}{12.1}$$

$$i = 128\%$$

PAGE NO.:
DATE: / /

Q6. (i) Let the annual repayment $= X$

$$1,60,000 = X\, a_{\overline{10}|} @ 8\%$$

$$X = \frac{1,60,000}{a_{\overline{10}|} @ 8\%}$$

$$X = 23,844.65209$$

(ii) Loan outstanding after 4^{th} installment $= PV_4$

$$PV_4 = X\, a_{\overline{6}|} @ 8\%$$

$$= 1,10,231.4422$$

Now i is increased to 10%.
Let annual repayment $= Y$

$$1,10,231.4422 = Y a_{\overline{6}|} @ 10\%$$

$$\cancel{Y = 25,309.72428}$$

$$Y = 25,309.72428$$

(iii) Loan outstanding after 7^{th} installment $= PV_7$

$$PV_7 = Y a_{\overline{3}|} @ 10\%$$

$$= 62,942.75331$$

Now i is reduced to 9%.
Let annual repayment $= Z$.

$$62,942.75331 = Z a_{\overline{3}|} @ 9\%$$

$$Z = 24,865.78174$$

Total amount repaid = $4X + 3Y + 3Z$

$= 2,45,905.1264$

Let, effective rate of interest = I.

$$(1+I)^{10} = \frac{2,45,905.1264}{1,60,000}$$

$I = 4.3914\%$.

PAGE NO.:
DATE: / /

Q7. (i) Disadvantages of investment in property

(a.) **Low liquidity**: Government securities are highly liquid and can be bought and sold for a profit in a fraction of a second. Real investments are comparably illiquid because properties can't be quickly and easily sold without a substantial loss in value.

(b) **Requires Management & Maintenance**
Financing payments, real estate taxes, insurance, management fees and maintenance costs can add up quickly, especially if the property sits empty for extended periods of time.

(C) **Greater liabilities**
Real estate investing involves taking on a great deal of financial and legal liability. All the disadvantages mentioned above add to the liability a real estate investor takes on when purchasing, financing, rehabbing leasing, managing and maintaining a property.

(ii) (a) Tender usually refers to the process whereby governments ~~and fin~~ invite bids for large projects that must be submitted within a finite deadline. A tender loan is a loan issued to a bidder under special conditions. The type of loan is used to ensure the fulfillment of obligations based on the results of the tender.

PAGE NO.:
DATE: / /

(b) Advantages of Bond Tender offers

1. Decreasing the cost of capital

The interest payments or coupon payments made to the bondholders represent a cost to the debtor company. For companies with a highly leveraged capital structure, it is favourable to eliminate or decrease the cost of repurchasing debt.

2. Retiring existing bonds at less than face value. On the open market, many debt securities trade below their face value, thus ~~not~~ making the repurchasing of a ~~bo~~ debt attractive to a company. In the case of a bond tender offer, the company offers to buy the bonds above their market value.

3. Does not affect the company's liquidity
Any company can either repurchase bonds in exchange for either cash or issue ~~to~~ a new security to the bondholders. Thus companies with access to capital can use their retained earnings to make an offer.

Disadvantage: Tender offers for buyback of any security can be made without the consent of members of a company's board of directors. Such offers can represent the threat of a hostile ~~takeoff~~ takeover.

PAGE NO.:
DATE: / /

C. 3 securities which have uncertain income:

1. Stocks / equity: The returns on equity / shares are variable almost everyday as the markets rise and fall continuously. The investment income from stocks is very volatile.

2. Index linked bonds: In index linked bonds the amount is never certain because it varies with inflation and hence cannot be predicted.

3. Contingent annuities: ~~B~~ In contigent annuities payments are made ~~after~~ on a contigency such as survival or death. Hence payment is dependant on the happening of the event.

Q9. (i) TWRR

$$(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{39} \times \frac{37}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{43.1}$$

$$i = 7.53285\%$$

MWRR

$$30.5(1+i)^3 + 6(1+i)^{26/12} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$$

$$i = 7.23\%$$

(ii) LIRR

$$\cancel{(1+i)^3 = \left(\frac{33-30.5}{30.5}\right) \times \left(\frac{39-38.5}{38.5}\right)}$$

$$(1+i)^3 = \left(\frac{33-30.5}{30.5}\right) \times \left(\frac{38.5-39}{39}\right) \times \left(\frac{37-38.5}{38.5}\right) \times \left(\frac{45-41.05}{41.05}\right) \times \left(\frac{45.6-45}{45}\right) \times \left(\frac{47-43.1}{43.1}\right)$$

$$= \frac{2.5 \times (-0.5) \times (-1.5) \times 3.95 \times 0.6 \times 3.9}{30.5 \times 39 \times 38.5 \times 41.05 \times 45 \times 43.1}$$

$$= 0.$$

(iii) MWRR is sensitive to amounts and timing of the net cashflows. If, we are assessing the skill of the fund manager, this is not ideal, as the fund manager does not control the timing or amount of the cash flows, he or she is merely responsible for investing the positive net cash flows and realising cash to meet the negative net cash flows. A measure which tries to eliminate this effect is the TWRR.

(ii) LIRR, using sub intervals of one year.

$$(1+i)^3 = \left(1+\frac{38.5-30.5}{30.5}\right)\left(1+\frac{45-38.5}{38.5}\right)\left(1+\frac{47-45}{45}\right)$$

$$(1+i)^3 = (1.262295082)(1.68831169)(1.044444444)$$

LIRR = 30.567%

PAGE NO.:
DATE: / /

Q10. (i) $P = 2,00,000$

$Q = 20,000$

$P - Q = 1,80,000$

Let loan amount $= X$

~~$X = 1,80,000\, a_{\overline{15|}} + 2000\, Ia_{\overline{15|}}$ @ 12%~~

$\therefore X = 1,80,000\, a_{\overline{15|}} + 20,000\, Ia_{\overline{15|}}$ @ 12%

$= (1,80,000 \times 6.8109) + (20,000 \times 40.731)$

~~X ... $8,14,620$~~

$X = ₹\ 20,40,582$

Let ~~Amount of~~ cost of house $= C$

$0.8(C) = 20,40,582$

$C = ₹\ 25,50,727.5$

(ii) 9^{th} installment $= 2,00,000 + (20,000 \times 8)$

$= 3,60,000$

Loan outstanding after 8^{th} installment

$PV_8 = 3,40,000\, a_{\overline{7|}} + 20,000\, Ia_{\overline{7|}}$

$= (3,40,000 \times 4.5638) + (20,000 \times 16.208)$

$PV_8 = 18,75,852$

PAGE NO.:
DATE: / /

Loan Schedule

Year.	Loan at Start	Installment	Interest	Capital	Loan outstanding
9.	18,75,852	3,60,000	2,25,102.24	1,34,897.76	17,40,954.24
10.	17,40,954.24	3,80,000	2,08,914.5088	1,71,085.4912	15,69,868.749

(iii) 11^{th} installment $= 2,00,000 + (20,000 \times 10)$
$= 4,00,000$.

~~PV~~ Loan outstanding after 10^{th} installment

$$PV_{10} = 3,80,000\, a_{\overline{5|}} + 20,000\, Ia_{\overline{5|}}$$

$$= (3,80,000 \times 3.6048) + (20,000 \times 10.0018)$$

$$= 15,69,860.$$

New loan amount $= 15,69,860 + (1\%$ of $15,69,860)$

$$= 15,85,558.6$$

Let revised annual payment be X.

$$15,85,558.6 = X\, a_{\overline{5|}} \text{ @ } 10\%$$

$$X = 4,18,264.9045$$

→ Total amount paid in Scenario I $= 2,00,000 + 2,20,000 + \dots + 4,80,000$

$$= \frac{15}{2}(2,00,000 + 4,80,000)$$

$$= 51,00,000$$

Total amount paid in Scenario II $= 2,00,000 + 2,20,000 + \dots + 3,80,000 + 4(X)$

$$= \frac{10}{2}(2,00,000 + 3,80,000) + 4(4,18,264.9045)$$

= 45,73,059.618

Thus, it is advisable for Sam to transfer the loan to new bank as he pays less in installments i.e. total value paid by him in scenario 2 is less than that of Scenario 1

PAGE NO.:
DATE: / /

Q11. (i) TWRR = 20%

$$(1+i)^2 = \frac{500}{460} \times \frac{550}{500} \times \frac{600}{550} \times \frac{X}{600}$$

$$\frac{(1.2)^2 \times 460 \times 500 \times 550 \times 600}{500 \times 550 \times 600} = X$$

$$X = 662.4$$

(ii) $460(1+i)^2 + 40(1+i)^{21/24} - 50(1+i) = 662.4$

$$i = 21.19\%$$

(iii) $$(1+i)^2 = \left(1 + \frac{550-460}{460}\right)\left(1 + \frac{662.4-550}{550}\right)$$

$$= (1.19565217)(1.204363636)$$

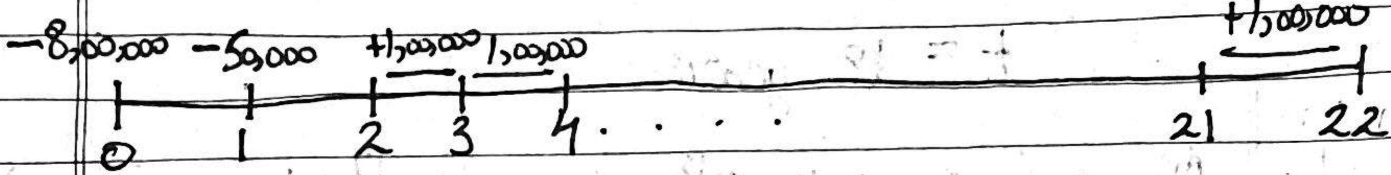
$i = 20\%$

LIRR = 20%

(iv) If the fund performance is reasonable stable in the period of assessment, then TWRR and MWRR will give similar results.

(V) TWRR ~~MWRR~~ is a better measure of investment as MWRR is sensitive to amount and timing of cashflows whereas TWRR eliminates this effect.

Q12. Cashflows



(i) The Discounted Payback period (DPP) tells us how long it takes for the project to move into position of profit. In many practical problems, the net cashflow changes sign only once, this change being from negative to positive. In these circumstances the balance in the investors account will change from negative to positive at a unique time, t_1, or it will always be negative, in which case the project is not viable. t_1 is referred to as the DPP. It is the smallest value of t such that $A(t) \geq 0$, where:

$$A(t) = \sum_{s \leq t} C_s (1+j_1)^{t-s} + \int_0^t \rho(s)(1+j_1)^{t-s}\,ds$$

(ii) $i = 7\%$

Let DPP $= t$

$AV_t \geq 0$

$$-8{,}00{,}000(1+i)^t - 50{,}000(1+i)^{t-1} + 1{,}00{,}000\left(\bar{s}_{\overline{t-2}|}\right) = 0$$

$$-80(1.07)^t - \frac{5(1.07)^t}{1.07} + 10\left(\frac{(1.07)^{t-2} - 1}{\ln(1.07)}\right) = 0$$

$$-80(1.07)^t - 4.672897196(1.07)^t + 10\left(\frac{(1.07)^t - (1.07)^2}{\ln(1.07)(1.07)^2}\right) = 0$$

$$-84.6728972(1.07)^t + 129.0949122(1.07)^t - 147.800765 = 0$$

$$44.42201498(1.07)^t = 147.800765$$

$$(1.07)^t = 3.327196325$$

PAGE NO.:
DATE: / /

$t \geq 17.77$ years

$t = 18$ years

b. $AV_{22} = (-8{,}00{,}000(1.07)^{18} - 50{,}000(1.07)^{17} + 1{,}00{,}000\,\bar{S}_{\overline{16}|})$ @ 7% $\times (1.06)^4 + 1{,}00{,}000\,\bar{S}_{\overline{4}|}$ @ 6%

$= (-27{,}03{,}945.821 - 1{,}57{,}940.7605 + 28{,}85{,}312.953) \times (1.06)^4 +$

$= 29{,}575.25488 + 4{,}50{,}458.0032$

$AV_{22} = 4{,}80{,}033.258$

(iii) DPP

$AV_t = 0$

$-8{,}00{,}000(1+i)^t - 50{,}000(1+i)^{t-1} + 1{,}00{,}000\,S_{\overline{t-2}|} = 0$

$-80(1.07)^t - \dfrac{5(1.07)^t}{1.07} + 10\left(\dfrac{(1.07)^{t-2}-1}{0.07}\right) = 0$

$-84.6728972(1.07)^t + 10\left(\dfrac{1.07^t - (1.07)^2}{0.07(1.07)^2}\right) = 0$

$-84.6728972(1.07)^t + 124.7769612(1.07)^t - 142.8571429 = 0$

$40.104064(1.07)^t = 142.8571429$

$t \geq 18.77$ year

DPP or $t = 19$ years

~~...~~ 19

$$AV_{22} = \left(-8{,}00{,}000\,(1.07)^{19} - 50{,}000\,(1.07)^{18} + 1{,}00{,}000\, S_{\overline{17}|}\right) @ 7\% \times (1.06)^3 + 1{,}00{,}000\, S_{\overline{3}|} @ 6\%$$

$$= (-28{,}93{,}222.028 - 1{,}68{,}996.6138 + 30{,}84{,}021.73) \times (1.06)^3 + 3{,}18{,}360$$

$$= 25{,}967.82684 + 3{,}18{,}360$$

$$AV_{22} = 3{,}44{,}327.827.$$

Q13. (i) Let annual repayment = X

$$9,88,000 = X \cdot a^{(12)}_{\overline{25|}} \ @ \ 7\%$$

$$= X\left(\frac{1-v^{25}}{i^{(12)}}\right)$$

$$X = 82,176.89243$$

Monthly installment = 6848.07437

(ii) Loan outstanding is given by $PV_{10/03/29}$

$\cancel{PV'_{10/03/29} = X}$

$$PV_{10/03/29} = 6848.07437 \ a_{\overline{136|}} \ @ \ \frac{i^{(12)}}{12}$$

$$\frac{i^{(12)}}{12} = 0.005654167$$

Loan outstanding = 6,48,577.7278

(iii) Loan outstanding after 10/09/2026

$$PV_{10/09/2026} = 6848.07437 \ a_{\overline{166|}} \ @ \ \frac{i^{(12)}}{12}$$

$$= 7,36,123.4937 \quad -① $$

168 Loan outstanding after 10/10/2026

$$PV_{10/10/2026} = 6848.07437 \ a_{\overline{165|}}$$

$$= 7,33,437.5845 \quad -②$$

Capital repaid on 10th october, 2026 = ① − ②

$$= 2685.909224.$$

PAGE NO.:
DATE: / /

iv) Loan outstanding after 10th April 2033,

$$PV_{10/04/33} = 6848.07437\, a_{\overline{87}|} \text{ @ } \frac{i^{(12)}}{12}$$

$$= 4,69,560.5748.$$

~~Data~~
Loan Schedule

Date	Loan ~~outstanding~~ at start	Installment	Interest	Capital repaid	Loan ~~outstanding~~ outstanding
1. 10/04/33	469560.5748	6848.07437			
2. 10/05/33		68488.07437			
3. 10/06/33		68488.07437			
4. 10/07/33		6848.07437			
5. 10/08/33		6848.07437			
6. 10/09/33		6848.07437			
7. 10/10/33		6848.07437			
8. 10/11/33		6848.07437			
9. 10/12/33		6848.07437			
10. 10/01/34		6848.07437			
11. 10/02/34		6848.07437			
12. 10/03/34		6848.07437			

0.005654167

Loan schedule

Date	Loan at start	Installment ~~6848.07437~~	Interest	Capital Repaid	Loan outstanding
10/04/33	469560.5748	6848.07437	2654.973907	4193.100463	462712.5004
10/05/33	462712.5004	6848.07437	2616.25375	4231.82062	455864.4261
10/06/33	455864.4261	6848.07437	2577.533594	4270.540776	449016.3517
10/07/33	449016.3517	6848.07437	2538.813438	4309.260932	442168.2773
10/08/33	442168.2773	6848.07437	2500.093282	4347.981088	435320.203
10/09/33	435320.203	6848.07437	2461.373126	4386.701244	428472.1286
10/10/33	428472.1286	6848.07437	2422.65297	4425.4214	421624.0542
10/11/33	421624.0542	6848.07437	2383.932814	4464.141556	414775.9798
10/12/33	414775.9798	6848.07437	2345.212658	4502.861712	407927.9055
10/01/34	407927.9055	6848.07437	2306.492501	4541.581869	401079.8311
10/02/34	401079.8311	6848.07437	2267.772345	4580.302025	394231.7567
10/03/34	394231.7567	6848.07437	2229.052189	4619.022181	387383.6824