ASSIGNMENT - 1

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

6
18
× 18

1180
+ 144

2

Mode → 18

400 − 80 + 4

$$\text{Median} \rightarrow \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2} = \frac{5^{th} + 6^{th}}{2}$$

$$= \frac{11+15}{2}$$

$$= \frac{26}{2} = 13$$

$$\text{Mean} = \frac{\Sigma x_i}{n} = \frac{67}{5} = 13.4$$

$$\sigma = \sqrt{\frac{\Sigma (x_i)^2}{n} - (\bar{X})^2} = \sqrt{217 - 179.56}$$

$$= 6.118823410$$

(2) $\lambda = 0.5$

(i) $P(X = 0) = e^{-0.5}$

$= 0.606530660.$

(ii)

Page No. :
Date :

3. $X \sim \text{Gamma}(2, \frac{1}{2})$ $\quad 0 < x < \infty$

(i) Mean,

$$E(X) = \frac{\alpha}{\lambda}$$

$$= \frac{2}{1/2}$$

$$= 4$$

→ For standard deviation,

$$V(X) = \alpha/\lambda^2$$

$$= \frac{2}{1/4}$$

$$= 8$$

$\Rightarrow \quad \sigma = \sqrt{8} = 2.828427125$

$= 2.83$

(ii) The shape of the curve of the given distribution will be right skewed.

(iii) $F_X(x) = \int_0^{\infty} f_X(x)$

Page No. :

Date :

(4) 10 male staff & 8 female staff.

$\downarrow$ $\downarrow$

6 $\rightarrow$ qualified actuaries $\leftarrow$ 7

$\rightarrow$ A team of two workers is selected randomly.

Page No.:
Date:

6. $X \sim U(1,2)$

$Y = \frac{3}{6-X} \rightarrow f_X(x) = \frac{1}{2-1} = 1$

$P_Y(y) = ?$

7.

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3) = P(X \leq 3)$

$= 0.05 + 0.15 + 0.35$

$= \boxed{0.55}$

(ii) $E(X) = \sum x \, P(X=x)$

$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$= 3.25$

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Page No. :

Date :

(iii) $V(X) = E(X^2) - [E(X)]^2$

$= (0.05)^2 \cdot (1) + (0.15)^2 (2) + (0.35)^2 (3) + \dots - (3.25)^2$

$= 0.8875$

(iv) Coefficient of skewness $= E(X_i - \bar{X})^3$

8. $P(X > 1000) = 0.02$

9. $P(A) = 0.01$

Using negative binomial distribution,

Here, $p = 0.01$

$n = 7$

$r = 3.$

$\Rightarrow$ Probability $= {}^{n-1}C_{r-1}\, p^{r} q^{n-r}$

$= {}^{6}C_{2} (0.01)^{2} (0.99)^{4}$

$= \frac{6 \times 5}{2}$

$= 0.001440894.$

10. No of claims per working week = 2.

11.) A woman has 5 boys and 2 girls

Probability that the next three children are boys $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{8}$

12) $\lambda = 10$

$F_X(0.1) = 1 - e^{-\lambda x} = 1 - e^{-10(0.1)} = 0.6321205$

$f_X(0.1) = \lambda e^{-\lambda x} = 10\,e^{-10(0.1)} = 3.678794$

13.) $a = 75$

$b = 120$

1) $f_X(x) = \dfrac{1}{120-75} = \dfrac{1}{45} = 0.02222$

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Page No. :

Date :

14) $X \sim \text{Gamma}(5, 0.4)$

$\alpha = 5, \quad \lambda = 0.4$

$P(X < 22.89)$

$$P(X = 22.89) = \frac{(0.4)^5 e^{-(5)(0.4)} x^4}{4 \times 3 \times 2 \times 1}$$

$$= 0.022413627 \; x^4.$$

$$= 6153.1185952$$

$$\Rightarrow F_X(X) = \int_0^{22.89} 0.022413627 \frac{x^5}{5}$$

$$= 28168.97856$$

15) $f_X(x) = \frac{k}{(x+5)^4}, \quad x > 0$

$$F_X(x) = \int_0^{\infty} f_X(x)\, dx$$

$$= \int_0^{\infty} \frac{k}{(x+5)^4}\, dx$$

$$= k \int_0^{\infty} (x+5)^{-4}\, dx$$