FE-1 Assignment 2

1)(i) given :- $S_0 = 65$, $\sigma = 0.25$, $r = 0.02$, $X = 55$, $T = 0.5$

$$C_t = S_0\,\Phi(d_1) - X e^{-rT}\,\Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad ; \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(0.02 + \frac{1}{2}(0.25)^2\right)0.5}{0.25\sqrt{0.5}} = 1.089957499$$

$$d_2 = \cancel{d_1}\ 1.089957499 - 0.25\sqrt{0.5} = 0.9131808036.$$

$$C_t = 65 \times \Phi(1.09) - 55 \times e^{-0.02 \times 0.5} \times \Phi(0.91)$$

$$= 65 \times 0.86214 - 55 \times e^{-0.02 \times 0.5} \times 0.81859$$

= £11.4646 is the price of the call option.

(ii) Delta of ~~an~~ call option is the change in the price of call ~~der~~ option with respect to change in the price of underlying

= Change in price of call / Change in price of underlying.

(iii) $\Delta = \Phi(d_1) = 0.86214$

(iv) $$\frac{\partial P}{\partial S} = -\Phi(-d_1) = -\Phi(-1.09) \Rightarrow -[1 - \Phi(1.09)]$$

$$\Rightarrow -[1 - 0.86214] = \boxed{-0.13786}$$

2)(i) Delta is the change in price of an option ~~derivative~~ with respect to change in price of underlying. Vega is the change in price of an option with respect to the volatility ~~volume~~ of the underlying.

(ii) Put - call parity :- $C - P = S - Ke^{-rT}$

diff. w.r.t σ :- $V_c - V_p = \frac{\partial}{\partial \sigma}\left(S - Ke^{-rT}\right)$

the underlying S and present value K are independent of volatility

RHS will become 0

$\therefore\ V_c - V_p = 0 \Rightarrow V_c = V_p$

$\therefore$ Hence shown vega for call & put with same strike price & time to maturity is same.

(iii) given :- $S_0 = 55$, $X = 50$, $\sigma = 0.25$, $r = 0.05$, ~~cont comp~~

$T = 1$

$$c_t = S_0\,\Phi(d_1) - Xe^{-rt}\,\Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{55}{50}\right) + \left(0.05 + \frac{1}{2}\times 0.25^2\right)1}{0.25\sqrt{1}} = 0.7062407192$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.7062407192 - 0.25\sqrt{1} = 0.4562407192$$

$$c_t = 55\times\Phi(0.71) - 50\times e^{-0.05\times 1}\times\Phi(0.46) = \$9.652\cancel{8}5$$

$$P_t = Xe^{-rT}\,\Phi(-d_2) - S_0\,\Phi(-d_1)$$

$$= 50\times e^{-0.05\times 1}\times\Phi(-0.46) - 55\times\Phi(-0.71)$$

$$= \$2.2140$$

(iv) A portfolio with overall delta equal to 0 is said to be delta-hedged. A portfolio with overall vega equal to 0 is said to be vega-hedged.

(v) no of call options $= a$, no of put options $= b$, no of forwards $= c$

delta of forward $= 1$, vega ~~delta~~ of forward $= 0$.

$P_c = 9.65\cancel{8}27$, $P_p = 2.2141$, $P_f = 0$.

(1) delta neutral : $\Delta_c \cdot a + \cancel{\Delta}\,\Delta_p \cdot b + c = 0$.

$\Delta_p = \Delta_c - 1$ & $a = -b$. so

$\Delta_c \cdot a + (\Delta_c - 1)(-a) + c = 0 \Rightarrow a\Delta_c - a\Delta_c + a + c = c \Rightarrow \boxed{c = -a}$

(2) Vega neutral : $V_c \cdot a + V_p \cdot b = 0$. $V_c = V_p$

so $V_c \cdot (a + b) = 0$

$a + b = 0 \Rightarrow \boxed{a = -b}$ $\Rightarrow b = c$

(3) price : $a \times 9.652\cancel{4}5 + b \times 2.214\cancel{0}3 = 1000$

$a \times 9.652\cancel{3}5 - 2.2140a = 1000$

$a = 134.433$

$b = c = -134.433$.

Portfolio: long 134.433 call options

Short 134.433 put options

Short 134.433 1 year forwards on underlying

3)(i) $C_t = E\left[e^{-r(T-t)} \cdot C_T \mid F_t\right]$

$F_t \Rightarrow$ filtration at time t

$C_T \Rightarrow$ payoff of derivative at time T

$C_t \Rightarrow$ derivative value at time t.

(ii) given:- $S_0 = 50$, $X = 49$, $r = \cancel{5\%}\ 0.05$, $\sigma = 0.25$, $T = 0.5$

$$C_t = S_0\, \Phi(d_1) - Xe^{-rT}\, \Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad ; \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln\left(\frac{50}{49}\right) + \left(0.05 + \frac{1}{2}\times 0.25^2\right)0.5}{0.25\sqrt{0.5}} = 0.3440934746$$

$$d_2 = 0.3440934746 - 0.25\sqrt{0.5} = 0.1673167793$$

$$C_t = 50 \times \Phi(0.344) - 49\times e^{-0.05\times 0.5}\,\Phi(0.167)$$

$$= 50\times 0.6346 - 49\times e^{-0.05\times 0.5} \times 0.5664 = \$4.6616$$

(iii) price of american call will be same as price of european call so $4.6616

(iv) $C - P = S - Ke^{-rT}$ $\Rightarrow$ Put-call parity

$$4.6616 - P = 50 - 49\times e^{-0.05\times 0.5}$$

$$P = 4.6616 + 49\times e^{-0.05\times 0.5} - 50$$

$$P = \$2.4518$$

(v) Dividends will cause the price of shares to fall. A holder of call will not have the share in hand as he/she decides to purchase the share in future at predetermined price & as this income is not realised due to holding call option, its price decreases. American call will still have more price than european call because we can exercise it any time in the tenure. In put, on the other hand, holders decides to sell the stock at fixed price & so ~~alrea~~ already holds the asset & so the holder gets the dividends & for that, put's price ~~increas~~ increases

4) (i) $Z_t \Rightarrow$ SBM under P & ~~$\tilde{Z}_t$~~ $\to$ $\gamma_t \to$ previsible process

there exists a Q equivalent to P, $\tilde{Z}_t = Z_t + \int_0^t \gamma_t \, ds$ is a SBM under Q. If Z_t is SBM under P & Q is equivalent to P then there exists a previsible process γ_t such that.

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds \text{ is a Brownian motion under } Q.$$

(ii) the discounted value of asset prices are martingales.

5) (i) delta $= \Phi(d_1) = \dfrac{\text{~~price of d~~ change in price of derivative}}{\text{change in price of underlying}}$

(ii) given :- $S_0 = 40$, $r = 0.02$, $K = 45.91$, $T = 5$.

$\Phi(d_1) = 0.6179. = \Delta$

from table ~~d_1~~

~~0.3 0.61791~~

~~0.31 0.62172.~~ $d_1 \approx 0.3$

$$0.3 = \frac{\ln\left(\frac{40}{45.91}\right) + \left(0.02 + \frac{1}{2}\sigma^2\right)5}{\sigma\sqrt{5}}$$

$$\left(\frac{3\sqrt{5}}{10}\right)\sigma = \ln\left(\frac{40}{45.91}\right) + 0.02 \times 5 + \frac{5}{2}\sigma^2$$

$$0 = \frac{5}{2}\sigma^2 - \left(\frac{3\sqrt{5}}{10}\right)\sigma + \left[\ln\left(\frac{40}{45.91}\right) + 0.1\right]$$

σ = between 31.5625%. & 31.625%.

$\boxed{\sigma = 32\%.}$

(iii) $ce^{-rT} Q(S_1/S_0 < K_S)\, Q(R_1/R_0 < K_R)$

(iv) perfect coorelation means $\dfrac{R_1}{R_0} = \dfrac{S_1}{S_0}$

$$\text{Price} \begin{cases} ce^{-rT} Q(S_1/S_0 < K_S) & K_S < K_R \\ ce^{-rT} Q(R_1/R_0 < K_R) & K_R < K_S \end{cases}$$

if $K_S = K_R \Rightarrow$ price $= ce^{-rT} Q(R_1/R_0 < K_S) =$
$ce^{-rT} Q(S_1/S_0 < K_S)$

So price $= ce^{-rT} \cdot Q(R_1/R_0 < \min(K_S, K_R))$
$= ce^{-rT} \cdot Q(S_1/S_0 < \min(K_S, K_R))$.

(v) price $= c \cdot e^{-rT} \cdot Q(S_1/S_0 < K_S) \cdot Q(R_1/R_0 < K_R)$

$= 50 \times e^{-0.02 \times 1} \cdot Q(S_1 < 0.8 \times 40) \cdot Q(R_1 < 0.6 \times 30)$

$= 50e^{-0.02} \cdot Q(S_1 < 32) \cdot Q(R_1 < 18)$

$= 50e^{-0.02} \cdot Q(\ln(S_1) < \ln(32)) \cdot Q(\ln(R_1) < \ln(18))$

~~$\ln(S_1) \sim N(\ln 40 + (0.02 - 0.5 \times 0.32^2)1, 0.32^2)$~~

~~$\ln(R_1) \sim N(\ln 30 + (0.02 - 0.5 \times 0.15)1, 0.15)$~~

~~$= 50e^{-0.02} \cdot Q(Z < \ln(32) - \ln(40) + 0.02 - 0.05$~~

$= 50e^{-0.02} \left(1 - \Phi\left(\log(1/0.8) + 0.02 - 0.032^2 \times 0.5 / 0.32\right)\right) \cdot$

$\left(1 - \Phi\left(\log(1/0.6) + 0.02 - 0.05 \times 0.15) / \sqrt{0.15}\right)\right)$.

$= 50e^{-0.02} \times (1 - 0.7257)(1 - 0.88039$

$= \$1.61$

6)(i)(a) $\Delta = -\Phi(-d_1) = \Phi(d_1) - 1$

(b) Δ of option $= \Delta$ m.

Δ of a share will be 1 cause ~~\$~~ change in share / change in share = 1

we ~~to~~ delta hedge using shares so overall delta = 0.

so $100\,000\,\Delta + 24830 = 0$.

$\Delta = \dfrac{-24830}{100{,}000} = -0.2483\text{m}$.

(ii) $\Phi(-d_1) = 0.2483 \Rightarrow 1 - \Phi(d_1) = 0.2483$

$\Phi(d_1) = 0.7517 \Rightarrow d_1 = 0.68$

$$0.68 = \frac{\ln\left(\frac{0.64}{0.63}\right) + \left(0.03 + \frac{1}{2} \times \sigma^2\right)1}{\sigma}$$

$$0 = \frac{1}{2}\sigma^2 - 0.68\sigma + \left[\ln\left(\frac{0.64}{0.63}\right) + 0.03\right]$$

$$\sigma = \frac{0.68 \pm \sqrt{0.68^2 - 4 \times 0.5 \times \left(\ln\left(\frac{0.64}{0.63}\right) + 0.03\right)}}{2 \times 0.5}$$

$\sigma = 0.670981 \approx 67.1\%$.

(iii)(a) $Xe^{-rT}\,\Phi(-d_2) - S_0[\Phi(-d_1)] \Rightarrow 0.2493.$

$\Rightarrow \cancel{0.2483 - 0.071}$

$= \cancel{0.1773}$

$0.63 \times e^{-0.03} \times (1 - \Phi(\overset{0.609}{\cancel{0.1773}})) - 0.64 \times (\overset{0.2483}{\cancel{1-0.76}}$

$= 6.894\,p$

(b) total cash holding

$= [24830 \times 0.64] + [100000 \times 0.06894]$

$= £\ 165806$

7)(i) delta is the change in price of derivative with respect to change in price of underlying.

gamma is the change in value of delta with respect to change in value of underlying.

Vega is the change in price of derivative with respect to change in volatility of the underlying.

(ii) delta $= \Phi(d_1)$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad \cancel{\ln\left(\frac{60}{50}\right) + \left(0.03 + \frac{1}{2}(0.25)^2\right)}$$

$$= \frac{\ln\left(\frac{60}{50}\right) + \left(0.03 + \frac{1}{2}(0.25)^2\right)3}{0.25\sqrt{3}} = 0.8454060474$$

delta $= \Phi(0.8454) = 0.801$

(iii) 0.801 shares & cash = 17.91 − 0.801 * 60 = \$30.15 short in cash.

(iv) vega = $\frac{d(\text{price of derivative})}{d(\text{volatility})}$ = \$29 × 0.02 = 0.58

the change will change price by 0.58.

option price = 17.91 + 0.58 = \$18.59

8) (i) delta is the change in price of derivative with respect to change in price of underlying.

(ii) $\Phi(d_1) = 0.42074$.

$$d_1 = -0.\cancel{184}\,2 = \frac{\ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{1}{2}(\sigma)^2\right)1}{\sigma\sqrt{1}}$$

$$0 = \frac{1}{2}\sigma^2 + 0.1\cancel{84}2\,\sigma + \left[\ln\left(\frac{100}{109.42}\right) + 0.03\right]$$

$$\sigma = \frac{-0.\cancel{184}2 \pm \sqrt{(0.\cancel{184}2)^2 - 4 \times 0.5 \times \left[\ln(100/109.42) + 0.03\right]}}{2 \times 0.5}$$

$$= \frac{-0.\cancel{184}2 \pm 0.3923047352}{} = 0.2000587523 \quad \cancel{0.2083\ldots}$$

$$\boxed{\sigma = 20\cancel{24}\%}$$

9) (i) $\frac{dg}{dt} + (r-q)\cdot S_t \cdot \frac{dg}{dS_t} + \frac{1}{2}\sigma^2 S_t^2 \frac{d^2g}{dS_t^2} = rg.$

$(r-q)$: r continuously compounded risk free rate; q continuously compounded dividend rate

boundary Condition ⇒ $g(T,x) = f(x)$

(ii) $g(t,x) = (x^n / S_0^{\,n-1})\, e^{u(T-t)}$

$$x g_x = ng, \quad x^2 g_{xx} = n(n-1)g, \quad g_t = -ug$$

we will need $u = \frac{1}{2}\sigma^2 n(n-1) + (n-1)r - nq$

$$g(T,x) \Rightarrow \cancel{u}\, -\frac{1}{2}\sigma^2 n(n-1) + (n-1)r - nq + (r-q)\cdot S_t \cdot \frac{ng}{x} + \frac{1}{2}\sigma^2 S_T^2 \cdot \frac{n(n-1)g}{x^2} \quad \cancel{= g(T,x)}$$

$$\boxed{g(T,x) = S_T^{\,n} / S_0^{\,n-1}}$$

10)(i) Consider portfolio with long one call plus cash of $Ke^{-r(T-t)}$ & short one put. the portfolio has a payoff at the time of expiry of S_T. Since this is the value of the stock at time T, the stock price should be the value at any time $t < T$, i.e. $C_t + Ke^{-r(T-t)} - P_t = S_t$.

$\therefore C_t - P_t = S_t - Ke^{-r(T-t)}$ Hence proven.

(ii)

given :- $S_0 = 110$, $X = 120$, $T = 1$, price $= 10.09$,

$r = 0.02$, $\sigma = ?$

$$C_t = S_0 \Phi(d_1) - Xe^{-rT} \Phi(d_2).$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad ; \quad d_2 = d_1 - \sigma\sqrt{T}$$

~~(1) lets take σ = 25%~~

~~$d_1 = -0.2556$, $d_2 = -0.5055$, $C_t =$~~

(1) lets take $\sigma = 33\%$

0.7852

~~$d_1 = 0.05$, $d_2 = -0.46$, C_t~~

$d_1 = -0.04$, $d_2 = -0.37$

$\Phi(d_1) = 0.484$, $\Phi(d_2) = 0.3557$

$C_t = 11.40738$

(2) lets take $\sigma = 28\%$

$d_1 = -0.0933$, $d_2 = -0.3793$

$\Phi(d_1) = 0.46044$, $\Phi(d_2) = 0.3522$

$C_t = 9.2185$

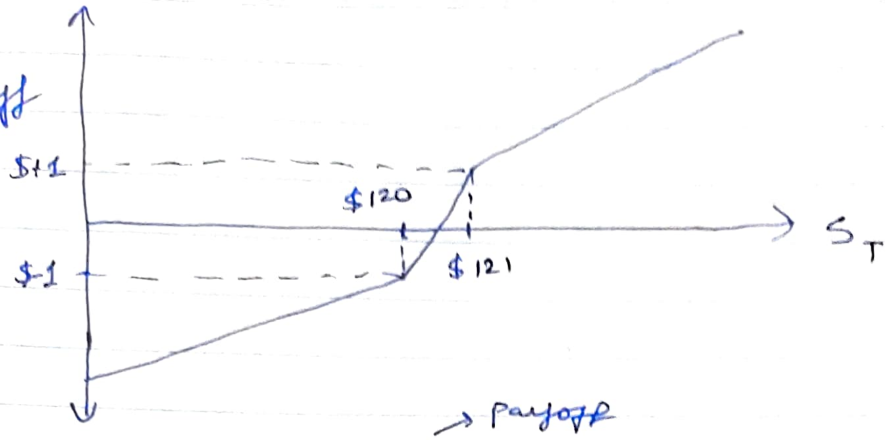
$\sigma = 28\%$, $C_t = 9.2185$

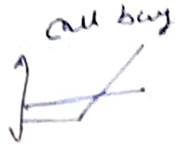
$\sigma = 33\%$, $C_t = 11.40738$

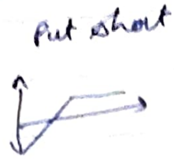
by linear interpolation :- $\frac{X - X_1}{X_2 - X_1} = \frac{Y - Y_1}{Y_2 - Y_1} \Rightarrow \frac{10.09 - 9.2185}{11.40738 - 9.2185} = \frac{Y - 0.28}{0.05}$

$Y = 0.299907.. \approx \boxed{30\%} = \sigma$

(iii)







(iv) (a) $S_1 - 121 \le \underset{\text{payoff}}{D} \le S_1 - 120$

$\Rightarrow S_0 - 121e^{-rt} \le V \le S_0 - 120$ initial price.

$-8.604 \le V \le -7.624$,

(b) $17.714 \le P_0 \le 18.694$.

(v) $S_0 = 110$, $X = 121$, $T = 1$, $\sigma = 30\%$, $r = 2\%$.

$$P_t = Xe^{-rt}\,\Phi(-d_2) - S_0\,\Phi(-d_1)$$

$$= 121 \times e^{-0.02} \times 0.5402 - 110 \times 0.6558$$

$$= \$18.354$$

11) i) ~~given: $X = \overset{150}{50}$, $r = 0.02$, $S_0 = 117.98$~~

so $100\,000\,\Delta_c \cancel{+} - 18673 = 0.$

$\Delta_c = 18673 / 100\,000 = 0.18673$

(ii) $d_1 = \cancel{0.574064}\ -0.89 \ \cancel{\delta} = \dfrac{\ln\left(\frac{117.98}{150}\right) + \left(0.02 + \frac{1}{2}\sigma^2\right)1}{\sigma}$

$$0 = \frac{1}{2}\sigma^2 + \overset{-0.89}{\cancel{0.574064}}\,\sigma + \left[\ln\left(\frac{117.98}{150}\right) + 0.02\right]$$

$$\sigma = \underset{-0.89}{\cancel{0.574064}} \pm \sqrt{(\underset{+0.89}{\cancel{0.574064}})^2 - 4 \times 0.5 \times \left[\ln\left(\frac{117.98}{150}\right) + 0.02\right]}$$

$$\sigma = \cancel{0.574074} \pm \cancel{0.87733766735}\ 1.11010826$$

$$\sigma = \cancel{1.45144 \approx 145.144\%}\ 22.0108\%$$

(iii) ~~$C - S_0 + Ke^{-rT} = P$~~

$$d_2 = -0.89 - 0.220108 = -1.110108261$$

$$P_t = 150 \times e^{-0.02} \times \Phi(+1.11) - \cancel{58}\,117.98 \times \Phi(0.89)$$

$$= \$31.4556$$

(iv) $\gamma_c = \gamma_p$, ~~$\Delta_c - \Delta_p = 1$~~ $\Delta_p = \Delta_c - 1$

$\Delta_p = -0.81327$, $\gamma_p = 0.01033m$

$-0.81327 \times x + \underbrace{y}_{\text{no of stock}} = 0.$

x: no of puts.

$\gamma_p = 0.01033m$

$x \times 0.81327 = y$

cash m

???

12)(i) volatility is constant, direction of stock can't be predicted & is completely random, returns on risky asset are normally distributed, no dividends on stock, risk free rate is constant & known in advance, zero transaction costs, all markets ~~are~~ are perfectly liquid, no arbitrage principle, unlimited buying & selling.

(ii)

$$P_t = Xe^{-rt}\,\Phi(-d_2) - S_0\,\Phi(-d_1).$$

$$d_1 = \frac{\ln\left(\frac{8}{9}\right) + \left(0.02 + \frac{1}{2}(0.2)^2\right)0.25}{0.2\sqrt{0.25}} = -1.0778.$$

$$d_2 = -1.0778 - 0.2\sqrt{0.25} = -1.1778.$$

$$P_t = 9e^{-0.02\times 0.25}\,\Phi(1.1778) - 8\,\Phi(1.0778).$$

$$P_t = \$1.0100$$

(iii) Put option premium falls when interest rate rises. Buying a put has similar benefit as shorting cash, which benefits from share price decline. With inc in interest rate, shorting stocks become more profitable than buying puts as former generates income & latter does the opposite. Hence, put option prices ~~are~~ are impacted negatively by increasing interest rates.