Deepika Sharewala (73)

NMA Assignment

1\. observed radius = 12.65 cm
Actual radius = 12.5 cm

observed area = πr^2
$A_0 = \pi \times (12.65)^2$
$= 507.725\ cm^2$

Actual area = πr^2
$A_1 = \pi \times (12.5)^2$
$= 490.875\ cm^2$

i Absolute ~~at~~ error = observed value - actual value
$= 507.725 - 490.874$
$= 11.851\ cm^2$

ii Relative error = absolute error / actual value
$= 11.851 / 490.875$
$= 0.02414$

iii Percentage error = Relative error x 100
$= 2.414\%$

FOR EDUCATIONAL USE

2. i

$x_1 = 4$ $\quad$ $y_1 = 0.60206$

$x = 5$ $\quad$ $y = ?$

$x_2 = 6$ $\quad$ $y_2 = 0.7781513$

$$\therefore \; y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{(x_2 - x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\therefore \; y = 0.6901$$

ii

$x_1 = 4.5$ $\quad$ $y_1 = 0.6532125$

$x = 5$ $\quad$ $y = ?$

$x_2 = 5.5$ $\quad$ $y_2 = 0.7403627$

$$\therefore \; y = \frac{y_2(x - x_1) + y_1(x_2 - x)}{(x_2 - x_1)}$$

$$= \frac{0.7403627(0.5) + 0.6532125(0.5)}{1}$$

$$\therefore \; y = 0.6901$$

3. $x^4 - x - 10 = 0$

$a = 1.5$, $b = 2$

$\therefore$ $f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$

$= -0.4375$

$f(b) = f(2) = 2^4 - 2 - 10$

$= 4$

$\therefore$ 1^{st} ~~it~~ iteration :

$x_1 = \frac{2 + 1.5}{2} = 1.75$

$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -2.3711$

2^{nd} iteration :

$x^2 = \frac{1.75 + 2}{2} = 1.875$

$f(x^2) = 1.875^4 - 1.875 - 10 = 0.4846$

3^{rd} iteration :

$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

4^{th} iteration:

$$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$$

$$f(x_4) = 1.84375^2 - 1.84375 - 10 = -0.2877$$

5^{th} iteration:

$$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$$

$$f(x_5) = 1.859375^4 - 1.859375 - 10$$
$$= 0.093$$

$\therefore$ root of $x^4 - x - 10 = 0$ is 1.859375
which is approximately 1.86

4. $f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$$\therefore x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = 0.875$ $f'(x_0) = 5.75$

$\therefore x_1 = x_0 - \frac{(f(x_0))}{(f'(x_0))}$

$= 1.5 - \frac{0.875}{5.75} = 1.34783$

$f(x_1) = 0.10068$ $f'(x_1) = 4.44991$

$x_2 = 1.34783 - \frac{0.10068}{4.44991} = 1.3252$

$\therefore f(x_2) = 0.00205$ $f'(x_2) = 4.2684$

$x_3 = 1.3252 - \frac{0.00205}{4.2684}$

$\therefore x_3 = 1.3247$

$f(x_3) = 0.000007545$ $f'(x_3) = 4.2645$

$\therefore x_4 = 1.3247 - \frac{0}{4.2645}$

$\therefore x_4 = 1.3247$

$\therefore$ Root of $x^3 - x - 1 = 0$ is 1.3247

5. $A^2 = A$

$7A - (1+A)^3$: To find

$\therefore \; 7A - (1+A)^3 = 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$

$= 7A - I^3 - A^3 - 3A^2I - 3AI^2$

$= 7A - I^3 - A^2.A - 3A^2 - 3A$

$= 7A - I - A.A - 3A - 3A$

$= 7A - I - A - 6A$

$= 7A - 7A - I$

$\therefore \; 7A(I+A)^3 = -I$

6. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore \; |A| = 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(-6) + 1(-4) + 2(4) = -2$

$|A| \neq 0 \quad \therefore$ inverse exists

$\therefore$ co-factor matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

$\therefore$ adjoint matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \times$ adjoint

$= -\frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & -1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$

9. The smallest 3 digit no. that leaves remainder of 2 is 101 and the largest is 998.

$\therefore a = 101 \quad , \; l = 998 \quad , \; d = 3$

$\therefore \; t_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$\frac{998 - 101}{3} = n - 1$

$\therefore n = 300$

$S_n = \frac{n}{2}(a + l) = \frac{300}{2}(101 + 998) = 164850$

$\therefore$ sum of all numbers that leave a remainder of 2 when divided by 2 is 164850

10. $t_0 = 32$

$t_8 = 128$

$\therefore \quad t_6 = ar^5$

$32 = ar^5 \quad \ldots \quad (i)$

$t_8 = ar^7$

$128 = ar^7 \quad \ldots \quad (ii)$

Dividing (ii) & (i)

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$r^2 = 4$

$r = 2$

11. Expand $(4+3x)^5 = (3x+4)^5$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}}{2!} + \frac{n(n-1)(n-2)a^{n-3}}{3!}b^3 + \ldots$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 4 + \frac{5\times4}{2}(3x)^3 4^2 + \frac{5\times4\times3}{3\times2}(3x)^2 4^3 + (3x)4^4 + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^3 + 3840x + 1024$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\therefore \lambda^3 - (0)\lambda^2 + (-13)\lambda - (-30 + 18 + 0) = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$

$\therefore \lambda = 1$ satisfies

1	1	0	-13	12
	$\downarrow$	1	1	-12
	1	1	-12	$\boxed{0}$

$\therefore \lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$(\lambda+4)(\lambda-3) = 0$

$\lambda = -4$ or $\lambda = 3$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 13$

$\therefore x_1 = 5 - \left(\frac{-13}{13}\right) = 6$

$f(6) = 9$
$f'(6) = 32$

$x_2 = 6 - \frac{9}{32}$

$\therefore x_2 = 5.71875$

14. $(1 - x + x^2)^4$

$\therefore [x^2 + (1-x)]^4$
Let $1 - x = y$
$\therefore [x^2 + y]^4$
using binomial theorem

$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + b^2$

$\therefore [x^2+y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{4 \times 3}{2} (x^2)^2 y^2 + \frac{4 \times 3 \times 2}{3 \times 2} \times (x^2) y^3 + y^2$

$= x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$

$\therefore [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$

FOR EDUCATIONAL USE

15. $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \qquad b = 2$

$\therefore f(a) = -0.367879$

$f(b) = 1.944106$

$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 3\log(1.25) - e^{-1.25}$

$= 0.382925$

$x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = 3\log(1.125) - e^{-1.125}$

$= 0.02869664$

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$

$= -0.16372$