FINANCIAL MATHEMATICS

ASSIGNMENT - 1

1. $d^{(4)} = 8\%$

$$\left[1 - \frac{d^{(4)}}{4}\right]^4 = 1 - d$$

$$\left[1 - \frac{2}{100}\right]^4 = 1 - d$$

$$\left[\frac{98}{100}\right]^4 = 1 - d$$

$$d = 1 - \left[\frac{98}{100}\right]^4$$

$$d = 1 - 0.92236816$$

$$d = 0.07763184$$

ii) $i = \frac{d}{1-d} = \frac{0.07763184}{0.92236816}$

$$= 0.0841657847$$

or

$$\underline{\underline{8.41657847\%}}$$

i) $\delta = \ln(1+i)$

$$\delta = \ln(1.0841657847)$$

$$= 0.080810829$$

or

$$\underline{\underline{8.0810829\%}}$$

iii) $\frac{d^{(12)}}{12} = ?$ (To find)

$$\left[1-\frac{d^{(4)}}{4}\right]^{4} = \left[1-\frac{d^{(12)}}{12}\right]^{12}$$

$$\left[\frac{98}{100}\right]^{4} = \left[1-\frac{d^{(12)}}{12}\right]^{12}$$

$$\left[\frac{98}{100}\right]^{1/3} = 1-\frac{d^{(12)}}{12}$$

$$1- 0.\overset{993288388}{\cancel{32666667}} = \frac{d^{(12)}}{12}$$

$$\frac{d^{(12)}}{12} = \cancel{0.673} \quad 0.006711612$$

$$\frac{d^{(12)}}{12} = \underline{\underline{0.6711612\,\%}}$$

2. $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = 1000 \times \dfrac{1}{e^{\int_0^{10}\delta(t)}}$

$$= \frac{1000}{e^{\int_0^{10}\delta(t)}}$$

$$= \frac{1000}{e^{[0.266667]}}$$

$$\doteq \frac{1000}{1.30560561}$$

$$= \underline{\underline{765.928081}}$$

$$\cancel{e}\; e^{\int_0^{10}\delta(t)} = e^{\int_0^{10}(0.604t + 0.0002t^2)}$$

$$\cancel{=}\;\cancel{e} = e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= e^{\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= e^{\left[0.002\cancel{t}\times10^2 + \frac{0.0002\times10^3}{3}\right]} - 0$$

$\cancel{[0.00}$

$$= e^{[0.2 + 0.066667]}$$

ii) $AV = PV(1+i)^{10}$ $\quad [AV = 1000 \text{ (given)}]$

$1000 = 765.928081(1+i)^{10}$

$(1+i)^{10} = \frac{1000}{765.928081}$

$(1+i)^{10} = 1.30560561$

$(1+i) = (1.30560561)^{1/10}$

$1+i = 1.02702544$

$i = 0.02702544$

or

$\underline{\underline{2.702544\ \%}}$

3. i) $1+i = \frac{1}{1-d}$

$\frac{1}{1+i} = 1-d$

$d = 1 - \frac{1}{1+i}$

$d = \frac{1+i-1}{1+i}$

$d = \frac{i}{1+i}$

$d = iv$ $\quad \left[\because \frac{1}{1+i} = v\right]$

ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\left[1 - \frac{d^{(4)}}{4}\right]^4 = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$\left[1 - \frac{225}{\cancel{400}}\right]^4 = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$400$$

$$\left[1 - 0.0225\right]^2 = 1 - \frac{d^{(2)}}{2}$$

$$0.95550625 = 1 - \frac{d^{(2)}}{2}$$

$$d^{(2)} = (1 - 0.95550625)2$$

$$d^{(2)} = 0.04449375 \times 2$$

$$= 0.0889875$$

or

$$\underline{8.89875\ \%}$$

b) $\frac{d^{(1/2)}}{(1/2)} = 5\%$

$$\left[1 - \frac{d^{(1/2)}}{(1/2)}\right]^{1/2} = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$\left[1 - \frac{5}{100}\right]^{1/2} = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$\left[\frac{95}{100}\right]^{1/2} = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$(0.95)^{1/4} = 1 - \frac{d^{(2)}}{2}$$

$$2(1 - 0.9872585) = \cancel{d^{(2)}}\ d^{(2)}$$

$$d^{(2)} = 0.02548291 \text{ or } \underline{2.548291\ \%}$$

iii) $i = 5\%$

Bill amount is 100

$$PV = 100 \times \frac{1}{(1+i)^{91/365}}$$

$$= 100 \times \frac{1}{(1.05)^{91/365}}$$

$$= \frac{100}{1.0122384}$$

$$PV = 98.7909568$$

$$\left(\frac{100}{PV}\right)^{365/91} = \frac{1}{1-d}$$

$$\left(\frac{100}{98.7909568}\right)^{365/91} = \frac{1}{1-d}$$

$$(1.0122384)^{365/91} = \frac{1}{1-d}$$

$$1.0499997 = \frac{1}{1-d}$$

$$d = 1 - \frac{1}{1.0499997}$$

$$= 1 - 0.95238098$$

$$d = \underline{\underline{0.04761902}} \quad \text{or} \quad \underline{\underline{4.761902\%}}$$

4. (i) $1 - d = \frac{1}{1+i}$

$d = \frac{1+i-1}{1+i}$

$d = \frac{i}{1+i}$ $\left[\because v = \frac{1}{1+i}\right]$

$d = iv$

(ii) a) $i = 0.08$

$d = \frac{i}{1+i}$

$d = \frac{0.08}{1.08} = \underline{0.0740740741}$

$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+i}$

$1 - \frac{d^{(12)}}{12} = \frac{1}{(1.08)^{1/12}}$

$1 - \frac{d^{(12)}}{12} = 0.993607102$

$\frac{d^{(12)}}{12} = 0.006392898$

$d^{(12)} = 0.076714776$ or $\underline{\underline{7.6714776}}\ \%$

b) $\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1 + 0.08$

$1 + \frac{i^{(365)}}{365} = (1.08)^{1/365}$

$i^{(365)} = 0.00021087 \times 365 = 0.07696755$

$= \underline{\underline{7.696755}}\ \%$

c) $\delta = \ln(1+i)$

$\delta = \ln(1.08)$

$\delta = 0.07696104114$ or $\underline{\underline{7.696104114}}$ %

d) $1 + \frac{i^{(1/2)}}{1/2} = 1.08$

$1 + \frac{i^{(1/2)}}{1/2} = (1.08)^2$

$i^{(1/2)} = \left[1 - (1.08)^2\right] \times \frac{1}{2}$

$i^{(1/2)} = \left[\frac{1 - 1.1664}{2}\right]$

$i^{(1/2)} = \frac{0.1664}{2} = 0.0832$ or $\underline{\underline{8.32}}$ %

5. i) $d^4 = 6\%$

$\left(1 - \frac{d^4}{4}\right)^4 = 1 - d$

$\left(1 - \frac{0.06}{4}\right)^4 = 1 - d$

$(1 - 0.015)^4 = 1 - d$

$(0.985)^4 = 1 - d$

$d = 1 - 0.941336551$

$d = 0.0586634494$

$1 + i = \frac{1}{1-d}$ $\Rightarrow i = \frac{1 - 1 + d}{1-d} = \frac{d}{1-d} = \frac{0.0586634494}{0.941336551}$

$\Rightarrow i = 0.0623193154$

a) $\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i$

$\frac{i^{(2)}}{2} = \cancel{1.062}\ (1.06231931 54)^{1/2} - 1$

$i^{(2)} = (1.03068876 - 1)2$

$i^{(2)} = 0.06137752$

or

$\underline{6.137752}\ \%$

b) $\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d$

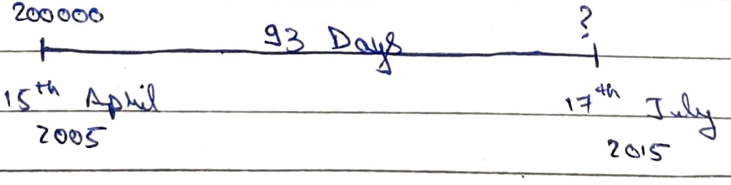
$\frac{d^{(12)}}{\cancel{12}} = \left(1-(1-d)^{1/12}\right) \times 12$

$d^{(12)} = \left[1-(0.413365506)^{1/12}\right]12$

$= 0.07097 4034 \times 12$

$= 0.851688408$ or $\underline{85.1688408}\ \%$

ii)



If paid on 15th April 2006

$\frac{220000}{200000} = 1+i$

$i = \frac{11}{10} - 1$

$i = 0.1$

Interest for 93 day $= 0.1 \times \frac{93}{365}$

$= 0.0254794521$

Amount paid on 17th July 2005 $= 200000 + [200000 \times 0.0254794521]$

$= 200000 + 5095.89041$

$= 205095.89041$

6) i) a) Retail price of the toy ₹100

Cash payment $= 100 - \frac{30}{100} \times 100$

$= 70$

b) Six months credit $= 100 - \frac{25}{100} \times 100$

$= 75$

Objective rate of interest

$70 = 75(1-d)^{0.5}$

$\left(\frac{70}{75}\right)^2 = 1-d$

$d = 1-\left(\frac{70}{75}\right)^2$

$= 0.128888889$ or 12.8888889%

ii) $d^{(12)} = \left[1-(1-d)^{1/12}\right] \times 12$

$= \left[1-(1-0.128888889)^{1/12}\right] \times 12$

$= 0.137195439$ or 13.7195439%

- $i = \frac{d}{1-d} = \frac{0.128888889}{0.871111111} = 0.147959184$

$i^{(2)} = \left[(1+i)^{1/2} - 1\right] \times 2$

$= \left[(1+0.147959184)^{1/2} - 1\right] \times 2$

$= 0.14285714$ or 14.285714%

- $\delta = \ln(1+i)$

$= \ln(1.147959184)$

$= 0.137985743$ or 13.7985743%

~~7.~~ ~~$AV = PV\left(1+\frac{i^{(4)}}{4}\right)^4$~~

~~$26000 = 20000\left(1+\frac{i^{(4)}}{4}\right.$~~

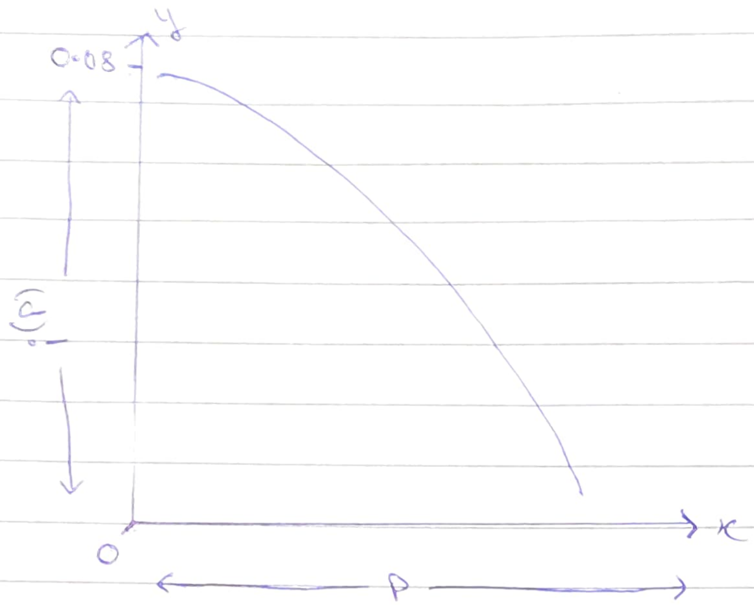
8. $i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$

$i = 8\%$

$p = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

p	$i^{(p)}$
1	~~8%~~ 0.08
2	0.07846096908
3	0.07795670402
4	0.07770618763
5	0.07755639199
6	0.0774567418
7	0.0773856674
8	0.0773324187
9	0.0772910367
10	0.07725795243
365	0.07696915541

$i^{(1)} > i^{(2)} > i^{(3)} > \ldots\ldots\ldots > i^{(4)}$



9. i) Force of interest = It is nominal rate of interest ~~rate~~ per unit time convertible continously

ii) $i^{(2)} = 0.0775$

• $i = \left(1+\frac{i^{(2)}}{2}\right)^2 - 1$

$= 0.079001563$ or 7.9001563%

• $\delta = \ln(1+i)$

$= \ln(1.079001563)$

$= 0.076036135$

• $d^{(12)} = 12\left(1-(1-d)^{1/12}\right)$

$= 12\left(1-(1-0.073217283)^{1/12}\right)$

$= 0.075795747$

• $d = \frac{i}{1+i}$

$= \frac{0.079001563}{1.079001563}$

$= 0.073217283$

- $i^{(1/2)} = \left[(1+i)^2 - 1\right] \times 1/2$

$$= \frac{(1.07900 1563)^2 - 1}{2}$$

$$= 0.0821222186 \text{ or } \underline{8.21222186}\ \%$$

10. $v(t) = e^{-\delta}$ or $\frac{1}{A(t)}$

$$= \frac{P \quad 1}{A(0,5)\,A(5,10)\,A(10,t)}$$

$$= e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$$

$$= e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04t]_{10}^{t}}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t + 0.4}$$

$$= e^{-[0.3 + 0.04t]}$$

11. a) $PV = AV\,(v(t))$

$PV = AV\,(1-d)^n$

$= 50000\,(1 - 0.18)^{9/12}$

$= 43085.42623$

~~b) i) $\delta = \ln\left(\frac{1}{1-d}\right)$~~

~~$\frac{1}{100} =$~~

b) i) $e^{\delta} = 1+i$

$e^{0.06} = 1+i$

$e^{0.06} - 1 = i$

$i = 0.061836547$

$d = \frac{i}{1+i} = \frac{0.061836547}{1.061836547} = 0.058235467$

$d^{(12)} = \left[1-(1-d)^{1/12}\right] \times 12$

$= \left[1-(1-0.058235467)^{1/12}\right] \times 12$

$= \left[1-0.995012479\right] \times 12$

$= 0.059850250$ or 5.9850250

ii) $1+i = \frac{1}{1-d}$

$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)^{4}$

$1+\frac{i^{(12)}}{12} = \left[\frac{4}{\text{[illegible]}.3094}\right]^{1/3}$

$1+\frac{i^{(12)}}{12} = 2.532568791$

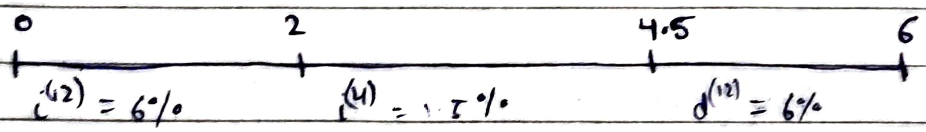
$\frac{i^{(12)}}{12} = 1.532568791$

$i^{(12)} = 18.39082549$

c) $d < \delta < i^{(4)} < i$

d)

12.



$$AV = 7049 \times \{A(0,2) \times A(2,4.5) \times A(4.5,6)$$

$$= 7049 \times \left(1 + \frac{i^{(12)}}{12}\right)^{24} \times (1 + 10 \times i) \times \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{\frac{3}{2} \times 12}}$$

$$= 7049 \times (1.127159776) \times (1.15) \times (1.09421325)$$

$$\simeq 9999.893613$$

13. (i)

$$AV = PV \times A(0,t)$$

$$2x = x(1+i)^n$$

$$2 = (1+i)^n$$

$$\log 2 = \log(1.1)^n$$

$$\log 2 = n \log(1.1)$$

$$\frac{\log(2)}{\log(1.1)} = n$$

$$n = \underline{7.272540897}$$

(ii)

$$d^{(12)} = 10\%$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - d$$

$$\left(1 - \frac{0.10}{12}\right)^{12} = 1 - d$$

$$\left(\frac{12 - 0.10}{12}\right)^{12} = 1 - d$$

$$(0.991666667)^{12} = 1 - d$$

$$1 - 0.904458374 = d$$

$$0.095541626 = d$$

$$AV = PV \left[\frac{1}{(1-d)^n}\right]$$

$$2x = x \times \frac{1}{(1-d)^n}$$

$$(1-d)^n = \frac{1}{2}$$

$$(0.904458374)^n = \frac{1}{2}$$

$$\log(0.904458374)^n = \log(1/2)$$

$$n \log(0.904458374) = \log(1/2)$$

$$n = \frac{\log(1/2)}{\log(0.904458374)}$$

$$n = 6.902550381$$