Probability & Statistics

Assignment - 1

Ques 1) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$\Rightarrow$ Mean $= \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$\bar{x} = 13.4$

$\Rightarrow$ Median $= \dfrac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$

$= \dfrac{11+15}{2}$

$= 13$

Mode $= 18$

IQR $=$ $q_1 = 8$ $\qquad q_3 - q_1$

$q_2 = 13$ $\qquad \Rightarrow 18 - 8$

$q_3 = 18$ $\qquad \Rightarrow 10$

ii)

No. of households	0	1	2	3	4
No of people	5	11	26	15	3

$Q_1 = \dfrac{15^{th} + 16^{th}}{2} = \dfrac{1+1}{2} = 1$

$Q_2 = 2$ $\qquad$ IQR $= Q_3 - Q_1$

$Q_3 = \dfrac{3+3}{2} = 3$ $\qquad = 2$

$$\Rightarrow \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{0\times5 + 1\times11 + 2\times26 + 3\times15 + 4\times3}{5 + 11 + 26 + 15 + 3}$$

$$= \frac{11 + 52 + 45 + 12}{60}$$

$$= \frac{120}{60}$$

$$= 2 \text{ Ans}$$

$\Rightarrow$ Median :

$$n = 60$$

$$= \frac{\left(\frac{n}{2}\right)^{th} + \left[\frac{n}{2} + 1\right]^{th}}{2}$$

$$= \frac{(30)^{th} + (31)^{st}}{2}$$

$$= (30.5)^{th} \text{ term}$$

$(30.5)^{th}$ term lies in the x_i, 2

Thus, median = 2

~~$\Rightarrow$ Mode = 2~~

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Standard deviation

x	f	$\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	5	2	4	20
1	11	2	1	11
2	26	2	0	0
3	15	2	1	15
4	3	2	4	12

$$S = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n-1}} = \sqrt{\frac{58}{59}} = 0.99149$$

Ques 1) a) Standard Deviation

x	$\bar{x}$	$(x-\bar{x})^2$
4	13.4	88.36
7	13.4	40.96
9	13.4	19.36
9	13.4	19.36
11	13.4	5.76
15	13.4	2.56
18	13.4	21.16
18	13.4	21.16
18	13.4	21.16
25	13.4	134.56

$$\Sigma(x_i-\bar{x})^2 = 374.4$$

$$\sigma^2 = \frac{374.4}{10}$$

$$= 37.44$$

$$\sigma = \sqrt{37.44}$$

$$= 6.12$$

Ques 2) $x \sim P(0.5)$ $\lambda = 0.5$

(i) $P(x=0) = \cancel{\frac{e^{-\lambda} x^{n}}{n!}}$ $\frac{e^{-\lambda} \lambda^{n}}{n!}$

$= \frac{e^{-0.5}\ 0.5^{0}}{0!}$

$= e^{-0.5}$

$= 0.6065$

ii) Let y = no. of years with a claim.
then $y \sim$ binomial $(3, 0.3935)$

$\therefore P(y=1) \Rightarrow {}^{3}C_{1} \times (0.3935)^{1} (0.6065)^{2}$
$\Rightarrow 0.434$

iii) $X \sim \exp(0.5)$
$P(X>2) = \int_{2}^{\infty} \lambda e^{-\lambda x}\, dx$

OR

$0.5 \int_{2}^{\infty} e^{-0.5x} = \frac{e^{-0.5x}}{-0.5} \times 0.5$

$= -\left[e^{-0.5x}\right]_{2}^{\infty}$

$= -\left[e^{-\infty} - e^{-1}\right]$

$= -\left[0 - \frac{1}{e}\right]$

$= \frac{1}{e}$

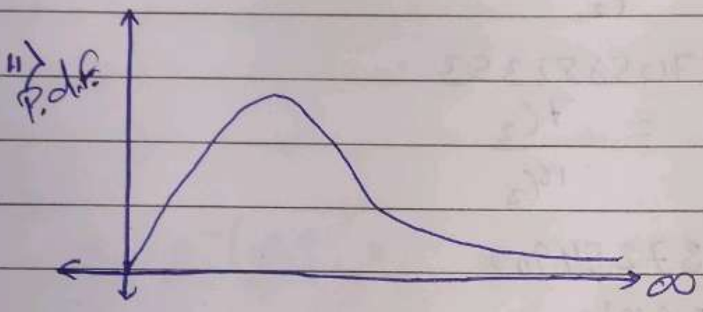
$= 0.368$

Ques 3> i> $X \sim$ Gamma $(2, 1/2)$ $\boxed{\alpha = 2}$ $\boxed{\lambda = \frac{1}{2}}$

$E(x) = \alpha/\lambda \Rightarrow 2 \times \frac{2}{1} = 4$

$V[x] = \frac{\alpha}{\lambda^2} = 2 \times \frac{1}{1/4} = 8$

ii>



$= \int_0^x \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\sqrt{\alpha}} = $ P.d.f.

increases in the begining and then decreases when $\alpha = 2$, $\lambda = \frac{1}{2}$

ii> CDF of Gamma distribution

$$\begin{cases} 0 & , x < 0 \\ \int_0^x \frac{\lambda^\alpha x^{\alpha-1}}{\sqrt{\alpha}} \times e^{-\lambda x} & , x > 0 \end{cases}$$

Ques 4) P(Choosing 2 members from the group)

$\Rightarrow {}^{18}C_2 = P(A)$

P (atleast one female) = 1 - P(both are male)

$P(F) = 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$

$= 0.705882353$

P(2 qualified females) $= \frac{{}^{7}C_2}{{}^{18}C_2}$

$P(F \cap Q) = 0.137254902$

$P(Q|F) = \frac{P(F \cap Q)}{P(F)}$

$= \frac{0.137254902}{0.705882353}$

$= 0.1945$

Ques 5) P(Delivering via level 1) = $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

P(Arriving late using level 1) $= 0.2 = P(L/L_1)$

$P(L/L_2) = 0.4$

$P(L/L_3) = 0.1$

$P(L_2/L) = \frac{P(L_2)\, P(L/L_2)}{P(L_1)\, P(L/L_1) + P(L_2) P(L/L_2) + P(L_3)\, P(L/L_3)}$

$= \frac{0.5 \times 0.4}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$

$= \frac{0.2}{}$

Ques 6) $x \sim U(1,2)$

$f_X = \frac{1}{\beta - \alpha} = \frac{1}{2-1} = 1$

$Y = \frac{3}{3 \times x}$ $\quad Y(6-x) = 3$

$6 - x = \frac{3}{y}$

$x = 6 - \frac{3}{y}$

$\therefore g^{-1}(y) = 6 - \frac{3}{y}$

$\frac{d}{dy} g^{-1}(y) = -3\left(\frac{-1}{y^2}\right)$

$= \frac{3}{y^2}$

$f_X g^{-1}(y) = 1$ {As RV x can ~~still~~ still only take value b/w 0 & 1}

$f_Y(y) = f_X g^{-1}(y) \left| \frac{d}{dy} g^{-1}(y) \right|$

$= 1 \times \frac{3}{y^2} = 1 \times \frac{3}{y^2} = 3y^{-2}$

Q7) i) $F(3) = P(x \le 3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(x) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.40) + 5(0.05)$

$= 3.25$

iii) $E(x^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$
$= 11.45$

$V(x) = E(x)^2 - [E(x)]^2$
$= 11.45 - (3.25)^2$
$= 0.8875$

iv) Coefficient of skewness $= E(x - \bar{x})^3$

Ques 8) $x \sim B(15, 0.2)$
$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$
$= 1 \times (0.8)^{15} + 15 \times (0.2)(0.8)^{14} + 105 (0.2)^2 (0.8)^{13}$
$= 0.39802$

Ques 9) So, he won two prizes in 6 weeks

So, $P(x=7) = {}^6C_2 (0.01)^3 (0.99)^4$
$= 0.00001409$

Ques 10) It will follow a Poisson distribution
$\lambda = \frac{2}{5} = 0.4$

$P(x > 2) = 1 - P(x \leq 2)$
$P(x=0) = \frac{\lambda^x - e^{-\lambda}}{x!} = \frac{(0.4)^0 e^{-0.4}}{0!}$
$= e^{-0.4}$
$= 0.670320046$

$P(x=1) = \frac{(0.4)^1 e^{-0.4}}{1!}$

$$P(X=2) = \frac{(0.4)^2 e^{-0.4}}{2!} = 0.053625604$$

$$P(X \le 2) = 0.992073668$$

$$P(X>2) = 1 - P(X \le 2)$$

$$= 0.007926332$$

Ques 11) $P(b) = P(g)$

$P(b) + P(g) = 1$

$2P(b) = 1$

$P(b) = \frac{1}{2}$

$\therefore P(g) = \frac{1}{2}$

Let E be the event of having 3 boys

$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$P(E) = \frac{1}{8}$

12) It is following exponential distⁿ. as exp(10).

$\lambda = 10$

$\alpha = 1$

$t = 0.1$ days

$P(T < 0.1) = F(0.1)$

$= 1 - e^{-\lambda t}$

$= 1 - e^{-1}$

$= 0.632120559$

13) $x \sim U(75, 120)$

$$f_X(x) = \frac{1}{120 - 75}$$

$$= \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100 - 80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444$$

14) $X \sim Gamma(5, 0.4)$

$\alpha = 5$

$\lambda = 0.4$

$X \sim Gamma(\alpha, \lambda)$

then $2\lambda X \sim \chi^2_{2\alpha} = \lambda$

$2 \times 0.4x \sim \chi^2_{10}$

$P(X < 22.89)$

$= P[2\lambda x < 2\lambda(22.89)]$

$= P[\chi^2_{10} < 22.89 \times 0.8]$

$= P[\chi^2_{10} < 18.312]$

$18.0 \rightarrow 0.945$

$18.5 \rightarrow 0.9529$

$0.5 \rightarrow 0.0079$

$1 \rightarrow \frac{0.0079}{0.5}$

$0.31 \rightarrow 0.0079 \times 0.31$

$$18.31 \rightarrow 0.945 + \frac{0.0079}{0.5} \times 0.31$$

$$18.31 \rightarrow 0.949898$$

$$P(\chi^2_{10} < 18.312) = 0.949898$$