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Saathi

$$\begin{aligned} \sum xy &= 182560 & \sum y &= 1737 \\ \sum x &= 810 & \sum x^2 &= 92000 \\ S_{xy} &= 34915 \end{aligned}$$

$$S_{xx} = 2040$$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = 1.72418$$

$$\bar{y} = 173.7 \quad \bar{x} = 27.14317$$

$$\hat{y} = 27.14317 + 1.72418x$$

Q2) x = share price of JPL y = share price of IBM

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$$

$$= 875.16$$

$$S_{xx} = 301.66$$

$$\hat{\beta} = \frac{875.16}{301.66} = 2.90115$$

$$\hat{\alpha} = 3.74804$$

$$95\% \text{ conf. int. for } \beta = \hat{\beta} \pm t_{0.025} \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$$

$$S_{yy} = 6409.7125$$

$$\hat{\sigma}^2 = \frac{1}{10} \left(6409.7125 - \frac{S_{xy}^2}{S_{xx}} \right) = 387.07471$$

$$\begin{aligned} \beta &= \left(2.90115 \pm 2.52379 \right) \\ &= (0.37736, 5.42494) \end{aligned}$$

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(ii) mean IRN = μ_0

$$se(\mu_0) = \sqrt{\left(\frac{1}{n} + \frac{(\bar{x} - \mu_0)^2}{S_{xx}}\right) \sigma^2}$$

$$= 10.59844$$

$$95\% \text{ conf Int} = \mu_0 \pm t_{10, 2.5} se(\hat{\mu}_0)$$

$$= 119.5405 \pm 23.61443$$

$$= (96.17966, 143.4812)$$

(3)

Assumption

Plot must be normal

Population have common variance

The observations are independent

(iii)

H_0 : Mean of Difference is same

H_1 : Mean of it is not same

$$SS_{TOT} = \sum y_i^2$$

$$= \sum y_i^2 - \left(\frac{\sum y_i}{n}\right)^2$$

$$1495 - \frac{(103)^2}{28}$$

$$SS_{TOT} = 1116.11$$

$$SS_{BET} = \left(64^2 + 44^2 + 8^2 + 17^2\right) - \frac{103^2}{28}$$

$$= 576.11$$

$$SS_{RES} = SS_{TOT} - SS_{BET} = 610$$

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Anova

Under H_0 $F = \frac{MSS_{reg}}{MSS_{res}} = \frac{172.04}{25} = 6.88$

Test statistic = 6.88

$F_{3, 24, 0.05} = 3.009$

Since we have sufficient evidence to reject H_0 at 5% LOS. The factors are underlying difference b/w.

i) $\bar{y}_1 = 9.14$

$\bar{y}_2 = 6.249$

$\bar{y}_3 = 1.14$

$\bar{y}_4 = -1.86$

$\bar{y}_1 > \bar{y}_2 > \bar{y}_3 > \bar{y}_4$

$\sigma^2 = \frac{SS_{res}}{n-1} = 25$

Least significant diffn b/w any 2 of mean is

$t_{24, 0.025} \times \sigma \sqrt{\frac{1}{7} + \frac{1}{7}} = 2.064 \times \sqrt{25} \times \sqrt{\frac{1}{7} + \frac{1}{7}} = 5.52$

$\bar{y}_1 - \bar{y}_2 = 2.85$

$\bar{y}_2 - \bar{y}_3 = 5.11$

$\bar{y}_3 - \bar{y}_4 = 3$

$\bar{y}_1 > \bar{y}_2 > \bar{y}_3 > \bar{y}_4$

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a) $y_{ij} = \mu + \tau_i + e_{ij} \quad \begin{matrix} i = 1, 2, \dots, k \\ j = 1, 2, \dots, n_i \end{matrix}$

k = no. of treatments

n_i = no. of replicates

y_{ij} = Response from i^{th} treatment

Assumption e_{ij} is iid $N(0, \sigma^2)$

b) $n_1 = 7$

$n_2 = 8$

$n_3 = 6$

$n_4 = 7$

$n_5 = 5$

$n = 33$

H_0 : Mean claim amounts of five cars are equal

H_1 : Mean claim amount of five components are not equal

$$\sum_{i=1}^5 \sum_{j=1}^{n_i} y_{ij} = 1856$$

$$\sum_{i=1}^5 \sum_{j=1}^{n_i} y_{ij}^2 = 174310$$

$$CF = \left(\sum \sum y_{ij} \right)^2 / n = 104385.94$$

$$SS_{TOT} = 69930.06$$

$$SS_{BET} = 227825.67$$

$$SS_{RES} = 64604.37$$

$$\text{Test Stat} = \frac{MS_{BET}}{MS_{RES}} = 3.283$$

$$F_{4, 28, 0.05} = 2.714$$

We have sufficient evidence to reject H_0

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c) Observed Frequencies (O_i)

	3-5	5-8	8-10	10-12	
0-3	45	20	6	6	76
4-6	7	20	3	5	42
7-9	5	8	15	12	40
	57	48	30	23	188

Expected Frequency

	3-5	5-8	8-10	10-12	
0-3	27.41	23.08	14.43	11.07	
4-6	15.15	12.75	7.57	6.11	
7-9	14.43	12.15	7.57	5.88	

Contingency Table

H_0 : Sales are independent w.r.t. no. of actual prof.
 H_1 : Sales are dependent on the actual prof.

$$\text{Test Stat} = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\text{Total Stat} = 49.91148$$

$$\text{D.O.F} = (r-1)(c-1) = 2 \times 3 = 6$$

$$p\text{-value} = 12.59$$

$$0.15$$

Hence, we have sufficient evidence to reject H_0

Sales are dependent on no. of actual prof.

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(i) The Homoscedasticity assumption is very different from each other.

(ii)
$$\text{mean at rate} = \frac{\sqrt{24} + \sqrt{13} + \sqrt{4}}{3} = 4.52443$$

Var at Rate 2 =
$$\frac{\sum (x_i - \bar{x})^2}{n-1} = 0.12127$$

for large n

(iii) $y_{ij} = \mu_i + \tau_j + e_{ij}$ $i=1,2,3,4$ $j=1,2,3$
 y_{ij} has transformed version of j th observation with overall group mean
 τ_j is the deviation of the j th mean from e_{ij} are the independent error terms
 $e_{ij} \sim N(0, \sigma^2)$

for Anova

$H_0: \tau_i = 0$ $i=1,2,3,4$
 $H_1: \tau_i \neq 0$ for atleast one i
 in n

66

Rate	mean	variance	y_i
1	2.992	0.163	80.882
2	4.027	0.121	208.878
3	5.716	0.186	294.053
4	6.0575	0.1	389.062
Total	20.110	0.618	973.375

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$$\text{Here } y_i^2 = \left(\sum_{j=1}^3 y_{ij} \right)^2 = \text{by memory}^2$$

$$SS_{\text{Rate}} = \sum_{j=1}^3 \frac{y_j^2}{3} - \frac{y^2}{12} = \frac{973.373}{3} - \frac{(3 \times 2 \times 14)^2}{12}$$

$$SS_{\text{Rate}} = 21.153$$

$$SS_{\text{Res}} = \sum_{i=1}^4 \left(\sum_{j=1}^3 (y_{ij} - \bar{y}_i)^2 \right)$$

$$\sum_{i=1}^4 2 \times \text{Var} = 2 \times 0.618 = 1.236$$

Source of Variation	df	SS	MSS
Rate	3	21.153	
Residual	8	1.236	0.155
Total	11	22.39	

$$\text{Test Stat} = 45.638$$

$$F_{3,8,1\%} = 7.977$$

d.f.]

16	5	9	2	3	1	6	5	4
73	120	34	46	24	26	93	45	66

$$n=10$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

$$S_{xy} = 661.2$$

$$S_{xx} = 54.9$$

$$S_{yy} = 8923.6$$

$$r = 0.94466$$

Fitted Regression eqⁿ

$$\hat{y} = \bar{y} - \beta^1 x$$

$$\bar{y} = 9.2245$$

$$\beta^1 = \frac{S_{xy}}{S_{xx}} = 12.04371$$

$$\hat{y} = 9.2245 + 12.04371x$$

$$S_{TOT} = S_{yy} = 8923.6$$

$$S_{Reg} = \frac{S_{xy}^2}{S_{xx}} = 7963.80492$$

$$S_{Res} = S_{TOT} - S_{Reg} = 960.24808$$

$$R^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = 89.238705$$

Comment

Model is good fit as model explains 89% of the variability is the output.
Better models can be considered considering more variables.

Q7)

$$S_{xx} = 472$$

$$S_{xy} = 1778.421$$

$$S_{yy} = 1189.205$$

$$\beta^1 = 0.45451$$

$$\hat{y} = \bar{y} - \beta^1 x$$

$$= 10.78086$$

$$\hat{y} = 10.78086 + 0.45451x$$

ii)

$$H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

$$\frac{\frac{\hat{\beta} - \beta}{\sqrt{\frac{S_{Res}}{S_{xx}}}}}{\sqrt{n-2}}$$

$$F^2 = \frac{1}{n-2} \left(S_{yy} - \frac{S_{xy}^2}{S_{xx}} \right) = 22.20136$$

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$$\text{Test Stat } \frac{\hat{\beta} - 0}{\sqrt{\sigma^2 / S_{xx}}} = 6.7568 =$$

$$t_{9, 2.5} = 2.262$$

Hence we have sufficient evidence to reject $H_0: \beta = 0$
Hence a linear relation exists

Assumption:

Populations have common Variance

Populations follows the normal distribution

Errors are independent

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = 0.91213$$

10) for Mean response

$$x_0 = 25$$

$$\hat{y}_0 = \hat{\alpha} + \hat{\beta} x_0 = 22.1524$$

$$se(\hat{y}_0) = \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} \hat{\sigma}^2 = 1.85781$$

$$\frac{\hat{y}_0 - y_0}{se(\hat{y}_0)} \sim t_9$$

$$95\% \text{ CI for } \hat{y}_0 = y_0 \pm t_{9, 2.5} se(\hat{y}_0) \\ = (18.84313, 25.66349)$$

Individual response

$$(10.932, 33.37462)$$

7) There is clearly some relationship b/w the residuals and the explanatory variable values. Hence our assumption of normality might be wrong

Principle Component	Diagonal / Eigen / Var	PC / CV
PC ₁	0.936	65.7
PC ₂	0.137	19.5
PC ₃	0.08	11.4
PC ₄	0.0165	2.4
PC ₅	0.012	1.7
		100%

8) As it is 3 principle component explain 95% of total variance, dataset can be reduced to 3 per this dimension. The 3rd principle component can be used for building further classified regression modelling purpose.

9) i) The appraiser take a negative correlation b/w the market and the spot per day on commodity

$$S_{xx} = 75$$

$$S_{yy} = 2482.4$$

$$S_{xy} = -296$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = -0.88$$

Hence they have -ve weak correlation

$$S_{xx} = 0.00011$$

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$$H_0: \rho = 0$$

$$H_1: \rho < 1$$

$$\text{trans}^{-1}(A) = \text{trans}^{-1}(\rho) \sim N(0, \frac{1}{n})$$

$$Z_{cal} = -2.07893$$

$$Z_{cr} = -1.649$$

$$|Z_{cal}| > Z_{cr}$$

We have sufficient evidence to reject H_0

Hence $\rho < 0$

$$ii) \hat{\beta} = \frac{S_{xy}}{S_{xx}} = -3.94667 \quad \hat{y} = \hat{y} - \hat{\beta}x$$

$$= 82.16$$

$$\hat{y} = 82.16 - 3.94x$$

$$iii) R^2 = 47.05983\%$$

Hence model explains 47% of variation

model not a good fit

$$iv) \text{with } n=1 \quad \hat{y} = \hat{y} - \hat{\beta}x_0 \quad \hat{y}_0 = 78.2225$$

d) i)	x	0.27	0.36	0.3
	y	0.05	0.1	0.0

$$S_{xx} = 0.0042$$

$$S_{yy} = 0.00126$$

$$S_{xy} = 0.0022$$

$$S_{STO} = 0.00126 \quad S_{Reg} = 0.00111 \quad S_{Res} = 0.00015$$

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Anova Table

Run	1	0.00115
Run	1	0.00000
Run	2	0.00000

$$H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

$$\text{Test Stat } t_{\text{obs}} = 10.4000$$

$$F_{1, 8} = 161.4$$

for 1 df

Do not reject H_0