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Assignment-1

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Subject : Numerical Methods And Algebra
Section : FY-B

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① The radius of a circular plate is measured as 12.65 cm instead of actual length 12.5 cm. Find the following in calculating area of the circular plate.

(i) Absolute error
(ii) Relative error
(iii) Percentage error.

Soln

measured radius = 12.65 cm
actual radius = 12.5 cm

Area = $\pi r^2 = \frac{22}{7} \times (12.65)^2 = 502.9279\ cm^2$

Actual Area = $\pi r^2 = \frac{22}{7} \times (12.5)^2 = 491.0714\ cm^2$.

∴ Absolute error = $|502.9279 - 491.0714| = \boxed{11.8465}$

Relative error = $\cancel{\frac{491.0714}{11.8465}}$ $\frac{11.8465}{491.0714} = \boxed{0.0241}$

Percentage error = $\boxed{2.41\%}$

② The following table is given

x	$f(x) = \log(x)$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6.	0.7781513

(a) Interpolate $\log(5)$ using point $x = 4$ & $x = 6$

(b) Interpolate $\log(5)$ using point $x = 4.5$ & $x = 5.5$

Soln

(a) $$f(5) = \frac{(5-6)}{(4-6)}(0.60206) + \frac{(5-4)}{(6-4)}(0.7781513)$$

$$\boxed{f(5) = 0.6901}$$

(b) $$f(5) = \frac{(5-5.5)}{(4.5-5.5)}(0.6532125) + \frac{(5-4.5)}{(6.5-4.5)}(0.7403627)$$

$$\boxed{f(5) = 0.6967}$$

(3) Find the root of $x^4 - x - 10 = 0$ approximately upto 5 iteration using Bisection method. let $a = 1.5$ & $b = 2$

Sol^n

x	$f(x)$
1.5	-6.4375
2	4

$\bar{x}_1 = \frac{1.5 + 2}{2} = 1.75$ $\quad f(x_1) = -2.3711$

$\bar{x}_2 = \frac{1.75 + 2}{2} = 1.875$ $\quad f(x_2) = 0.4842$

$\bar{x}_3 = \frac{(1.875 + 1.75)}{2} = 1.8125$ $\quad f(x_3) = -1.0202$

$\bar{x}_4 = \frac{(1.8125 + 1.875)}{2} = 1.84375$ $\quad f(x_4) = -0.2877$

$\bar{x}_5 = \frac{(1.84375 + 1.875)}{2} = 1.859375$ $\quad f(x_5) = 0.0934$

$\bar{x}_6 = \frac{(1.859375 + 1.84375)}{2} = 1.8515625$ $\quad f(x_6) = -0.0984$

$\bar{x}_7 = \frac{(1.8515625 + 1.859375)}{2} = 1.85546875$ $\quad f(x_7) = -0.0028$

$\bar{x}_8 = \frac{(1.85546815 + 1.859375)}{2} = 1.857421575$ $\quad f(x_8) = 0.0452$

$\bar{x}_9 = \frac{(1.857421575 + 1.8515625)}{2} = 1.856445163$ $\quad f(x_9) = 0.0211$

$\bar{x}_{10} = \frac{(1.856445163 + 1.85546815)}{2} = 1.855956957$ $\quad f(x_{10}) = 0.0091$

$\bar{x}_{11} = \frac{(1.855956957 + 1.85546815)}{2} = 1.855712854$ $\quad f(x_{11}) = 0.0032$

$\bar{x}_{12} = \frac{(1.855712854 + 1.85546815)}{2} = 1.855590802$ $\quad f(x_{12}) = 0.0002$

$\overline{x_{13}} = \left(\frac{1.855590802 + 1.85546875}{2}\right) = 1.855529776 \quad f(x_{13}) = -0.0013$

$\overline{x_{14}} = \left(\frac{1.855529776 + 1.855590802}{2}\right) = 1.855560289 \quad f(x_{14}) = -0.00059$

$\overline{x_{15}} = \left(\frac{1.855560289 + 1.855590802}{2}\right) = 1.855575546 \quad f(x_{15}) = -0.00022$

$\overline{x_{16}} = \left(\frac{1.855575546 + 1.855590802}{2}\right) = 1.855583174 \quad f(x_{16}) = -0.00003$

Hence, the approximate value of the x is 1.855583174.

④ Find a root of an equation $f(x) = x^3 - x - 1$ using Newton Raphson method.

Soln

x	y
0	-1
1	-1
2	5

$f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

$x_0 = 1$
$f(x_0) = -1$
$f'(x_0) = 2$

$x_1 = 1 + \frac{1}{2} = 1.5$

$x_1 = 1.5$
$f(x_1) = 0.875$
$f'(x_1) = 5.75$

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$$x_2 = 1.5 - \frac{0.875}{5.75} = 1.3478$$

$f(x_2) = 0.1006$
$f'(x_2) = 4.4497$

$$x_3 = 1.3478 - \frac{0.1006}{4.4497} = 1.3252$$

$f(x_3) = 0.0019$
$f'(x_3) = 4.2684$

$$x_4 = 1.3252 - \frac{0.0019}{4.2684} = 1.3247$$

$f(x_4) = -0.000076$

Hence, the approximate value of x is 1.3247.

5) If A is a square matrix such that $A^2 = A$, then find the value of $7A - (I+A)^3$, here I is an Identity matrix.

Soln $7A - (I+A)^3$
$= 7A - (I^3 + 3I^2A + 3A^2I + A^3)$
$= 7A - I - 3A - 3A - A^3$
$\boxed{= -I}$

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(6) Find out the inverse of

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

Solⁿ

$$|A| = 1(-6) + 1(-4) + 2(4)$$
$$= -6 - 4 + 8$$
$$= -2$$

$$A^{-1} = \frac{1}{(-2)} \begin{bmatrix} -6 & +1 & -6 \\ +4 & -1 & +2 \\ +4 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}}$$

(7) Using the defination of the scalar product, find the angle between the following vectors (Find the smallest nonnegative angle)

(a) $A = 8\hat{i} - 9\hat{j}$
$B = 7\hat{i} - 9\hat{j}$.

Solⁿ $A = 8\hat{i} - 9\hat{j}$
$B = 7\hat{i} - 9\hat{j}$

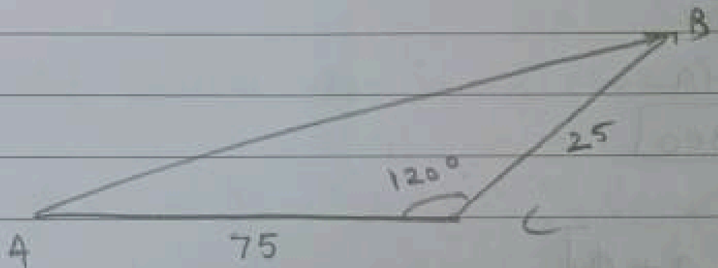
$$\cos\theta = \left[\frac{(8\hat{i} - 9\hat{j})\cdot(7\hat{i} - 9\hat{j})}{|8\hat{i} - 9\hat{j}| \times |7\hat{i} - 9\hat{j}|}\right]$$

$$\cos\theta = \left[\frac{56+81}{\sqrt{145}\times\sqrt{130}}\right]$$

$$\boxed{\theta = \cos^{-1}\left[\frac{137}{5\sqrt{754}}\right]}$$

⑧ Sabrina walked 75 metre to the east, Then she turned 30 degree to left & walk 25 metre. Determine the magnitude of Sabrina displacement vector.

Sol^n //



$$AB = \sqrt{AC^2 + BC^2 + 2(AC)(BC)\cos\theta}.$$

$$= \sqrt{(75)^2 + (25)^2 + 2(75)(25)\cos 30}$$

$$\boxed{AB = 97.46 \text{ metre.}}$$

(9) What is the sum of all 3 digit number that leave remained 2 when dividend by 3.

Sol^n 101, 104 998

$a = 101$

$d = 3$

$t_n = 101 + (n-1)d$

$998 = 101 + (n-1)d$

$(n-1)d = 897$

$n-1 = 299$

$\boxed{n = 300}$

$S_n = \frac{n}{2}[a + t_n]$

$= \frac{300}{2}[101 + 998]$

$\boxed{S_n = 164850}$

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(10) The 6^{th} term of a GP is 32 & its 8^{th} term is 128, then find the common ratio of GP.

Soln $T_6 = ar^5 = 32$

$T_8 : ar^7 = 128$

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\boxed{r = 2}$$

(11) Use Binomial Theorem to expand $(4+3x)^5$.

Soln $(4+3x)^5 = {}^5C_0(4)^5 + {}^5C_1(4)^4(3x) + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)(3x)^4 + {}^5C_5(3x)^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

$$\boxed{(4+3x)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024}$$

(12) Find all the eigenvalues and eigenvector for matrix A if

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Soln $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(3-\lambda)(-5-\lambda)-0\right] + 3\left[2(3-\lambda)-0\right] + 0 = 0$$

$$(2-\lambda)\left[-15-3\lambda+5\lambda+\lambda^2\right] + 3\left[6-2\lambda\right] = 0$$

$$(2-\lambda)\left[\lambda^2+2\lambda-15\right] + 3\left[6-2\lambda\right] = 0$$

$$\cancel{2\lambda^2}+4\lambda-30-\lambda^3-\cancel{2\lambda^2}+15\lambda+18-6\lambda = 0$$

$$-\lambda^3+13\lambda-12=0$$

1	-1	0	13	-12
		-1	-1	12
	-1	-1	12	0

$$(\lambda-1)(-\lambda^2-\lambda+12)=0$$

$$(\lambda-1)(-\lambda^2-4\lambda+3\lambda+12)=0$$

$$(\lambda-1)\left[-\lambda(\lambda+4)+3(\lambda+4)\right]=0$$

$$(\lambda-1)(-\lambda+3)(\lambda+4)=0$$

Hence eigen value are 1, 3 & -4.

for $\lambda = 3$.

$$(A-\lambda I)X=0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_2 \rightarrow R_2 + 2R_1$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_1 \rightarrow R_1 + \tfrac{3}{14} R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_2 \rightarrow R_2 / -14$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x_1 = 0$, $y_1 = 0$ & $z_1 = \alpha$

eigen vector = $\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

(ii) for $\lambda = 1$

$$(A - \lambda I)X = 0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ [illegible] & [illegible] & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} x-3y \\ 0 \\ 2z \end{bmatrix} = 0$$

eigen vector is $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} y.$

(iii) for $\lambda = -4$

$(A - \lambda I) X = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ 2x - y \\ 7z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x - y = 0$

$7z = 0$

Hence the eigen vector is $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} X$.

(13) Use Newton Rapshon Method to determine x_2, for $f(x) = x^3 - 7x^2 + 8x - 3$, if $x_0 = 5$.

Sol^n/

$f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 13$

$$x_1 = 5 - \frac{(-13)}{13}$$

$$x_1 = 6$$

$x_1 = 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$$x_2 = 6 - 9/32$$

$$\boxed{x_2 = 5.71875}$$

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(14) Expand the following. $(1-x+x^2)^4$

Soln

$$(1-x+x^2)^4 = (\underbrace{x^2-x}_{a}+\underbrace{1}_{b})^4$$

$$= (x^2-x)^4 + 4(x^2-x)^3 + 6(x^2-x)^2 + 4(x^2-x) + 1$$

$$= [x^8 - 4x^7 + 6x^6 - 4x^5 + x^4] + 4[x^6 - 3x^5 + 3x^4 - x^3] + 6[x^4 - 2x^3 + x^2] + 4[x^2 - x] + 1$$

$$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4x^6 - 12x^5 + 12x^4 - 4x^3 + 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4x + 1$$

$$\boxed{(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1}$$

(15) Find the approximated value of x till 4 iteration for $e^{-x} = 3\log(x)$ using Bisection Method.

Soln

$3\log(x) - e^{-x} = 0$

X	y
1	-0.3679
2	1.9441

$x_1 = \frac{1+2}{2} = 1.5 \rightarrow f(x_1) = 0.9933$

$x_2 = \frac{1+1.5}{2} = 1.25 \rightarrow f(x_2) = 0.3829$

$x_3 = \frac{1+1.25}{2} = 1.125 \rightarrow f(x_3) = 0.0287$

$x_4 = \frac{1+1.125}{2} = 1.0625 \rightarrow f(x_4) = -0.1637$

Hence the value of x is 1.0625.

x x x