PROB & STATS ASSIGNMENT

10) $kx^{-a} \cdot e^{-y/\beta}$, $a > 1$, $\beta > 0$

We know $\Rightarrow \int_1^\infty \int_1^\infty k x^{-a} e^{-y/\beta} \, dx dy = 1$

$\int_1^\infty k \left(\frac{x^{-a+1}}{-a+1} \right)_1^\infty e^{-y/\beta} dy = 1$

$\Rightarrow k \int_1^\infty \frac{+\beta e^{-y/\beta}}{-a+1} = 1.$

$\Rightarrow k(-\beta)(-e^{-1/\beta}) = a-1.$

$k \beta e^{1/\beta} = 1.$

$k = \frac{a-1}{\beta e^{1/\beta}}.$

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a)

	μ	σ	σ^2
X	3	2	4
Y	4	1	1.

$\rho(x, y) = -0.3$.

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\,\text{var}(Y)}}$$

$\text{cov}(X, Y) = -0.3 \times 2 = -0.6$.

$Z = X + Y$

(i) $\text{cov}(X, Z) = \text{cov}(X, Y + X)$

$= \text{cov}(X, X) + \text{cov}(X, Y)$

$= \text{var}(X) + \text{cov}(X, Y)$

$= 4 - 0.6$

$= 3.4$

(ii) $\text{Var}(Z) = \text{Var}(X + Y)$

$\text{Var}(X+Y) = \text{var}(X) + \text{var}(Y) + 2\,\text{cov}(X, Y)$

$= 4 + 1 - 0.6$

$= 4.4$

8) $a = 3$ $\rightarrow X \sim Ga(3,2)$
$\lambda = 2$.

$E(Y|X=x) = 3x+1$.
$V(Y|X=x) = 2x^2+5$

$E(X) = \frac{a}{\lambda} = \frac{3}{2}$.

$V(X) = \frac{a}{\lambda^2} = \frac{3}{4}$

$E(Y) = E(E(Y|X))$
$= E(3X+1)$
$= 3E(X)+1$.
$= 4.5+1$
$= 5.5$

$V(Y) = E(V(Y|X)) + V(E(Y|X))$
$= E(2x^2+5) + V(3x+1)$
$= 2E(X^2)+5+9V(X)$

$V(X) = E(X)^2 - (E(X))^2$
$\frac{3}{4} = E(X)^2 - \frac{9}{4} \Rightarrow 3$.

$= 6+5+6.75$
$= 17.75$

7)

		X			
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

(i) Det $var(X|Y=2)$

$var(X|Y=2) = E(X^2|Y=2) - (E(X|Y=2))^2$.

$E(X|Y=2) = \Sigma x\, P(X=x, Y=2)$

$= 1(0) + 2(0.3) + 3(0.1) + 4(0.2)$

$= 1.7$.

$E(X^2|Y) = \Sigma x^2\, P(X=x, Y=2)$

$= 1(0) + 4(0.3) + 9(0.1) + 16(0.2)$

$= 5.3$.

$V(X|Y=2) = 5.3 - (1.7)^2$

$= 2.41$

Arthur:

6) $X \sim Poi(\lambda)$

$Y \sim Poi(\mu)$

$X - Y \sim Poi(\lambda - \mu)$

MGFs:

$M_x(t) = e^{\lambda(e^t - 1)}$

$M_y(t) = e^{\mu(e^t - 1)}$

$M_{x-y}(t) = E(e^{(X-Y)t})$

$= E(e^{Xt - Y(t)})$

$= E(e^{Xt}) E(e^{-Y(t)})$

$= M_x(t) M_y(t)$

$= e^{\lambda(e^t - 1)} \cdot e^{-\mu e^t - 1}$

$= e^{(e^t - 1)(\lambda - \mu)}$

$X - Y \sim Poi(\lambda - \mu)$

Hence proved

Christine

$X \sim Poi(\lambda)$ & $Y \sim Poi(\mu)$

$X + Y \sim Poi(\lambda + \mu)$

$M_x(t) = e^{\lambda(e^t - 1)}$

$M_y(t) = e^{\mu(e^t - 1)}$

$M_{x+y}(t) = E(e^{(x+y)(t)})$

$= E(e^{tx}) E(e^{yt})$

$= M_x(t) \cdot M_y(t)$

$= e^{(\lambda+\mu)(e^t - 1)}$

5) $f(x,y) = ax(x+y)$, $0 < x < 5$ & $10 < y < 20$

$\Rightarrow \int_0^5 \int_{10}^{20} ax(x+y)\, dy\, dx = 1.$

$\Rightarrow \int_0^5 a\left(x^2y + \frac{xy^2}{2}\right)_{10}^{20} dx = 1$

$\Rightarrow \int_0^5 a(10x^2 + 150x)\, dx = 1$

$\Rightarrow a\left(\frac{10x^3}{3} + \frac{150x^2}{2}\right)_0^5 = 1$

$a\left(\frac{1250}{3} + 1875\right) = 1 \Rightarrow a = \frac{3}{\cancel{3}\cancel{5000}}.$

$a = \frac{\cancel{3}}{\cancel{5745}} \quad \frac{3}{6875}$

$P(Y > \frac{1}{3}x + 10) = \int P(Y > \frac{1}{3}x + 10 \mid Y = y) f_Y(y) dy$

$0 < x < 5 \Rightarrow 0 < \frac{1}{3}x + 10 < 5$

$-10 < 1/3x < -5 \Rightarrow -30 < x < -15$

$\Rightarrow \int_{10}^{20} (3Y - 10 > x \mid Y = y)$

4) PGF of $V \Rightarrow G_V(t) = \left(\dfrac{p}{1-qt}\right)^k.$

$p+q=1$

$G_V(t) = p^k (1-qt)^{-k}.$

$$G_X(t) = E(t^X)$$

$$= \sum_{\text{all } x} t^x P(X=x)$$

$$= t^0 p_0 + t^1 p_1 + t^2 p_2 + t^3 p_3 + t^4 p_4$$

$\Rightarrow$ $G_V(t) = E(t^V)$

$P(X=x) = p^k (1-qt)^{-k}.$

$\Rightarrow t^x \left(\dfrac{p}{1-qt}\right)^k$

$$P(X=n) = \frac{1}{n!} G_X^{(n)}(0) = \frac{1}{n!} \frac{d^n}{ds^n} (G_X(s)) \Big|_{s=0}$$

$$P(V=4) = \frac{1}{4!} G''''(X)(0)$$

~~$g'(x) = k($~~

3) $f(x) = \lambda e^{-\lambda(x-a)}$

(i) $M_x(t) = E(e^{tx})$

$$= \int_a^\infty e^{tx} \lambda e^{-\lambda(x-a)} \, dx$$

$$= \int_a^\infty \lambda e^{\lambda a} e^{-(\lambda - t)} \, dx$$

$$= -\left[\frac{\lambda e^{\lambda a} . e^{-(\lambda - t)}}{\lambda - t} \right]_a^\infty$$

$$= \frac{\lambda}{\lambda - t} e^{ta}$$

(ii) $M_x(t) = \left(1 - \frac{t}{\lambda}\right)^{-1} e^{ta}$

$M'x(t) = \left(\frac{+1}{\lambda}\right)\left(1 - \frac{t}{\lambda}\right)^{-2} e^{ta} + a\left(1 - \frac{t}{\lambda}\right)^{-1} e^{ta}$

$E(X) = M'(x)(0) = \frac{1}{\lambda} + a$

$E(X^2) = M''(x)(0)$

$M''x(t) = \frac{+2}{\lambda^2}\left(1 - \frac{t}{\lambda}\right)^{-3} e^{ta}$.

$M''x(0) = E(X^2) = \frac{2}{\lambda^2} + \frac{2a}{\lambda} + a^2$

$var(x) = \frac{2}{\lambda^2} + \frac{2}{\lambda} + a \left(\frac{1}{\lambda} + a\right)^2 = \frac{1}{\lambda^2}$

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Q.2 $P(N=n) = \binom{n+2}{n} 0.9^3 0.1^n$

$X \sim$ Gamm $(6,2)$ ($X =$ claims amt)

$Y \sim$ total claim amt

Negative binom $= \binom{x-1}{k-1} p^k (1-p)^{x-k}$.

$x-1 = n+2 \Rightarrow x = 5$

$k = 3$ & $k-1 = n \Rightarrow n = 2$

$N \sim$ No of claims $\Rightarrow$ Neg BP$(0.9, 8)$

Mgf of $Y = M_Y(t) = E(e^{ty})$.

$= E(e^{tx_1} e^{tx_2})$ $\times \sim$ Gamma $\underset{a\ \lambda}{(6,2)}$

$= E(e^{tx}) E(e^{tx_1}) \Rightarrow (M_{x_1}(t))^n$

$= (1 - t/2)^{-6n}$

(ii) standard deviation $= \sqrt{6^2}$

$V(S) = V(E(S|N)) + E(V(S|N))$

$= V[E(X_1 + X_2 + X \ldots X_n (N))]$

$= V(n. E(X) + E(V(X).n)$

$= V(X) E(N) + (E(X))^2 V(N)$.

$= \frac{6}{4} \times \frac{(2)(0.1)}{0.9} + \frac{6}{2} \times \frac{6}{2} \times \frac{2}{0.3}$

$= 570/27$

$= 20.57$

$sd = 4.513$

Q.1