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Assignment 1

1) If $S_x \sim U(0,1)$

$$P(S_x \leq s) = \frac{x}{365}$$

$\therefore$ For $u.q:x$

$$_{u}q_x = S_u \times q_x$$
$$= u \times q_x$$

2) a) 20 weeks

b) ~~2~~ 52 weeks

3) Left censoring: When previous record is unknown
Not present

Right censoring: When future record is unknown
Not present

Interval censoring: When censoring is intermediate
Calender years of surrender

Informative censoring: When censoring is part of an investigation ~~Not~~ Present as all unhealthy insured people would opt to stay while healthy ones surrender

6a 30 year old, female, smoker

b) $h_1(t) = h_0(t) \times \exp(0 + \cancel{30}0 - 0.05)$

$$= h_0(t) \times e^{-0.05}$$

$h_2(t) = h_0(t) \times \exp(0.01(40-30) + 0 + 0)$

$$= h_0(t) \times e^{0.1}$$

$$S(t) = \exp\left\{-\int_{s=0}^{t} h(s)\, ds\right\}$$

$$S_1(t) = \exp\left\{-\int_{s=0}^{t} h(s)\, e^{-0.05}\right\} = \exp\left[-e^{-0.05}\int_{s=0}^{t} h_0(s)\, ds\right]$$

$$\therefore S_2(t) = \exp\left[-e^{0.1}\int_{s=0}^{t} h_0(s)\,ds\right]$$

The ratio is $\dfrac{\exp(-e^{-0.05})}{\exp(-e^{0.1})} = 1.16642$

c) The survival function for $M, S, 30 = \exp\left(-e^{-0.05}\int_{s=0}^{t} h_0(s)\,ds\right)$

$$\therefore h_{M,S,40}(t) = h_0(t) \times \exp[0.1 + 0 - 0.05] = h_0(t) \times e^{0.05}$$

$$S_{M,S,40}(t) = \exp\left[-e^{0.05}\int_{s=0}^{t} h_0(s)\,ds\right]$$

The ratio between the two is $\dfrac{\exp(-e^{-0.05})}{\exp(-e^{0.05})} = 1.10522$

$$\frac{h_1(t)}{h_2(t)} = \frac{e^{-0.05}}{e^{0.05}} = 0.904$$

$$S(t) = \exp\left(-\int_0^t h(s)\,ds\right)$$

$$\therefore S_1(t) = \{S_2(t)\}^{0.904}$$

Which implies that $S_1(t) < S_2(t)$ for all $t > 1$

Q7 i) We will use age next birthday as both the given variables are as per that convention

Since curtate period is given, we will use our mortality rates for $\left(x - \frac{1}{2}\right)$.

ii) $E_x^c = \int_0^{10} P_x(t) = \frac{1}{2}\sum_{x=0}^{9} P_x(t) + P_x(t+1)$

$E_x = E_x^c + \frac{1}{2}\sum_{x=0}^{10} \theta_{x,t}$

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Q8 i) Right censoring: We don't know what happens to the patients after the observation ends. There's also a chance that the patients die after discharge

Informative censoring: We don't know for sure if the deaths were due to the infection

ii) Yes.

iii)

j	d_j	c_j	n_j	λ_j	$1-\lambda_j$	$S(t) = \prod(1-\lambda_j)$
5	1		13	0.0769	0.9231	0.9231
7	1		12	0.0833	0.9167	0.8462
14	1		11	0.0909	0.9090	0.7693
18	~~1~~	~~2~~ 1	10	0	1	0.7693
25		1	9	0	1	0.7693
28	1		8	0.125	0.875	0.6731
30		1	7	0	1	0.6731
34		1	6	0	1	0.6731
35	1		5	0.2	0.8	0.5385
36		1	4	0	1	0.5385
45		3	3	0	1	0.5385

Assumptions: censoring just after death
Discharge only because of recovery

iv) Only 2 of 11 died so their survival rate is right but:
1) They are ignoring deaths in first 2 weeks
2) They are ignoring status after discharge
3) The overall survival rate is around 54% as per iii)

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Q9 UDD

$\int_0^1 {}_tp_x \, dt = \int_0^1 1 - t q_x \, dt = 1 - 0.5 q_x$

$q_x = 0.3$

$m_x = \frac{0.3}{1 - 0.15} = 0.352941$

CFM:

$q_x = 1 - e^{-\mu}$

$\int_0^1 {}_tp_x \, dt = \int_0^1 e^{-\mu t} = \frac{1}{\mu}(1 - e^{-\mu}) = \frac{q_x}{\mu}$

$\therefore m_x = \mu = -\ln(1 - q_x) = 0.356675$

Q10 i) Because of proportionality to baseline hazard

ii) $h(t) = h_0(t) \times \exp(\beta_F z_f + \beta_D z_d + \beta_M z_m)$

iii) Annual premium, online distribution and direct debit

iv) $\exp(\beta_D) = (1 - 0.25) \exp(\beta_F + \beta_D + \beta_M)$

$\exp(\beta_D \cancel{\beta_F}) = \exp(\beta_F)$ — (2)

$\exp(\beta_M) = \frac{3}{4}(2\beta_D)$

From (2) $\beta_D = \beta_F$

$\exp(\beta_F) = \cancel{\frac{1}{4}}\, 0.75 \exp(2\beta_F + \beta_M)$

$\frac{4}{3} = \frac{e^{2\beta_F} \times e^{\beta_M}}{e^{\beta_F}}$

$\frac{4}{3} = e^{\beta_F + \beta_M}$

From (3) $\frac{4}{3} = \frac{e^{2\beta_F - \beta_M}}{\cancel{e^{\beta_M}}}$

$\therefore \beta_F + \beta_M = 2\beta_F - \beta_M$

$\beta_F = 2\beta_M = \beta_D$

$\frac{4}{3} = e^{3\beta_M}$

$\beta_M = 0.0959$

$\beta_F = \beta_D = \cancel{0.04795}\ 0.19179$

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11 ii)

T	S(t)	λ(t)	nt	dt	ct
0	1	0	12	0	
1	0.9167	0.0833	12	1	2
3	0.713	0.22	9	2	2
6	0.428	0.4	5	2	3

Total deaths = 5

Total censored = 7

12 i) It is a kind of exponential function

$\lambda_x = Bc^x$ where λx : force of mortality at age x

iii) Non-smoker female

iv) $\exp(-4 + 0.65) \times 1.05^4$

$= 0.04266$

v) $\lambda_s = \exp(B_0 + B_1 x_1)\, c^s$

$\lambda_t = \exp(B_0 + B_1 x_1 + 0.65)\, c^t$

$\cancel{\&}\; c^{s-t} = e^{0.65}$

$1.05^{s-t} = 1.9155$

$s - t = 13.32$

∴ Approx 13 years

14) i) $h(t) = h_0(t) \times \exp(\beta_P x_P + \beta_G x_G + B_L x_L + B_A x_A)$

$h_1(t) = h_0(t) \times \exp(-0.1\beta_P + 0.3\beta_G - 0.4 B_A)$

$S(t) = \int_0^2 h_i(t) = 0.3$

$h_0(t) = \dfrac{0.3}{\exp(-0.1B_P + 0.3B_G - 0.4B_A)}$

$h_2(t) = \dfrac{0.3}{\exp(-0.1B_P + 0.3B_G - 0.4B_A)} \times \exp(0.5B_P + 0.2B_L + 0.7B_A)$

$= 0.3 \times \exp(0.6B_P - 0.3B_G + 0.2B_L + 0.3B_A)$

15) i) 1) Simple to understand
2) More logical

ii) Rita : central exposed → 3 months
initial expose → 1 year

Sita : central exposed → 1 year
initial exposed → 1 year

Nita : central exposed → 9 month
initial exposed → 9 months

Gita : null

iii) Central exposed = 2 years
Initial exposed = 2.75 years