

Assignment

p. i)

(i) Constant force of mortality is an assumption to model fractional ages. It says the force of mortality between the age x & $x+1$ is (constant) constant.

(ii)

Under CFM.

$$p_x = e^{-\mu_x}$$

$$\therefore \mu = -\ln(p_x)$$

$$\mu_{55} = -\ln(1 - q_{55})$$

$$\mu_{55} = 0.00470 \approx 0.0047$$

Q.2)

(i) A select mortality is death rate of individual who have recently undergone some medical test. It is generally lower mortality rates as individual has passed ~~(a)~~ certain medical test.

(ii)

$$20P[25]:[30] = 20P[25] \times 20P[30]$$

$$= \frac{l_{us}}{l_{[25]}} \times \frac{l_{[30]}}{l_{[30]}}$$

$$= \frac{9801.3123}{9951.9913} \times \frac{9712.0728}{9923.7497}$$

$$= 0.9638520314$$

Q.3)

VDD

$$1.4 \text{ pA} = 1.4 \text{ pA}$$

$$= 1 - 0.5 \text{ pA} \times 0.9 \text{ pA}$$

$$= 1 - [(1 - 0.5 \text{ pA}) (1 - 0.9 \text{ pA})]$$

$$= 1 - \left[\left(1 - \frac{0.5 \times 1.4}{1 - 0.5 \text{ pA}} \right) (1 - 0.9 \text{ pA}) \right]$$

$$= 1 - \left[\left(1 - \frac{0.5 \times 0.00714}{1 - 0.5 \times 0.00714} \right) (1 - 0.9 \times 0.00714) \right]$$

$$= 0.0107309121$$

Q4)

$$P \ddot{a}_{35:\overline{30}|} = 5000000 \ddot{A}_{35:\overline{30}|} + 20P \ddot{A}_{35:\overline{30}|}$$

$$\ddot{a}_{35:\overline{30}|} = 14.35 \quad \text{[from tables]}$$

$$\ddot{A}_{35:\overline{30}|} = \ddot{A}_{35} - v^{30} \ddot{A}_{65}$$

$$= 0.04488 - A \times 0.40177 = 0.03251 \quad (A)$$

$$\ddot{A}_{35:\overline{30}|} = v^{30} \ddot{P}_{65}$$

$$= (1.02)^{-30} \left(\frac{8821.2612}{9894.4299} \right)$$

$$= 0.1552258147 \quad (A)$$

$$\therefore P(14.35 - 30 \times A) = 5000000 \times B$$

$$P = 16771.98381$$

Q. 5)

(i) $\bar{a}_x$

(ii)

$$\bar{a}_x = \frac{\int_0^{\infty} v^s \cdot s p_x \, ds}{\int_0^{\infty} v^s \cdot s p_x \, ds}$$

PV =

$$\bar{a}_{T_x}$$

, $T_x > 0$

$$EPV = \bar{a}_x = \int_0^{\infty} v^s \cdot s p_x \, ds$$

Q. 6)

$$(i) P = 5,000,000 A'_{45:\overline{20}|}$$

$$\begin{aligned} A'_{45:\overline{20}|} &= A_{45:\overline{20}|} - A_{45:\overline{20}|}^1 \\ &= 0.46998 - (1.04)^{-20} \left(\frac{8821.2612}{9801.3123} \right) \\ &= 0.05922801726 \end{aligned}$$

$$P = 296140.0863$$

$$\begin{aligned} \therefore \text{Total single premium} &= 10000 \times P \\ &= 2961400.863 \end{aligned}$$

$$(ii) N = 5000000 A'_{46:\overline{19}|} - 296140.0863$$

$$\begin{aligned} &= 5000000 \left[0.34599 - (1.06)^{-19} \left(\frac{8821.2612}{9786.9534} \right) \right] \\ &\quad - 296140.0863 \end{aligned}$$

$$= -55694.26417$$

$$\begin{aligned} \text{DSAR} &= 5000000 - (-55694.26417) \\ &= 5055694.26417. \end{aligned}$$

$$\begin{aligned} \text{EIS} &= 10000 \text{ g}_{45} \times \text{DSAR} \\ &= 74065920.97 \end{aligned}$$

$$\begin{aligned} \text{AOS} &= 10 \times \text{DSAR} \\ &= 50556942.6417. \end{aligned}$$

$$\begin{aligned} \text{Mental MP} &= \text{EIS} - \text{AOS} \\ &= 23508978.33 \end{aligned}$$

Q7)

$$FV = 100000 A_{\overline{20}|i} + 200000 A_{60|\overline{20}|i}$$

$$= 100000 \left(A_{\overline{20}|i} - v_{\overline{20}|i} A_{60|\overline{20}|i} \right) + 200000 v_{\overline{20}|i} A_{60|\overline{20}|i}$$

$$v_{\overline{20}|i} = (1.02)^{-20} \left(\frac{9287.2164}{9856.2863} \right)$$

$$= 0.2938021365 \quad (A)$$

$$A_{\overline{20}|i} = \cancel{0.73056} \cdot 0.12313$$

$$\cancel{FV} A_{60} = \cancel{0.45640} \cdot 0.32192$$

$$FV = \cancel{36465.1295} + 21917.97945$$

$$E(PV^2) = 100000^2 \left(2 A_{40}^{100} \right) + 200000^2 A_{60}^{200}$$

$$2 A_{40}^{100} = 2 A_{40} - 2 A_{40}^{100} A_{60}^{100}$$

$$= \frac{0.02907}{0.02907} - (1.08) \left(\frac{0.14098}{0.14098} \right) \left(\frac{0.287.2164}{0.287.2164} \right)$$

$$= 0.01415$$

$$E(PV^2) = 1798358788$$

$$SD = \sqrt{E(PV^2) - [E(PV)]^2}$$

$$= 36303.73211$$

Q.8. $A_{BS} = 0.5412$

$A_{CG} = 0.5591$

$$A_{BS} = q_{BS} + (1 - q_{BS}) A_{CG} V$$

$$A_{BS} = q_{BS} + A_{CG} V - A_{CG} V q_{BS}$$

$$A_{BS} - A_{CG} V = q_{BS} (1 - A_{CG} V)$$

$$\frac{0.5412 - 0.5591(1+0.08)}{1 - 0.5591(1.08)^{-1}} = q_{BS}$$

$$q_{BS} = 0.048756$$

$$A_{[GS]} = 0.75 q_{BS} + \frac{(1 - 0.75 q_{BS}) 0.5591}{1.08}$$

$$A_{[GS]} = 0.535321$$

$$P = 100000 \times 0.535321$$

$$P = 53532.12963$$

$$p.9) (i) \quad P = 120000 a_{\overline{10}|0.05}$$

$$P = 120000 (\overline{a}_{\overline{10}|0.05} - 1)$$

$$= 120000 (10.562)$$

$$P = 1267440.$$

$$(ii) \quad P(L_0 > 0) = 1$$

$$P(L_0 \leq 0 \mid G = G_M) = P(K_{\overline{10}|0.05} \geq n)$$

(10)

$$(i) PV = \bar{a}_{\overline{12}|i}$$

$$FPV = \int_0^{\infty} \bar{a}_{\overline{t}|i} \cdot (P_x - \mu_x) dt = \bar{a}_x$$

(ii)

we know $\mu = 0.0225$ and $S = 16$

$$\int_0^{\infty} e^{-(\mu+\delta)t} dt = 16$$

$$= \frac{1}{-(\mu+\delta)} \left[e^{-(\mu+\delta)t} \right]_0^{\infty} = 16$$

$$= \frac{1}{-(\mu+0.04)} \left[0 - 1 \right] = 16$$

$$1 = 16\mu + 0.64$$

$$\mu = \frac{1-0.64}{16}$$

$$\mu = 0.0225$$

$$V_m = \frac{1}{8^2} \left(2\bar{A}_m - (\bar{A}_m)^2 \right)$$

$$\bar{A}_m = 1 - 0.36 = 0.64$$

$$\bar{A}_m = 1 - 0.36 = 0.64$$

$$V_m = 0.0225$$

$$e^{-\bar{A}_m} = \int_0^{\infty} e^{-2\delta t} e^{-\mu t} \mu dt$$

$$= 0.0225 \left[\frac{e^{-(2\delta + \mu)t}}{-(2\delta + \mu)} \right]_0^{\infty}$$

$$= \frac{-0.0225}{(2 \times 0.0225 + 0.04)} [0 - 1]$$

$$\frac{9}{34}$$

$$= SD = \sqrt{\frac{1}{0.04} \left(\frac{9}{34} - (0.36)^2 \right)}$$

$$= 9.18918802$$

Q11)

- (i) In term assurance the premiums are paid annually in advance for the term. where if a policy holder fails to pay premium he or she will no longer be in the force with the contract where as the annuity contract is a single premium contract, the policy holder would have paid the premium at the start and is in force till he/she dies.

(ii)

Term Assurance

$$P \ddot{a}_{40:\overline{25}|} = 2,500,000 A'_{40:\overline{25}|}$$

$$A'_{40:\overline{25}|} = A_{40:\overline{25}|} - A_{40:\overline{25}|}^1$$

$$= 0.38907 - (1.04)^{-25} \left(\frac{8821.2612}{9856.2863} \right)$$

$$= 0.05334485019 \quad (a)$$

$$P = \frac{2,500,000 \times A}{15.814}$$

$$P = 8396.002296$$

(B)

$${}_{10}V = 250000 \cdot A'_{50:\overline{15}|} - P \ddot{a}_{50:\overline{15}|}$$

$$A'_{50:\overline{15}|} = A_{50:\overline{15}|} - A_{50:\overline{15}|}$$

$$= 0.56719 - (1.04)^{-15} \left(\frac{8821.2612}{9712.0728} \right)$$

$$= 0.062885$$

$${}_{10}V = 87334.42964 \quad (M)$$

Annuity.

$${}_{10}V = 250000 \ddot{a}_{60} - 355450 \quad (N)$$

$$= 3908000$$

$$\text{Total portfolio reserve} = 8500 \times M + 9900 \times N$$

$$= 8500 \times 87334.42964 + 9900 \times 3908000$$

$$= 39431542050$$

(iii) Premiums of term Assurance = 2000

Investments in different securities.

$$(1000 \cdot 10) + (1000 \cdot 10)$$

$$(1000 \cdot 10) + (1000 \cdot 10)$$

$$= 20000000$$

Q.12

$$\begin{aligned}
 (i) \quad S_{55.107} &= a_{55.107} (1+i)^{10} \cdot \rho_{55} \\
 &= \left(\frac{8421.2612}{9557.8179} \right) (1.06)^{10} \times 7.161 \\
 &= 12.57810568
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad \ddot{a}_{(u)}^{(u)} &= \ddot{a}_{(u)}^{(u)} \cdot v^{20} \cdot \ddot{a}_{60}^{(u)} \\
 &= \ddot{a}_{(u)}^{(u)} - \frac{3}{8} - v^{20} \left(\ddot{a}_{60}^{(u)} - \frac{3}{8} \right) \\
 &= \left(15.494 - \frac{3}{8} \right) - (1.06)^{-10} \left(11.891 - \frac{3}{8} \right) \\
 &= \frac{1287.2164}{9654.3036} \\
 &= 14.67350635
 \end{aligned}$$

0.13

$$(12) \quad P_{\vec{a}_{45:25}} = 100,000 A_{45:25} + 100,000 A_{45:25}$$

$$\vec{a}_{45:15} = 11.886$$

$$A_{45:25} = 0.46991$$

$$A_{45:25} = (1.04)^{-20} \cdot {}_{20}P_{45}$$

$$= 0.4107519827$$

$$P = 7734.608748$$

$$(13) \quad V = 100000 A_{58:\overline{7}} + 100000 A_{58:\overline{7}} \cdot P_{\vec{a}_{58:2}}$$

$$A_{58:\overline{7}} = 0.76516$$

$$A_{58:\overline{7}} = (1.04)^7 \times {}_7P_{58}$$

$$= 0.712085$$

$$\vec{a}_{58:2} = 1.955$$

$$13V = 132603.4289$$

$$DSAR = 100000 - \frac{1}{2}V$$

$$= -32603.42895$$

$$EOS = 1991 \cdot 0.00565 \times DSAR$$

$$= -36657.66534$$

$$ADS = 4 \times DSAR$$

$$= -130413.7158$$

$$M.P = EOS - ADS$$

$$= 93756.05046$$

Q.14)

(i)

$$5000 \bar{a}_{\overline{T_{63}-2}|} v^2 \quad ; \quad T_{63} \leq 2$$

$$5000 \bar{a}_{\overline{T_{63}}|} v^2 \quad ; \quad T_{63} > 2$$

(ii)

$$5000 \times 100 \times \frac{1}{2} p_{63} \times v^2 \bar{a}_{63}$$

$$= 500000 (14.871 - 0.5) (1.04)^{-2} \left(\frac{9703.108}{9775.888} \right)$$

$$= 6594347.317$$

(iii)

$$E(x^2) = 5000 \times 100 \times v^4 \times \frac{1}{2} p_{63}$$

$$\text{Var} = 5000^2 \times 100 \times \frac{1}{\delta^2} \left(v^{2n} n q_n + \delta^2 v^{2n} n p_n^2 \bar{A}_{n:n} - (v^n n q_n + v^n n p_n (\bar{A}_{n:n})^2) \right)$$

$$\Delta = 5000^2 \times 100 \times \frac{1}{\delta^2} \left(v^{2 \times 2} \frac{1}{2} p_{63} + v^{2 \times 2} \frac{1}{2} p_{63}^2 \bar{A}_{63} - (v^2 \frac{1}{2} p_{63} + v^2 \frac{1}{2} p_{63} (\bar{A}_{63})^2) \right)$$

$$\bar{A}_{65} = 1 - \delta \bar{a}_{65}$$

$$\bar{A}_{65} = 0.436359 \quad (B)$$

$$\begin{aligned} {}^2\bar{A}_{65} &= \frac{1}{\delta} (1 - \delta) (0.20847) \\ &= 0.2125985 \quad (C) \end{aligned}$$

$$\begin{aligned} {}^2q_{63} &= \frac{9703.708}{9703.708 + 19175.988} \\ &= 0.9928 \quad (D) \end{aligned}$$

$$\begin{aligned} {}^2q_{63} &= 1 - D \\ &= 0.007383 \quad (E) \end{aligned}$$

$$Var = 5000^2 \times 100 \times 13.53$$

$$= (182937.6604)^2$$

Q.15)

$$a = \frac{(1 + r + p)(1 + i)^{11/12} - (1 + i)^{10/12}}{(1 + i)^{10/12} - 1} \times \frac{1}{(1 + i)^{10/12}} = \frac{1 + r + p}{(1 + i)^{10/12} - 1} \times \frac{1}{(1 + i)^{10/12}}$$

(1 + r + p)

b

$$(1 + r + p)(1 + i) - 1 = \frac{R}{(1 + i)^{10/12} - 1} \times \frac{1}{(1 + i)^{10/12}}$$

$$R = \frac{(1 + r + p)(1 + i) - 1}{(1 + i)^{10/12} - 1} \times \frac{1}{(1 + i)^{10/12}}$$

(1 + r + p)(1 + i) - 1

(1 + r + p)(1 + i) - 1

$$R = \frac{(1 + r + p)(1 + i) - 1}{(1 + i)^{10/12} - 1} \times \frac{1}{(1 + i)^{10/12}}$$

18830.15501