

Maths Homework - ①

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Ans ①	x	y	Δy	Δy^2	Δy^3
	7	3	-2	2	6
	8	1	0	8	
	9	1	8		
$x_n \rightarrow$	10	9	$\nabla f(x_n)$	$\nabla^2 f(x_n)$	$\nabla^3 f(x_n)$

$$h = 1, \quad u = \frac{x - x_n}{h} \Rightarrow \frac{9.5 - 10}{1} \Rightarrow -0.5$$

$$\text{So, } f(9.5) = f(x_n) + u \nabla f(x_n) + \frac{u(u-1)}{2!} \nabla^2 f(x_n) + \frac{u(u-1)(u-2)}{3!} \nabla^3 f(x_n)$$

$$f(9.5) = 9 + (-0.5)(8) + \frac{(-0.5)(-0.5-1)}{2} (8) + \frac{(-0.5)(-0.5-1)(-0.5-2)}{6}$$

$$\Rightarrow 9 + (-4) + \frac{(-0.5)(-1.5)}{2} (8) + \frac{(-0.5)(-1.5)(-2.5)}{6}$$

$$\Rightarrow 5 + 3 - 1.875 \Rightarrow \boxed{6.125} \quad \boxed{6.125}$$

$$\text{So, } \boxed{f(9.5) = 6.125} \quad \text{Solution.}$$

Soln

Ques 2)

x	3	7	9	10
y	168	120	72	63

$$f(6) = ?$$

~~(6-3)(6-7)~~

$$\Rightarrow \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 - \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$\Rightarrow (12) + (180) + (-72) + (+27) \Rightarrow \boxed{20} \boxed{147}$$

$$\boxed{f(6) = 147}$$

sol ③

x	1	2	7	8
y	4	5	5	4

$$f(6) = ?$$

$$\left[\frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 \right] + \left[\frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5 \right] +$$

$$\left[\frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 \right] + \left[\frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4 \right]$$

$$\Rightarrow (-0.762) + (2.6667) + (6.6667) + (-1.904761905)$$

$$\text{so, } \boxed{f(6) = 3.666}$$

← solⁿ

Q4

x	1	2	3	4	7
y	2	4	8	16	128

; $f(5) = ?$

$$f(5) = \left[\frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 \right] + \left[\frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 \right]$$

$$+ \left[\frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 \right] + \left[\frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 \right]$$

$$+ \left[\frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128 \right]$$

$$\Rightarrow f(5) = -0.6667 + 6.4 + (-24) + 42.6667 + 8.5333$$

$$f(5) = 32.9333$$

Q5

x	10	20	14	16
y	0.25	0.2	0.15	0.10

; $f(15) = ?$

$$f(15) = \left[\frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25 \right] + \left[\frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2 \right]$$

$$+ \left[\frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15 \right] + \left[\frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.1 \right]$$

$$= -0.00417 - 0.00521 + 0.078125 + 0.052083$$

$$\Rightarrow 0.203233 \quad \boxed{0.180828} \leftarrow \text{solution}$$

Homework-2

Regular falsi.

Sol ①

$$y = x^3 + x - 1$$

x	$f(x)$
0.5	-0.375

1	1
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$$\bar{x} = \frac{0.5(1) - (-0.375)(1)}{1 - (-0.375)} = 0.63636.$$

$$\text{so, } \bar{y} = (0.63636)^3 + (0.63636) - 1 = -0.10594$$

x	$f(x)$
0.63636	-0.10594

1	1
---	---

$$\bar{x}_1 = \frac{0.63636(1) - (-0.10594)}{1 - (-0.10594)} = 0.6711937$$

$$\bar{y}_1 = (0.6711937)^3 + (0.6711937) - 1 = -0.0264$$

x	$f(x)$
0.67119	-0.0264
1	1

$$\bar{x}_2 = \frac{(0.67119) - (-0.0264)}{1 - (-0.0264)} = 0.67965$$

$$\bar{y}_2 = (0.67965)^3 + (0.67965) - 1 = -0.006522$$

→ so, $\therefore$ approx value of the root is 0.67965 ← Solⁿ.

Sol (2)

$$y = x^3 - 2x + 0.5$$

So,

x	$f(x)$
0	0.5
1	-0.5

$$x = \frac{-0.5(0) - 0.5}{-0.5 - 0.5} = 0.5$$

$$y = (0.5)^3 - 2(0.5) + 0.5 = -0.375$$

x	$f(x)$
0	0.5
0.5	-0.375

$$x_1 = \frac{0(-0.375) - (0.5)^2}{-0.375 - 0.5} \Rightarrow 0.2857$$

$$y = (0.2857)^3 - 2(0.2857) + 0.5 = -0.0481$$

x	$f(x)$
0.2857	-0.0481
0	0.5

$$x_2 = \frac{(0.5)(0.2857) - (-0.0481)(0)}{0.5 + 0.0481} = 0.26063$$

$$y = (0.26063)^3 - 2(0.26063) + 0.5 = -0.003556$$

The approx value of the required root is $\boxed{0.26063}$

Q1 (3)

$$y = x^3 + 2x - 20 = 0$$

x	$f(x)$
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2	-8
---	----

2.5	0.625
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$$\bar{x}_1 = \frac{2(0.625) - (-8 \times 2.5)}{0.625 + 8} = 2.46377$$

$$\bar{y} = (2.46377)^3 + 2(2.46377) - 20 = \cancel{2.46377} - 0.11697$$

x	$f(x)$
-----	--------

2.46377	-0.11697
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2.5	0.625
-----	-------

$$2.46948$$

$$\bar{x}_1 = \frac{2.46377(0.625) - (-0.11697 \times 2.5)}{0.625 + 0.11697} = \cancel{2.46377}$$

$$\bar{y}_1 = (2.46948)^3 + (2.46948)(2) - 20 = -0.001303$$

$\therefore$ Approx value of root is 2.46948

Q1 (4)

$$y = 3x - \cos x - 1$$

$$0 < 1$$

x	$f(x)$	$\Rightarrow \bar{x}_1 = \frac{2(0) - (-2)(1)}{2+2} \Rightarrow 0.5$
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0	-2
---	----

1	2
---	---

$$\bar{y} = -0.49996$$

x	$f(x)$
-----	--------

0.5	-0.49996
-----	----------

1	2
---	---

$$\bar{x}_1 = \frac{2(0.5) + (0.49996)(1)}{2 + 0.49996} \Rightarrow 0.5$$

$$\bar{y}_1 = -0.199975$$

x	$f(x)$	$\bar{x}_2 = \frac{0.59999(2) + 0.199975}{2.199975}$
0.59999	-0.199975	
1	2	= 0.63635

$$\bar{y}_2 = -0.090888$$

x	$f(x)$	$\bar{x}_3 = \frac{2(0.63635) + 0.090888}{2.090888}$
0.59999	-0.199975	
0.63635	-0.090888	
1	2	= 0.652156

$$\bar{y}_3 = -0.043467$$

x	$f(x)$	$\bar{x}_4 = \frac{2(0.652156) + 0.043467}{2.043467}$
0.652156	-0.043467	
1	2	

$$\bar{y}_4 = 0.659555$$

$$\bar{y}_4 = -0.02127$$

x	$f(x)$	$\bar{x}_5 = \frac{2(0.659555) + 0.02127}{2.02127}$
0.659555	-0.02127	
1	2	

$$\bar{y}_5 = 0.66314$$

$$\bar{y}_5 = -0.0105$$

x	$f(x)$	$\bar{x}_6 = \frac{2(0.66314) + 0.0105}{2.0105}$
0.66314	-0.0105	
1	2	

$$\bar{y}_6 = 0.6649$$

$$\bar{y}_6 = -0.00523$$

so, approx value of required root is $\Rightarrow \underline{\underline{0.6649}}$

Sol ⑥

$$y = x^3 - 3x - 5$$

2 and 2.5

x $f(x)$

2 -3

2.5 3.125

$$\bar{x} = \frac{3.125(2) + 3(2.5)}{6.125} = 2.244897$$

$$\bar{y} = -0.4213777$$

x

$f(x)$

2.244897

-0.4213777

2.5

3.125

$$\bar{x}_1 = \frac{2.244897(3.125) + 0.4213777(2.5)}{3.125 + 0.4213777}$$

$$= 2.2752$$

$$\bar{y}_1 = -0.04795$$

x

$f(x)$

2.2752

-0.04795

2.5

3.125

$$\bar{x}_2 = \frac{3.125(2.2752) + 0.04795(2.5)}{3.125 + 0.04795}$$

$$= 2.2786$$

$$\bar{y}_2 = -0.00526$$

So, $\therefore$ the approx value of required root is 2.2786

Sol ⑥

$$y = x \log_{10} x - 1.2$$

2 & 3

x

$f(x)$

2

-0.59794

3

0.23136

$$\bar{x}_1 = \frac{0.23136(2) + 0.59794(3)}{0.23136 + 0.59794}$$

$$= 2.72102$$

$$\bar{y}_1 = -0.017086$$

x

$f(x)$

2.72102

-0.017086

3

0.23136

$$\bar{x}_1 \Rightarrow \frac{0.23136(2.72102) + 3(0.017086)}{0.23136 + 0.017086}$$

$$\Rightarrow 2.7394$$

$$\bar{y}_1 = -0.001086$$

$\therefore$ The approx value of required root is 2.7394 ✓ Sol

Q1

$$y = x^3 - 4x + 1$$

$$\bar{x} = \frac{(-2)(0) - 1}{-2 - 1} = 0.3333$$

$$\bar{y} = -0.296174$$

$$\bar{x}_1 = \frac{-0.3333}{-1.296174} = 0.257141$$

$$\bar{y} = -0.01156$$

$$\bar{x}_2 = \frac{-0.257141}{-1.01156} = 0.254202$$

$$\bar{y}_2 = -0.0003818$$

So, the approx value is: 0.254202

Q2

$$y = x^3 + 2x^2 - 8$$

$$\bar{x} = \frac{8 - (-5 \times 2)}{8 + 5} = 1.3846$$

$$\bar{y} = -2.57636$$

$$\bar{x}_1 = \frac{8(1.3846) + (-2.57636)(2)}{8 + 2.57636} = 1.53451$$

$$\bar{y}_1 = 0.32278$$

$$\bar{x}_2 = \frac{1.3846(0.32278) + 2.57636(1.53451)}{0.32278 + 2.57636} = 1.51782$$

$$\bar{y}_2 = 0.10427$$

x	$f(x)$
1.3846	-2.57636
1.51782	0.10427

$$\bar{x}_3 = \frac{1.3846(0.10427) + 2.57636(1.51282)}{0.10427 + 2.57636}$$

$$= 1.51264.$$

$$\bar{y}_3 = 0.0372$$

x	$f(x)$
1.3846	-2.57636
1.51264	0.0373

$$\bar{x}_4 = \frac{0.0373(1.3846) + (-2.57636)(1.51264)}{0.0373 + 2.57636}$$

$$\Rightarrow 1.51082$$

$$\bar{y}_4 = 0.01372$$

x	$f(x)$
1.3846	-2.57636
1.51082	0.01372

$$\bar{x}_5 = \frac{(1.3846)(0.01372) + (-2.57636)(1.51082)}{0.01372 + 2.57636}$$

$$= 1.51015$$

$$\bar{y}_5 = 0.00508$$

So, approx value is : 1.51015

30) ⑨ $y = x^3 - 9x + 1 = 0$

x	$f(x)$
0	1
1	-7

$$\bar{x} = \frac{-7(0) - 1}{-7-1} \Rightarrow 0.125$$

$$\bar{y} = -0.12305$$

x	$f(x)$
0	1

$$\bar{x}_1 = \frac{-0.125}{-0.12305-1} = 0.111304$$

0.125	-0.12305
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$$\bar{y}_1 = -0.000357$$

So, the required approx value is 0.111304 Sol.

30) ⑩ $y = x^3 - x - 4 = 0$

x	$f(x)$
1	-4
2	2

$$\bar{x} = \frac{2(1) + 4(2)}{6} = 1.6667$$

$$\bar{y} = -1.03688$$

x	$f(x)$
1.6667	-1.0368
2	2

$$\bar{x}_1 = \frac{2(1.6667) + (1.0368)(2)}{2 + 1.0368}$$

$$= 1.7805$$

$$\bar{y}_1 = -0.136$$

x	$f(x)$
1.7805	-0.136
2	2

$$\bar{x}_2 = \frac{2(1.7805) + 2(0.136)}{2 + 0.136}$$

$$= 1.7945$$

$$\bar{y}_2 = -0.0158$$

x	$f(x)$
1.7945	-0.0158
2	2

$$\bar{x}_3 = \frac{1.7945(2) + 2(0.0158)}{2 + 0.0158} \Rightarrow 1.7961$$

$$\bar{y}_3 = -0.00184$$

So, the approx value is -0.00184 ← Sol.