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Numerical Methods & Linear Algebra - Practice Questions - Unit 1

1.

Ans Original value = 687
New value = 787
Absolute change = New value - Original value
= 787 - 687
Absolute change = 107
Then,
Relative change = $\frac{\text{Absolute change}}{\text{Original value}}$

$= \frac{107}{687}$

Relative change = 0.1471

2

Ans For Las Vegas

New value = 565,000
Original value = 478,000
Absolute change = New value - Original value
= 565,000 - 478,000
Absolute change = 87,000
Then,

For Dallas

$$\text{Relative change} = \frac{\text{Absolute change}}{\text{Original value}}$$

$$= \frac{87000}{478,000}$$

Relative change = 0.182

For Dallas,

New value = 1,045,000

Original value = 1,189,000

Absolute change = New value - Original value

= 1,246,000 - 1,189,000

Absolute change = 56,000

Then,

$$\text{Relative change} = \frac{\text{Absolute change}}{\text{Original value}}$$

$$= \cancel{0}\ \frac{56000}{1,189,000}$$

Relative change = 0.0471

Thus, Las Vegas has increased more than Dallas in both absolute and relative terms.

3

Ans (i) Mean value of the period of oscillation $= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.8}{5}$

Mean $= 2.624$ s

(ii) Absolute error in each measurement.

Using the formula, ~~Absolute error = $\frac{\text{Expressed value}}{\text{Mean value}}$~~

Absolute error = Expressed value − Mean value

(a) For measurement $= 2.63$ s,

Absolute error $= 2.63 - 2.624$

$= 0.006$ s

(b) For measurement $= 2.56$ s

Absolute error $= \cancel{2.63}\ 2.56 - 2.624$

$= -0.064$ s

(c) For measurement $= 2.42$ s

Absolute error $= 2.42 - 2.624$

$= -0.204$ s

(d) For measurement $= 2.71$ s

Absolute error $= 2.71 - 2.624$

$= 0.086$ s

(e) For measurement $= 2.80$ s

Absolute error $= 2.80 - 2.624 = 0.176$ s

(iii) Mean absolute error $= \frac{0.006 + (-0.064) + (-0.204) + 0.086 + 0.176}{5}$

Mean absolute error = 0.00 s.

(iv) Relative error = $\frac{\text{Mean absolute error}}{\text{Mean value of period of osillation}}$

$= \frac{0}{2.624}$

Relative error = 0

(v) Percentage error = Relative error × 100

= 0 × 100

Percentage error = 0%

4

Ans (a) Given that: $f(x) = x^3 - 7x^2 + 14x - 6$

[0, 1] (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	3.17188
0.5	0.75	0.625	-0.625	3.17188	0.25977
0.5	0.625	0.5625	-0.625	0.25977	-0.16187
0.5625	0.625	0.59375	-0.16187	0.25977	0.054047

0.5625	0.59375	0.578125	−0.16187	0.054047	−0.052624
0.578125	0.59375	0.585938	−0.052624	0.054047	0.001035
0.578125	0.585938	0.582032	−0.052624	0.001035	−0.025711

Approximate root is 0.582032

(b) [1, 3.2] (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1	3.2	2.1	2	−0.112	1.791
2.1	3.2	2.65	1.791	−0.112	0.552125
2.65	3.2	2.925	0.552125	−0.112	0.085828
2.925	3.2	3.0625	0.085828	−0.112	−0.05444
2.925	3.0625	2.99375	0.085828	−0.05444	0.006328
2.99375	3.0625	3.028125	0.006328	−0.05444	−0.026521
2.99375	3.028125	3.010938	0.006328	−0.026521	−0.010697
2.99375	3.010938	3.002344	0.006328	−0.010697	−0.002333
2.99375	3.002344	2.998047	0.006328	−0.002333	0.001961
2.998047	3.002344	3.0001955	0.001961	−0.002333	−0.000195
2.998047	3.0001955	2.9991213	0.001961	−0.000195	0.0008802
2.9991213	3.0001955	2.9996584	0.0008802	−0.000195	0.0003418

Approximate root is 2.9996584

c. [3.2, 4] (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
3.2	4	3.6	−0.112	2	0.336
3.2	3.6	3.4	−0.112	0.336	−0.016
3.4	3.6	3.5	−0.016	0.336	0.125
3.4	3.5	3.45	−0.016	0.125	0.046125
3.4	3.45	3.425	−0.016	0.046125	0.0130156
3.4	3.425	3.4125	−0.016	0.0130156	−0.001998
3.4125	3.425	3.41875	−0.001998	0.0130156	0.0053816

Approximate root is 3.41875

5.

Ans (a) Given that: $f(x) = 3(x+1)\left(x-\frac{1}{2}\right)(x-1)$
[−2, 1.5]

To find: p_3 (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
−2	1.5	−0.25	−22.5	3.75	2.109375
−2	−0.25	−1.125	−22.5	2.109375	−1.2949219
−1.125	−0.25	−0.6875	−1.2949219	2.109375	1.87866211

$p_3 = -0.6875$

(b) $[-1.25, 2.5]$

To find : p_3 (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
-1.25	2.5	0.625	-2.95313	31.5	-0.228516
0.625	2.5	1.5625	-0.228516	31.5	4.5944824
0.625	1.5625	1.09375	-0.228516	4.5944824	0.3496399

$p_3 \simeq 1.09375$

6 (a)

Ans. Given that : $f(x) = 3x - e^x$ for $1 \le x \le 2$

(Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1	2	1.5	0.281718	-1.389056	0.0183109
1.5	2	1.75	0.0183109	-1.389056	-0.504603
1.5	1.75	1.625	0.0183109	-0.504603	-0.203419
1.5	1.625	1.5625	0.0183109	-0.203419	-0.083233
1.5	1.5625	1.53125	0.0183109	-0.083233	-0.030203
1.5	1.53125	1.515625	0.0183109	-0.030203	-0.0053904
1.5	1.515625	1.5078125	0.0183109	-0.0053904	0.00659811
1.5078125	1.515625	1.51171875	0.00659811	-0.0053904	0.0006385
1.51171875	1.515625	1.513671875	0.0006385	-0.0053904	-0.0023673
1.51171875	1.513671875	1.51269513	0.0006385	-0.0023673	-0.00086227
1.51171875	1.51269513	1.51220702	0.0006385	-0.00086227	-0.00011137

1.51171875	1.512207032	1.511962891	0.0006385	-0.00011137	0.0002637
1.511962891	1.512207032	1.512084962	0.0002637	-0.0001137	0.0000762
1.512084962	1.512207032	1.512145997	0.0000762	-0.0001137	-0.0000176
1.512084962	1.512145997	1.512115480	0.0000762	-0.0000176	0.0000293
1.512115480	1.512145997	1.512130739	0.0000293	-0.0000176	0.0000059
1.512130739	1.512145997	1.512138355	0.0000059	-0.0000176	-0.0000059

Approximate root is 1.51213

7. (a)

Ans. $f(x) = x^2 - 5$, for $x_1 = 2$

Using Newton Ramphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $n = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$$

$$\therefore x_2 = 2 - \frac{(2)^2 - 5}{2(2)}$$

$$\therefore x_2 = 2.25$$

For $n = 2$,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\therefore x_3 = 2.25 - \frac{(2.25)^2 - 5}{2(2.25)}$$

$$\therefore x_3 = 2.236111111$$

Ans. Approximate solution to $f(x) = x^2 - 5$ is 2.236111111.

7. (b)

Ans. $f(x) = x^3 - 3$, $x_1 = 1.4$

Using Newton Ramphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.4 - \frac{(1.4)^3 - 3}{3(1.4)^2}$$

$$x_2 = 1.443537415$$

For $n = 2$,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\therefore x_3 = 1.443537415 - \frac{(1.448537415)^3 - 3}{3(1.448537415)^2}$$

$$\therefore \underline{x_3 = 1.442250719}$$

Ans Approximate solution to $f(x) = x^3 - 3$ is 1.442250719

8) (a)

Ans $f(x) = x^3 + 48$

To start with, assuming $x_0 = -3.63$

Using Newton Ramphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $n = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = -3.63 - \frac{(-3.63)^3 + 48}{3(-3.63)^2}$$

$$\therefore x_1 = -3.634246143$$

For $n = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\therefore x_2 = (-3.634246143) - \frac{(-3.634246143)^3 + 48}{3(-3.634246143)^2}$$

$$\therefore x_2 = -3.634241186$$

8 (b)

Ans $f(x) = 2 - x^3$

To start with, assuming $x_0 = 1.26$

Using Newton Ramphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $n = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = (1.26) - \frac{(2) - (1.26)^3}{-3(1.26)^2}$$

$$x_1 = 1.259921055$$

For $n = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\therefore x_2 = 1.259921055 - \frac{2 - (1.259921055)^3}{-3(1.259921055)^2}$$

$$\underline{x_2 = 1.259921050}$$