P&S - 2 Assignment
Roll no. 10.

1) $X \sim Poi(\theta)$

$$\frac{x-\theta}{\sqrt{\theta/n}} \sim N(0,1)$$

$$\therefore \theta = x \pm 1.96\left(\sqrt{\theta/n}\right).$$

2) $X_i \sim N(\mu, \sigma^2).$

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right).$$

$$E(\bar{x}) = E\frac{(\Sigma x_i)}{n} = \frac{n\mu}{n}$$

$$= \mu$$

$$Var(\bar{x}) = \frac{Var(\Sigma x_i)}{n}$$

$$= \frac{1}{n^2}(n\sigma^2).$$

$$= \frac{\sigma^2}{n}.$$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1} \qquad E(s^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$$

$$var(s^2) = \frac{2(n-1)\sigma^4}{(n-1)^2}$$

$$= \frac{2\sigma^4}{n-1}$$

Camlin Page
Date / /

3) $n=4$ $\quad Bin(4,p)$ $\quad$ 180 policies.

i)

No. of claims.	0	1	2	3	4
No. of policies	86	75	16	2	1

$$L(p) = \left((1-p)^4\right)^{86} \left(4p(1-p)^3\right)^{75} \left(6p^2(1-p)^2\right)^{16} \left(4p^3(1-p)\right)^2 \left(p^4\right)^1$$

$$= (1-p)^{603} \, p^{(117)} \text{ constant.}$$

$$\ln L(p) = 603 \ln(1-p) + 117 \ln p$$

$$\frac{d}{dp} \ln L(p) = \frac{-603}{1-p} + \frac{117}{p} = 0.$$

$$\therefore p = 0.1625$$

ii) $P(x=0) = (1-p)^4 = 0.492.$

$P(x=1) = 4p(1-p)^3 = 0.3818.$

$P(x=2) = 6p^2(1-p)^2 = 0.111$

$P(x=3) = 4p^3(1-p) = 0.0144.$

$P(x=4) = 0.0007.$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.59	0.126

After clubbing.

	0	1	2+
O_i	86	75	19
E_i	88.56	68.72	22.71
$\frac{(O_i - E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$$\frac{\Sigma (O_i - E_i)^2}{E_i} = 1.2557 \sim \chi^2_1$$

$\because$ $1.2357 < 3.841$

$\therefore$ do not reject H_0.

4) $f(x) = \dfrac{cB^3}{(x+B)^4}$; $x > 0$, $B > 0$.

$$\int_0^\infty f(x) \cdot dx = 1.$$

$$cB^3 \int_0^\infty \frac{1}{(x+B)^4} = 1$$

$$\frac{cB^3}{-3}\left(0 - B^{-3}\right) = 1.$$

$\therefore$ $c = 3$.

$$E(x) = \int_0^\infty x \cdot f(x)\, dx.$$

$$= 3B^3 \int_0^\infty \frac{x+B}{(x+B)^4} - \frac{B}{(x+4)^4}\, dx.$$

$$= 3B^3 \left[0 - 0 - \left[\left(\frac{B^{-2}}{-2}\right) - \left(\frac{B^{-2}}{-3}\right) \right] \right]$$

$$= 3B^3 \left[\frac{B^{-2}}{2} - \frac{B^{-2}}{3} \right].$$

$$= \frac{B}{2}.$$

$$E(x^2) = \int_0^{\infty} x^2 f(x)\, dx.$$

$$= 3\beta^3 \int_0^{\infty} \frac{x^2}{(x+\beta)^4}\, dx.$$

Using pareto $\rightarrow \frac{\cancel{\beta}\,\beta^2}{5} \Rightarrow v(x)$

where $\alpha = 3$ $\lambda = \beta$.

5) $U(a, b)$ $E(x) = \frac{a+b}{2}$ $var(x) = \frac{(b-a)^2}{12}$.

To prove

$a = \bar{x} - \sqrt{3}\, s$ $a = 2\bar{x} - b$ $a = b - 2\sqrt{3}\, s$

$2\bar{x} - b = b - 2\sqrt{3}\, s$

$b = \bar{x} + \sqrt{3}\, s$ $a = \bar{x} - \sqrt{3}\, s$.

Sample = 2, 4, 6, 8, 10. $x = 6$ $s = 3.16$

$a = 6 - \sqrt{3}(3.16)$ $b = 6 + \sqrt{3}(3.16)$

$= 0.527$ $= \cancel{0}1.47$.

$U(a, b)$ $F(x) = \frac{1}{b}$.

$L = \frac{1}{b^n}$

$\log L = -n \log b.$

$\frac{d}{dL} \log L = \frac{-n}{b}$ $b \rightarrow \alpha$.

Camlin Page
Date / /

6) H_0; $p = 0.1$ H_1; $p > 0.1$

12 missiles fire H_0 : 3 or more missiles fired.

or

24 missiles fires H_0 = 5 or more missile fired.

7). $\bar{x}_{Pub} = 30$ $\bar{x}_{Poi} = 35.056$

$s^2_{Pub} = 64.3125$ $s^2_{Poi} = 67.403$.

$s^2_p = 58.123$.

$$\cancel{\frac{(\bar{x}_{Pub} - \bar{x}_{Poi}) - (\mu_{Pub} - \mu_{Poi})}{S_p / \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}} \sim t_{n_1+n_2}$$

$$\left(\frac{30 - 35.058}{\sqrt{58.123} \times \sqrt{\frac{2}{9}}} \right)$$

$= -1.407$.

$\therefore$ do not reject H_0.

iii) $\therefore$ Private hospitals do not result in higher claims.

8.

i) unbias when $E(\hat{\theta}) = \theta$.

ii) bias is when $E(\hat{\theta}) - \theta \neq 0$

iii) MSE is when $\theta = \text{var} + \text{bias}^2$.

9. i) $t_k = \dfrac{N(0,1)}{\sqrt{\dfrac{t_k}{K}}}$

$$\frac{\bar{x} - \mu}{s\sqrt{n}} \sim t_{n-1}$$

ii) $\mu(t_k) = 0$, $\text{var}(t_k) = \dfrac{k}{k-2}$.

iii) $n = 10$ $\mu = 50$ $s^2 = 48.667$. CI = 99%

a) $t_{0.005,9} = 3.25$

$$\mu = \bar{x} \pm t_{0.005,9} \frac{s}{\sqrt{n}}$$

$$= 50 \pm 3.25 \sqrt{\frac{48.667}{10}}$$

$$= 57.17, 42.83$$

b) $t_{0.005,9} = 2.5750$

$$\mu = 50 \pm 2.5758 \sqrt{\frac{48.667}{10}}$$

$$= 55.68, 44.32$$

10. $n = 1000$ $\quad X \sim Poi(3)$ $\quad X \sim N(3000, 3000)$.

if $P(x < 3100$ then $x \sim Poi(3)$

else x doesn't follow $Poi(3)$.

i) Type 1 error $= P(H_0 \text{ is rejected} / H_0 \text{ is true})$

$= P(X > 3100 / \lambda = 3)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$= P(Z > 1.826)$

$= 3.39\%$.

ii) Type 2 error $P(H_0 \text{ is accepted} / H_0 \text{ is false})$

iii) Power $= 1 - \beta$

or $1 -$ Type 2 error

$= 1 - \left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$.

Date / /

11. $n = 12$ $\Sigma x = 213200$ $\Sigma x^2 = 4919860000$

$\bar{x} = 17767$ $s = 10144$.

$x_1 = 1100$ $x_2 = 4800$ $x_1' = 27500$

$x_2' = 31500$.

$\Sigma x = 213200 = A + 11000 + 48000$.

$A = 154200$.

$\Sigma x' = A + x_1' + x_2' = 213200$

$\bar{x}' = 17767$

$\Sigma x^2 = 4919860000 = B + 11000^2 + 48000^2$

$B = 2494860000$

$\Sigma' x^2 = B + x_1'^2 + x_2'^2$

$= 4243360000$

$$s = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$$

$= 6434.03$.