

Calculus Assignment

$$\textcircled{1} \lim_{n \rightarrow 0} \frac{2(-3+n)^2 - 18}{n}$$

$$= \frac{2((-3)^2 + (n)^2 + 2 \times (-3) \times n) - 18}{n}$$

$$= \frac{2(9 + n^2 - 6n) - 18}{n}$$

$$= \frac{18 + n^2 - 6n - 18}{n} = \frac{2(n^2 - 6n)}{n} = \frac{2n(n-6)}{n}$$

$$\lim_{n \rightarrow 0} 2(n-6) = 2(0-6) = 2(-6) = -12 //$$

$$\textcircled{2} \text{ a) } x = 4$$

$$\lim_{x \rightarrow 4^-} g(x)$$

$$= 2 \times 4 = 8$$

$$\lim_{x \rightarrow 4^+} g(x)$$

$$= 2 \times 4 = 8$$

LHS = RHS
Continuous

$$\text{b) } x = 6$$

$$\lim_{x \rightarrow 6^-} g(x)$$

$$= 2 \times 6 = 12$$

$$\lim_{x \rightarrow 6^+} g(x)$$

$$x-1 = 5$$

LHS $\neq$ RHS
Discontinuous

$$(3) \lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$$

$$= \frac{(\sqrt{x})^2 - 3^2}{3 - \sqrt{x}}$$

$$= \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{3 - \sqrt{x}} = - \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(\sqrt{x} - 3)}$$

$$= \lim_{x \rightarrow 9} -(\sqrt{x} + 3) = -(\sqrt{9} + 3) = -(3 + 3) = -6 //$$

(4)

a) $x = -1$

$$f(x) = \frac{4x+5}{9-3x} \quad f(-1) = \frac{4(-1)+5}{9-3(-1)}$$

$$= \frac{-4+5}{9+3} = \frac{1}{12}$$

continuous

b) $x = 0$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

continuous

$$c) x=3$$

$$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{12+5}{9-9} = \frac{17}{0} = \text{N.D}$$

Discontinuous

$$(5) \lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \lim_{x \rightarrow \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \lim_{x \rightarrow \infty} \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$$

$$= \lim_{x \rightarrow \infty} \frac{6-0}{8-0+0} = \frac{6}{8} = \frac{3}{4}$$

$$6) g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

$$a) \lim_{y \rightarrow 6} g(y)$$

$$= \lim_{y \rightarrow 6^-} 1-3(6) = -17$$

$$\lim_{y \rightarrow 6^+} 1-3(6) = -17$$

Continuous

$$b) \lim_{y \rightarrow -2} g(y)$$

$$\lim_{u \rightarrow -2^-} (-2)^2 + 5 = 4 + 5 = 9$$

$$\lim_{u \rightarrow -2^+} 1 - 3(-2) = 1 + 6 = 7$$

LHS $\neq$ RHS
Not continuous

$$f(2) = 1 - 3(2) = -7$$

$$\begin{aligned} \textcircled{7} \quad f(t) &= (4t^2 - t)(t^3 - 8t^2 + 12) \\ &= (4t^2 - t) \cdot \frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \cdot \frac{d}{dt}(4t^2 - t) \\ &= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1) \\ &= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12 \\ &= 20t^4 - 132t^3 + 24t^2 + 96t - 12 \end{aligned}$$

$$8) Q(u) = \frac{3u+u^4}{2u^2+1}$$

$$= \frac{(2u^2+1) \cdot \frac{d}{du}(3u+u^4) - (3u+u^4) \cdot \frac{d}{du}(2u^2+1)}{(2u^2+1)^2}$$

$$= \frac{(2u^2+1) \cdot (3+4u^3) - (3u+u^4) \cdot (4u)}{(2u^2+1)^2}$$

$$= \frac{(6u^2+8u^5+3+4u^3) - (12u^2+4u^5)}{4u^4+4u^2+1}$$

$$= \frac{-6u^2+4u^5+3+4u^3}{4u^4+4u^2+1}$$

$$9) F(x) = 2e^x \cdot 8^x$$

$$F'(x) = 2e^x - 8^x \log 8$$

$$10) y = z^5 - e^z \ln(z)$$

$$\frac{dy}{dz} = 5z^4 - \left[e^z \ln(z) + e^z \times \frac{1}{z} \right]$$

$$= 5z^4 - e^z \ln(z) - \frac{e^z}{z}$$

$$11) a) F(x) = (6x^2 + 7x)^4$$

$$F'(x) = 4(6x^2 + 7x)^3 \times (12x + 7)$$

$$b) F(t) = 5 + e^{4t+t^7}$$

$$F'(t) = \frac{d}{dt} 5 + \frac{d}{dt} e^{4t+t^7}$$

$$F'(t) = e^{4t+t^7} (4 + 7t^6)$$

$$12) a) z = \ln(7 - u^3)$$

$$= \frac{d}{du} \ln(7 - u^3)$$

$$= \frac{1}{(7 - u^3)} \times \frac{d}{du} (7 - u^3)$$

$$= \frac{1}{(7 - u^3)} \cdot (-3u^2) = \frac{-3u^2}{(7 - u^3)}$$

$$= F''(u) = \frac{(7 - u^3) \cdot \frac{d}{du} (-3u^2) - (-3u^2) \cdot \frac{d}{du} (7 - u^3)}{(7 - u^3)^2}$$

$$= \frac{(7 - u^3) \cdot (-6u) - (-3u^2) \cdot (0 - 3u^2)}{(7 - u^3)^2}$$

$$= \frac{-42u + 6u^4 - (9u^4)}{(7 - u^3)^2} = \frac{-42u - 3u^4}{(7 - u^3)^2}$$

$$b) Q(v) = \frac{2}{(6+2v-v^2)^4}$$

$$= \frac{(6+2v-v^2)^4 \cdot \frac{d}{dv} 2 - 2 \cdot \frac{d}{dv} (6+2v-v^2)^4}{((6+2v-v^2)^4)^2}$$

$$= \frac{-2 [4 (6+2v-v^2)^3 \cdot (0+2-2v)]}{(6+2v-v^2)^8}$$

$$= \frac{-2 \times 4 (6+2v-v^2)^3 \cdot 2(1-v)}{(6+2v-v^2)^8}$$

$$= \frac{16 (6+2v-v^2)^3 (1-v)}{(6+2v-v^2)^8} = \frac{16(1-v)}{(6+2v-v^2)^{8-3}} = \frac{16-16v}{(6+2v-v^2)^5}$$

$$F''(x) = \frac{(6+2v-v^2)^5 \cdot \frac{d}{dv} (16-16v) - (16-16v) \cdot \frac{d}{dv} (6+2v-v^2)^5}{((6+2v-v^2)^5)^2}$$

$$= \frac{(6+2v-v^2)^5 \cdot (0-16) - (16-16v) \cdot 5(6+2v-v^2)^4 \cdot (0+2-2v)}{(6+2v-v^2)^{10}}$$

$$= \frac{(6+2v-v^2)^4 [(6+2v-v^2)(-16) - (16-16v) \cdot 5(2-2v)]}{(6+2v-v^2)^{10}}$$

$$= \frac{[-96 - 32v + 16v^2 - (16-16v) \cdot (10-10v)]}{(6+2v-v^2)^6}$$

$$= \frac{-96 - 32v + 16v^2 - (160 - 160v - 160v + 160v^2)}{(6+2v-v^2)^6} = \frac{-144v^2 + 288v - 256}{(6+2v-v^2)^6}$$

$$= \frac{-16(9v^2 - 18v + 16)}{(6+2v-v^2)^6}$$

$$c) F(t) = \ln(1+t^2)$$

$$F'(t) = \frac{1}{(1+t^2)} \times (0+2t)$$

$$= \frac{2t}{(1+t^2)}$$

$$F''(t) = (1+t^2) \cdot \frac{d}{dt} 2t - 2t \cdot \frac{d}{dt} (1+t^2)$$

$$= \frac{(1+t^2) \cdot 2 - 2t \cdot (2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2}$$

$$13) x^3 - 3x^2 - 9x + 12$$

$$P'(x) = 3x^2 - 6x - 9$$

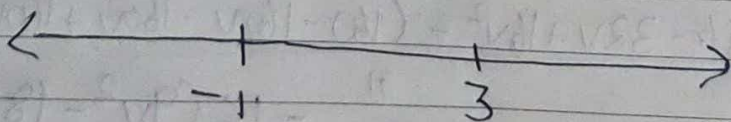
$$= 3(x^2 - 2x - 3)$$

$$= 3(x^2 - 3x + x - 3)$$

$$= 3(x(x-3) + 1(x-3))$$

$$= 3(x+1)(x-3)$$

$$x = -1 \text{ or } 3$$



$$f''(x) = 6x - 6$$

$$f''(-1) = 6(-1) - 6 = -12 \quad [\text{Maximum}]$$

$$f''(3) = 6(3) - 6 = 12 \quad [\text{Minimum}]$$

$$f(-1) = 17 \quad \text{Max.}$$

$$f(3) = -15 \quad \text{Min.}$$



$$14) f(x) = 4x^3 - 18x^2 + 24x - 7$$

$$f'(x) = 12x^2 - 36x + 24$$

$$= 12(x^2 - 3x + 2)$$

$$= 12(x - 2)(x - 1)$$

$$= 12(x(x-2) - 1(x-2))$$

$$= 12(x-1)(x-2)$$

$$x = 1, 2$$

$$f''(x) = 24x - 36$$

$$f''(1) = 24(1) - 36 = -12 \quad [\text{Maximum}]$$

$$f''(2) = 24(2) - 36 = 12 \quad [\text{Minimum}]$$

$$f(1) = 3$$

$$f(2) = 1$$



$$15) f(x) = x^2(x+3)$$

$$= x^3 + 3x^2$$

$$f'(x) = 3x^2 + 6x$$

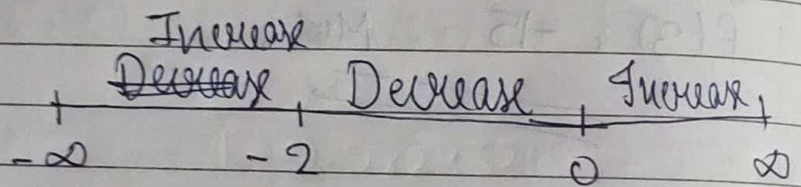
$$= 3x(x+2)$$

$$x = 0, -2$$

$$f(1) = 3(1)(1+2)$$

$$= 3 \times 3 = 9$$

(inc)



$$f''(x) = 6x + 6$$

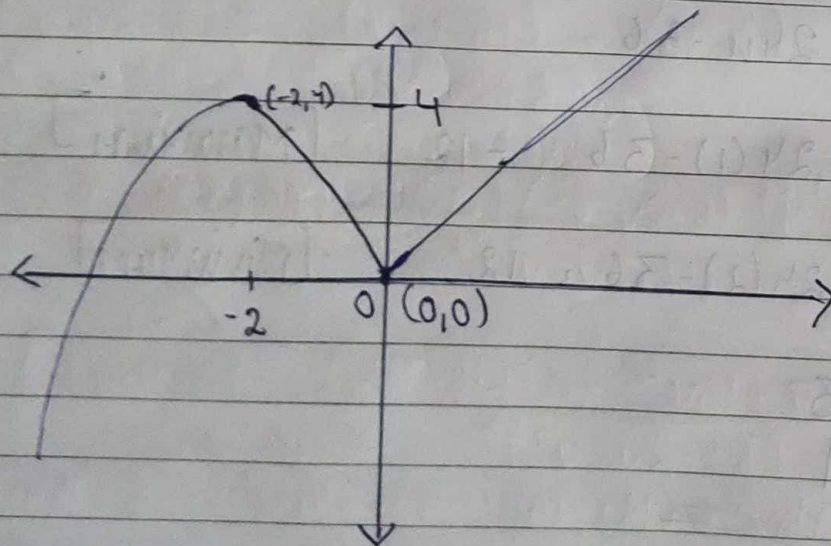
$$f''(0) = 6(0) + 6 = 6 \quad [\text{Minima}]$$

$$f''(-2) = 6(-2) + 6 = -12 + 6 = -6 \quad [\text{Maxima}]$$

$$f(0) = 0 \quad (0, 0)$$

$$f(-2) = (-2)^2(-2+3)$$

$$= 4(1) = 4 \quad (-2, 4)$$

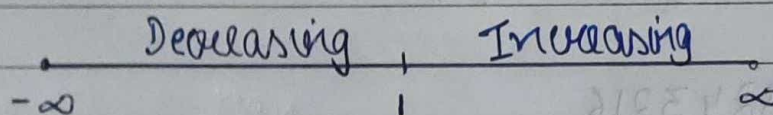


$$16) f(x) = 3x^2 - 6x$$

$$f'(x) = 6x - 6$$

$$= 6(x - 1)$$

$$x = 1$$



$$f'(0) = 6(0 - 1)$$

$$= -6 \text{ (Decreasing)}$$

$$f''(x) = 6$$

$$f''(1)$$

$$f(0) = 3(0)^2 - 6(0)$$

$$= 0$$

$$f(2) = 3(2)^2 - 6(2)$$

$$= 12 - 12 = 0$$

