Prob & Stat II

Assignment -

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Q 1) $X \sim Poi(\theta)$ $\quad X = 200$

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$$

$$\therefore \theta = X \pm 1.96\left(\sqrt{\theta/n}\right)$$

Q2) $X_i \sim N(\mu, \sigma^2)$

i) $\bar{X} \sim N(\mu, \sigma^2/n)$

$$E(\bar{x}) = E\left(\frac{\Sigma x_i}{n}\right) = \frac{\cancel{n}\mu}{\cancel{n}} = \mu$$

$$var(\bar{x}) = var\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n^{\cancel{2}}}\cancel{n}\sigma^2 = \frac{\sigma^2}{n}$$

ii) $$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$E(s^2) = \frac{(\cancel{n-1})\sigma^2}{(\cancel{n-1})} = \sigma^2$$

iii) $$\frac{(n-1)^2}{\sigma^4} var(s^2) = 2(\cancel{n-1})$$

$$var(s^2) = \frac{2\sigma^4}{n-1}$$

②

iv) $P(50 < s^2 < 150) = \left(\frac{\cancel{50} \times \cancel{9}^{4.5}}{\cancel{100}_2} < \chi^2_9 < \frac{\cancel{9}^{4.5} \times \cancel{150}^{3}}{\cancel{100}} \right)$

$= (4.5 < \chi^2_9 < 13.5)$

Ques3) $n = 4$ $\quad$ Bin $(4, P)$

$L(P) = (P(X=0))^{86} (P(X=1))^{75} (P(X=2))^{16} (P(X=3))^{2} (P(X=4))^{1}$

$= [(1-P)^4]^{86} [4P(1-P)^3]^{75} [6P^2(1-P)^2]^{16} [4P^3(1-P)]^2 [P^4]$

$L(P) = (1-P)^{603} \; P^{117} \times \text{constant}$

$\ln L = (603 \ln(1-P) + 117 \ln P) \ln \text{constant}$

$\frac{d \ln L}{dp} = \frac{603}{1-P} \times -1 + \frac{117}{P}$

$\frac{d \ln L}{dp} = 0$

$\frac{117}{P} = \frac{603}{1-P}$

$117(1-P) = 603P$

$117 - 117P = 603P$

$117 = 720P$

$P = \frac{117}{720}$

$P = 0.1625$

ii) $P(X=0) = (1-p)^4 = (1-0.1625)^4 = 0.492$

$P(X=1) = 4p(1-p)^3 = 4(0.1625)(1-0.1625)^3$
$= 0.3818$

$P(X=2) = 6p^2(1-p)^4$
$= 6(0.1625)(1-0.1625)^2$
$= 0.111$

$P(X=3) = 4p^3(1-p)$
$= 6(0.1625)^3(1-0.1625)$
$= 0.0144$

$P(X=4) = 0.0007$

que4)

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.96	2.59	0.126

By clubbing

	0	1	2
O_i	86	75	19
E_i	88.56	68.72	22.71
$C_i = \frac{\Sigma (O_i - E_i)^2}{E_i}$	0.07400	0.5739	0.6061

$\Sigma C_i = 1.2539 \sim \chi^2_1$

$\therefore 1.2539 < 3.841$

do not Reject H

Ques5) $U(a, b)$ $E(X) = \frac{a+b}{2}$ $V(X) = \frac{1}{12}(b-a)^2$

$a = \bar{X} - \sqrt{35}$ $a = 2\bar{X} - b$ $a = b - 2\sqrt{35}$

$2\bar{X} - b = b - 2\sqrt{35}$

$b = \bar{X} + \sqrt{35}$ $a = \bar{X} - \sqrt{35}$

$\bar{X} = 6$ $s = 3.16$

$a = 6 - \cancel{\sqrt{3 \times 3.16}}\ \sqrt{3}(3.16)$ $b = 6 + \cancel{\sqrt{3 \times 3.16}}\ \sqrt{3}(3.16)$

$a = \cancel{2.92104}\ 0.52672$ $b = 11.47329$

$U(a, b)$ $P(X) = \frac{1}{b}$

$L = \frac{1}{b^n}$

$\log L = -n \log b$

$\frac{d \ln L}{db} = \frac{-n}{b}$

$\boxed{b \longrightarrow \infty}$

Que6) H_0 $P_1 = 0.1$

H_1 $P > 0.1$

$P \rightarrow$ Prob. of missiles hittes

12 missiles fired

24 missiles fired total H_0 = 500 more hits observed

5

Q7) $\bar{X}_{pub} = 30$ $\bar{X}_{pri} = 35.056$

i) $S^2_{pub} = 64.3125$

$S^2_{pri} = 67.403$

$S_p^2 = 58.123$

$$\frac{(\bar{X}_{pub} - \bar{X}_{pri}) - (\mu_{pub} - \mu_{pri})}{S_p / \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

ii) $H_0 : \mu_{pub} = \mu_{pri}$
$H_1 : \mu_{pub} \neq \mu_{pri}$

$$\frac{30 - 35.056}{\sqrt{8.123}\sqrt{\frac{2}{9}}} = -1.407$$

$\therefore$ accept H

iii) Privet hospital $\boxed{\text{do not}}$ result in higher claim

que 8) i) unbias when $E(\hat{\theta}) = \theta$
ii) bias when $E(\hat{\theta}) - \theta \neq 0$
iii) MSE θ = var + bias^2

que 9) i) $t_k = \dfrac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$; $\dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

ii) $\mu(t_k) = 0$ $\quad$ $var(t_k) = \dfrac{k}{k-2}$

iii) $n = 10$ $\bar{X} = 50$ $s^2 = 48.667$ $C.I = 99\%$

a) $t_{0.005, 9} = 3.25$

$\mu = \bar{X} \pm t_{0.005, 9}$

$S/\sqrt{n} = 50 \pm 3.25 \sqrt{\frac{48.667}{10}}$

$\boxed{\mu = 57.17, \ 42.83}$

b) $t_{0.005, 9} = 2.5758$, $\mu = 50 \pm 2.5758 \left(\sqrt{\frac{48.667}{10}} \right)$

$= 55.68, \ 44.32$

Que 10) $n = 1000$ $X \sim Poi(3)$

$X \sim N(3000, \cancel{30}\ 3000)$

if $P(X < 3100)$ then $X \sim Poi(3)$

else don't follow $Poi(3)$.

i) $P(H_0$ is ~~red~~ reject $/ H_0$ is true

$= P(X > 3100, \lambda = 3)$

$\therefore X \sim N(3000, 3000)$

$P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$P(Z > 1.826)$

$1 - P(Z < 1.826) = 3.39\%$

ii) Type 2 Error

$P($ Ho accepted $/$ Ho is false $)$

iii) Power $= P(\text{reject } H_0 / H_0 \text{ false}) =$

$$= 1 - \left(z < \frac{3100 - n\mu}{\sqrt{n\mu}} \right)$$

$$= 1 - \left(z < \frac{3100 - 3000}{\sqrt{3000}} \right)$$

$$= 1 - \left(z < \frac{100}{10\sqrt{30}} \right)$$

$$= 1 - (z < 1.82)$$

$$= 0.0343$$

q ii) $n = 12$ $\quad \Sigma x = 213200$ $\quad \Sigma x^2 = 4919860000$

$\bar{x} = 17767$ $\quad s = 10144$

$x_1 = 11000$ $\quad x_2 = 48000$ $\quad x_1' = 27500$

$x_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 43000$

$A = 154200$

$\Sigma x' = A + x_i' + x_2' = 213200$

$\bar{x}' = 17767$

$\Sigma x^2 = A + x_1' + x_2' = 213200$

$\Sigma x = 4919860000 = B + 11000^2 + 48000^2$

$B = 2494860000$

(8)

$$\Sigma \mid x^2 = \beta + x_1' + x_2'^2$$

$$= 4243360000$$

$$s = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$$

$$= 6434.03$$