CALCULUS ASSIGNMENT

1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{2(h^2 - 6h + 9) - 18}{h}$

$= \lim_{h \to 0} \dfrac{2h^2 - 12h + 18 - 18}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{h}(2h - 12)}{\cancel{h}}$

$= -12$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$

(b) $x = 6$.

$\therefore \lim_{x \to 6^-} g(x) = \lim_{x \to 6^-} 2x = 2(6) = 12$

$\lim_{x \to 6^+} g(x) = \lim_{x \to 6^+} x - 1 = 6 - 1 = 5.$

LHL $\neq$ RHL.

$\therefore$ function is discontinuous.

PTO $\rightarrow$

3. $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})}$

$= \lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x} = \dfrac{-1\,\cancel{(9-x)}\,(3+\sqrt{x})}{\cancel{(9-x)}}$

$\therefore \lim_{x \to 9} (-1)(3+\sqrt{x})$

$= -3 - \sqrt{9} = \underline{\underline{-6}}$.

4. $f(x) = \dfrac{4x+5}{9-3x}$.

(a) $x = -1$.

$\therefore f(-1) = \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{1}{12}$.

~~limit exists~~

$\therefore$ ~~given function has a removable discontinuity~~.

$\therefore$ it is a continuous function.

(b) $x = 0$.

$f(0) = \dfrac{0+5}{9-0} = \dfrac{5}{9}$.

function is continuous.

(c) $x = 3$

$f(3) = \dfrac{12+5}{9-9} = $ N.D.

$\therefore$ function is discontinuous.

6. $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2. \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$

(b) $\lim\limits_{y \to -2} g(y) = \lim\limits_{y \to -2} 1 - 3y = 1 - (3)(-2) = \underline{\underline{7}}$

(a) $\lim\limits_{y \to 6} g(y) = \lim\limits_{y \to 6} 1 - 3y = 1 - (3)(6) = \underline{\underline{-17}}.$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\therefore f'(t) = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12.$

$= 12t^4 + 8t^4 - 64t^3 - 3t^3 - 64t^3 - t^3 + 16t^2 + 8t^2 + 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12.$

$\therefore f'(t) = \underline{\underline{4(5t^4 - 33t^3 + 6t^2 + 24t - 3)}}.$

8. $R(W) = \dfrac{3W + W^4}{2W^2 + 1}.$

$R'(W) = \dfrac{(2W^2 + 1)\frac{d}{dW}(3W + W^4) - (3W + W^4)\frac{d}{dW}(2W^2 + 1)}{(2W^2 + 1)^2}.$

$= \dfrac{(2W^2 + 1)(3 + 4W^3) - (3W + W^4)(4W)}{(2W^2 + 1)^2}.$

$= \dfrac{6W^2 + 8W^5 + 3 + 4W^3 - 12W^2 - 4W^5}{(2W^2 + 1)^2}.$

$= \underline{\underline{\dfrac{4W^5 + 4W^3 - 6W^2 + 3}{(2W^2 + 1)^2}}}$

9. $f(x) = 2e^x - 8^x$

$\therefore f'(x) = 2e^x - 8^x \log 8.$

10. $y = z^5 - e^z \ln(z)$

$= 5z^4 - \left[e^z \cdot \frac{d \ln(z)}{dz} + \ln(z) \frac{de^z}{dz}\right]$

$= 5z^4 - \frac{e^z}{z} - \ln(z) \cdot e^z$

$\therefore \frac{dy}{dz} = 5z^4 - \left[e^z\left(\frac{1}{z} + \ln(z)\right)\right]$

11. (a) $f(x) = (6x^2 + 7x)^4$

$= 4(6x^2 + 7x) \cdot \frac{d}{dx}(6x^2 + 7x)$

$\therefore f'(x) = 4(6x^2 + 7x)(12x + 7)$

$\cancel{= 4(72x^3}$

(b) $f(t) = 5 + e^{4t + t^7}$

$\therefore f'(t) = e^{4t + t^7} \cdot \frac{d}{dt}(4t + t^7)$

$\therefore f'(t) = e^{4t + t^7} \cdot (4 + 7t^6)$

12. (a) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{1}{(7-x^3)} \cdot \frac{d}{dx}(7-x^3)$$

$$\therefore \frac{dz}{dx} = \frac{1}{(7-x^3)} \cdot (-3x^2)$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) + (3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 + (-9x^4)}{(7-x^3)^2}.$$

$$\therefore \frac{d^2z}{dx^2} = \frac{-3x^4 - 42x}{(7-x^3)^2}$$

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$Q'(v) = 2(6+2v-v^2)^{-4}$$

$$= 2\left[-4(6+2v-v^2)^{-5} \cdot (2-2v)\right]$$

$$= \frac{-8(2-2v)}{(6+2v-v^2)^5} = \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$\therefore Q''(v) = -16 \frac{d}{dv}(1-v)(6+2v-v^2)^{-5}$$

$$= -16\left[(1-v)(2-2v)(-5)(6+2v-v^2)^{-6} + (6+2v-v^2)^{-5}(-1)\right]$$

$$\therefore Q''(v) = \frac{-16(1-v)(2-2v)(-5)}{(6+2v-v^2)^6} - \frac{1}{(6+2v-v^2)^5}$$

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{1}{(1+t^2)}(2t)$

$f''(t) = \dfrac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$

$= \dfrac{2+2t^2-4t^2}{(1+t^2)^2}$.

$\therefore f''(t) = \dfrac{2-2t^2}{(1+t^2)^2}$

13. $f(x) = x^3 - 3x^2 - 9x + 12$.

$\therefore f'(x) = 3x^2 - 6x - 9$

$\therefore \cancel{3x^2}\ 3(x^2 - 2x - 3) = 0$.

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$x - 3 = 0$ or $x + 1 = 0$

$x = 3$ or $x = -1$.

$\therefore$ at $x = 3$

$f(3) = 3^3 - 3(3)^2 - 9(3) + 12$.

$= -15 = -ve$.

$\therefore$ function is maximum at $x=3$.

$f(-1) = -1^3 - 3(-1)^2 - 9(-1) + 12$.

$= 17 = +ve$.

$\therefore$ function is minimum at $x=-1$.

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$\therefore f'(x) = 12x^2 - 36x + 24$

now, $f'(x) = 0$.

$\therefore 12(x^2 - 3x + 2) = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$x - 2 = 0$ or $x - 1 = 0$

$x = 2$ or $x = 1$.

at $x = 2$.

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 1 = +ve$.

$\therefore$ function is minimum at $x = 2$.

at $x = 1$.

$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 3 = +ve$.

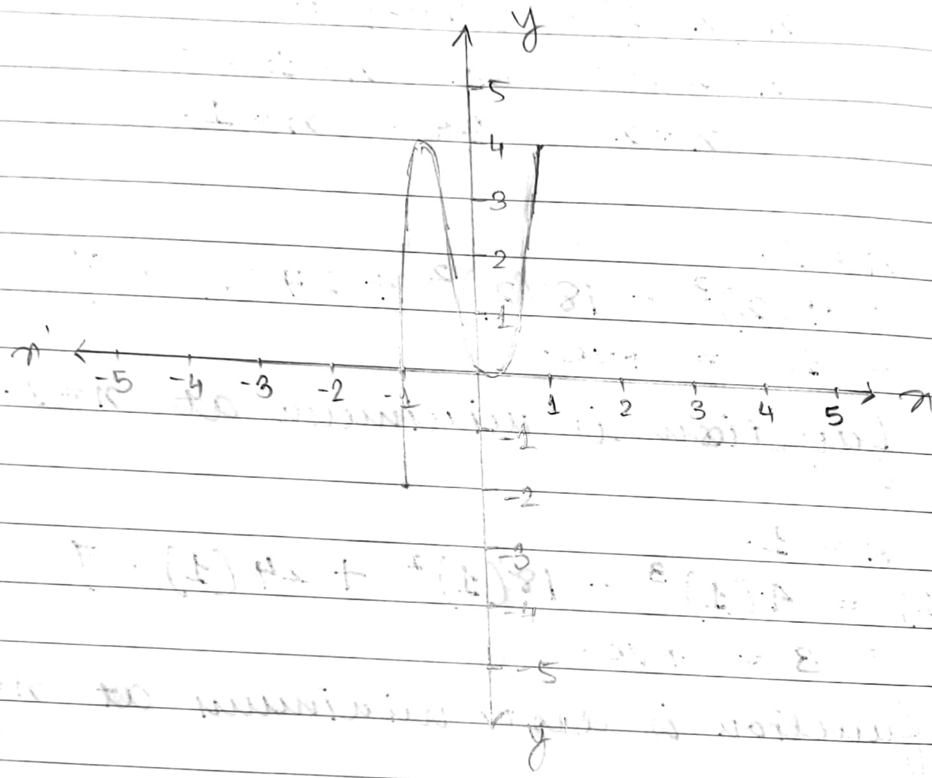
$\therefore$ function is also minimum at $x = 1$.

$\longrightarrow$ PTO

15. $f(x) = x^2 (x+3)$.

$\therefore \; y = x^2 (x+3)$

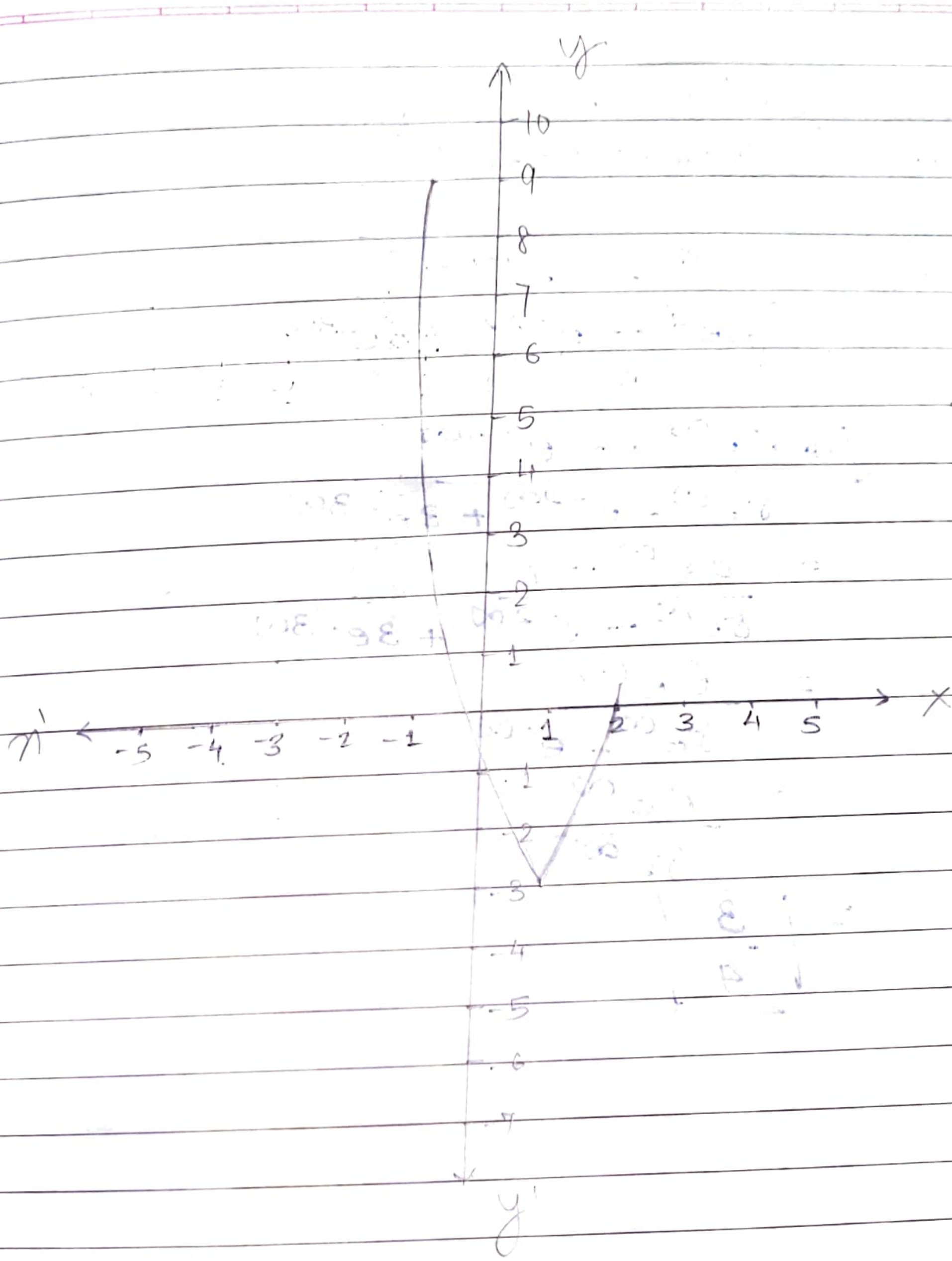
x	1	0	-1
y	4	0	2



16. $f(x) = 3x^2 - 6x$

$\therefore \; y = 3x^2 - 6x$

x	1	2	-1
y	-3	0	9



y
10
9
8
7
6
5
4
3
2
1
X
1
2
3
4
5
X'
-5
-4
-3
-2
-1
-1
-2
-3
-4
-5
-6
-7
y'

5. $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\lim_{x \to \infty} \frac{6e^{x} - e^{-6x}}{8e^{x} - e^{-2x} + 3e^{-3x}}$$

$$[e^{-x} = 0] \rightarrow \frac{6e^{\infty} - e^{-6\infty}}{8e^{\infty} - e^{-2\infty} + 3e^{-3\infty}}$$

$$= \frac{6e^{\infty} - 0}{8e^{\infty} - e^{-2\infty} + 3e^{-3\infty}}$$

$$= \frac{6e^{\infty}}{8e^{\infty} - e^{-\infty}}$$

$$= \frac{6\cancel{e^{\infty}}}{8\cancel{e^{\infty}}}$$

$$= \boxed{\frac{3}{4}}$$