# P&S Assignment (1) (unit 1 & 2)

Q1. i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\text{Mean} = \bar{x} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= 13.4.$$

ii)

| No. of households ($x_i$) | No. of people ($f_i$) | cf | $f_i x_i$ |
|---|---|---|---|
| 0 | 5 | 5 | 0 |
| 1 | 11 | 16 → $Q_1$ | 11 |
| Median class → 2 | 26 | 42 | 52 |
| 3 | 15 | 57 → $Q_3$ | 45 |
| 4 | 3 | 60 | 12 |
| | $\Sigma f_i = 60$ | | $\Sigma f_i x_i = 120$ |

$$\boxed{\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}}$$

$\Rightarrow \bar{x} = \text{Mean} = \frac{120}{60} = 2$

$N = 60$

$\frac{N}{2} = 30$

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 1.5 + \frac{30 - 16}{26} \times 1$$

$$= 1.5 + \frac{14}{26}$$

$$= 2.038461538$$

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= Mode = 3 Median − 2 Mean
= Mode = 6.115384615 − 4
= Mode = 2.115384615

= $Q_1 = \frac{1}{4}(\cancel{60})^{15}$ th observation
= 15th observation

$||^{ly}$, $Q_3$ = 45 th observation

$$Q_1 = l_1 + \frac{\left(\frac{1}{4}N - F_1\right)}{f\,Q_1} C$$

$$= 0.5 + \frac{(15 - 5)}{11} \times 1$$

$$= 0.5 + \frac{10}{11} = 1.4090$$

$$Q_3 = l_3 + \frac{\left(\frac{3}{4}N - F_3\right)}{f\,Q_3} (C)$$

$$= 2.5 + \frac{(45 - 42)}{15} \times 1$$

$$= 2.5 + \frac{\cancel{3}}{\cancel{15}} \frac{1}{5}$$

$$= 2.7$$

$$IQR = Q_3 - Q_1$$

$$= 2.7 - 1.4090$$

$$= 1.291$$

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Q2 i) 0 claims in one year $X \sim Pois(0.5)$.

$$P(X=0) = \frac{e^{-\lambda}\lambda^{x}}{0!} = \frac{e^{-0.5}(0.5)^0}{1} = e^{-0.5}$$

$$= 0.6065306597$$

ii) To find Prob that claims (one or more) occur in one of the three consecutive years and no claim in other 2 years.

$$\therefore P(X \geq 1) = 1 - P(X=0) = 1 - 0.60653065 9$$

$$= 0.393469 3$$

This is a binomial distribution.

$$P(X=1) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$$

$$= {}^{3}C_{1}\, p^{1}(1-p)^{3-1}$$

$$= {}^{3}C_{1}(0.3934693)(0.6065306597)^2$$

$$= \frac{3!}{2!\,1!} \times 0.1447492662$$

$$= 3 \times 0.1447492662$$

$$= 0.4342477986$$

iii) $P(X>2) = 1 - P(X=2) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x}$

$X \sim Exp(0.5)$ (Time) $\rightarrow$ time gap $\quad e^{-\lambda x} = e^{-0.5 \times 2}$

$$= e^{-1}$$

$$= \frac{1}{e}$$

$$= 0.367879441$$

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Q3. $\alpha = 2, \quad \lambda = 1/2$

$\longrightarrow \quad 0 < x < \infty$

$sd(x)^2 = \text{variance}$

$\therefore \quad sd(x) = \sqrt{\text{variance}}$

$= \sqrt{\alpha / \lambda^2}$

$= \sqrt{\frac{2 \times 4}{1}}$

$= 2\sqrt{2}$

Q4 $P\left(\frac{\text{two qualified females}}{\text{atleast one female}}\right)$

~~$\longrightarrow \quad P(F) =$~~

Q5. let A be the event that package arrives late

$\longrightarrow$
$E_1 \longrightarrow$ Arrives through level (1)
$E_2 \longrightarrow$ — " — level (2)
$E_3 \longrightarrow$ — " — level (3)

By Bayes' Theorem,

$$P(E_2/A) = \frac{P(A/E_2) \cdot P(E_2)}{P(A/E_1) \cdot P(E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{50}{100} \cdot \frac{40}{100}}{\frac{50}{100} \times \frac{40}{100} + \frac{40}{100} \times \frac{20}{100} + \frac{10}{100} \times \frac{10}{100}}$$

$$= \frac{\frac{20 \times 100}{100 \times 100}}{\frac{20 \times 100 + 800 + 100}{\cancel{100 \times 100}}}$$

$$= \frac{2000}{2000 + 900} = \frac{2000}{2900} = \frac{20}{29}$$

**Q6.** $X \sim U(1, 2)$

$Y = \frac{3}{6 - x}$

$\longrightarrow$ PDF of $Y$ =

**Q7.**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 |
| $P(X = x)$ | 0.05 | 0.15 | 0.35 | 0.4 | 0.05 |

$\longrightarrow$ i) $F(3) = P(X < x) = 0.35$

ii) $E(X) = \sum P(X = x) \cdot x\,dx$

$= 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 0.05 \times 5$

$= 3.25$

(iii) $V(X) = E(X^2) - [E(X)]^2$

$= 1 \times 0.05 + 4 \times 0.15 + 9 \times 0.35 + 16 \times 0.4 + 25 \times 0.05$

$(3.25)^2$

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$= \quad 0.8875$

iv) Skewness coefficient $= \dfrac{\mu_3}{\sigma^3}$

doubt $\quad = \cancel{E}$

Q8

$=$

Q9

$= /$

Q12 $\quad \lambda = 10$

$\alpha = 0.1$

$= \quad P\cancel{C} = \lambda e^{-\lambda x} = 10\, e^{-1}$

$= \dfrac{10}{e}$

$= 3.678794412$

Q13 $\quad 75 < x < 120$

$\quad\ \ \downarrow \qquad\quad \downarrow$

$\quad\ \ a \qquad\quad\ b$

$\longrightarrow \quad PDF = \dfrac{1}{b-a} = \dfrac{1}{120-75} = \dfrac{1}{45}$

$= \quad p(80 < x < 100) = \dfrac{100-80}{45}$

$= \dfrac{20}{45}$

$= \dfrac{4}{9}$

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Q11 $P(M) = P(F) = \frac{1}{2}$

= Woman has 5 boys & 2 girls

P(Next 3 children are boys) = $\frac{1}{2} \times \left(\frac{1}{2}\right)^{3-1}$

(geometric distn) $= \frac{1}{2} \times \frac{1}{2}^{2}$

Since there are many trials before the first success $= \frac{1}{8}$

Q10 $\lambda = \frac{2}{5}$

= $P(X \geq 2) = \lambda e^{-\lambda x} = \frac{2}{5} e^{-2/5 \times 2}$

$= \frac{2}{5} e^{-4/5}$

$= 0.1797315856$