FINANCIAL MATHEMATICS
ASSIGNMENT-2

1] i Loan amout = 20000
Monthly payments = 427.9
$n = 5$

$$20000 = 427.9(12)\, a^{(12)}_{\overline{5}|} @ i\%$$

$$20000 = 427.9(12)\left(\frac{1-(1+i)^{-5}}{12((1+i)^{1/12}-1)}\right)$$

At $i = 10\%$, value $= 20341$
$i = 11\%$, value $= 19916$
Using interpolation
$i = 10.8\%$
$\therefore APR = 10.8\%$

ii) $PV_1 = 427.9(12)\, a^{(12)}_{\overline{4}|} @ i = 10.8\%$

$$= 427.9(\cancel{12})\left(\frac{1-(1.108)^{-4}}{\cancel{12}((1.108)^{1/12}-1)}\right)$$

$= 427.9(39.20536)$
$= 16775.9773$

$$16775.9773 = 274.49(12)\, a^{(12)}_{\overline{t}|} @ 10.8\%$$

$$\frac{16775.9773}{274.49} = \frac{(1-(1.108)^{-t})\,\cancel{12}}{((1.08)^{1/12}-1)\,\cancel{12}}$$

$$1-(1.108)^{-t} = 0.524667$$
$$t = 7.4499$$

iii) Total interest of initial loan $= 427.9 \times 48 - 16775.9773$
$= 3763.22$

Total interest of restructed loan $= 7.2499 \times 274.49 \times 12 - 16775.9773$
$= 7104.3233$

$\therefore$ They will pay 3341.43 more interest

2] i) Payback period is the number of years necessary to recover funds invested in the project without considering the interest, both on borrowing and investing.

Discounted payback period is the number of years after which the cummilative discounted cash inflows cover the initial investment.

ii) Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is as they donot consider cash flows after the acc. value of zero is achieved.
The PP doesnot take time value of money into consideration.
Hence NPV is superior than either of the two.

iii) Project A

PV of outlay $= -170 - 20v - 10v^2$

PV of inflow $= 20v + 20v^2 + 200v^3$

$NPV = -170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$

$0 = -170 + 10v^2 + 200v^3$

IRR $\Rightarrow$

$0 = -170 + 10v^2 + 200v^3$

$120 = 10(1+i)^{-2} + 200(1+i)^{-3}$

When $i = 0.07$, value $= 171.99$

$i = 0.08$, value $= 167.33$

By using interpolation

$i = 7.4\%$

Project B:

PV of outlay $= -200$

PV of inflow $= 14a_{\overline{6}|} + 200v^{6}$

$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6} = 0$

~~When $i = 0.07$~~ By using interpolation & Trial

$i = 7\%$

b) $NPV_A = -170 + 10v^2 + 200v^3$

At $i = 6\%$.

$NPV_A = -170 + 10(1-0.06)^{2} + 200(1-0.06)^{3}$

$= 4.9528$

$NPV_B = -200 + 14a_{\overline{6}|} + 200v^{6}$

$= 5.099583$

3] PV of annuity 1 $= 200\,\ddot{a}_{\overline{22}|}\,(v^{3/12})$

$= 2161.365533$

PV of annuity 2 $= 320\,\ddot{a}^{(4)}_{\overline{16.75}|}\,v^{2/12}$

$= 2996.0488$

PV of annuity 3 $= 15\,\ddot{a}_{\overline{270}|}$ @ $\frac{d^{12}}{12}$

$= 1795.651419$

Total PV : 6953.0658

Revised annuity $= 6953.0658 = X\,\ddot{a}^{(2)}_{\overline{21.5}|}$ @ $d^{(2)}$

$X = 649.01356$

4] To Derive $(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

$PV = 1 + 2v + 3v^2 + \dots nv^{n-1}$

$(1-d)(PV) = v + 2v^2 + 3v^3 + \dots (n-1)v^n$

$PV - (1-d)PV = 1 + v + v^2 \dots v^{n-1} - nv^n$

$d(PV) : \ddot{a}_{\overline{n}|} - nv^n$

$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Page No.
Date

5] i) For Property Fund:

$$1+i = \frac{15.1}{17.4} \times \frac{\cancel{14.8}}{13.1} \times \frac{\cancel{15.8}}{\cancel{14.8}} \times \frac{16.4}{\cancel{15.8}}$$

$i = 32.25800$

For equity fund:

$$1+i = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$$

$i = 28.0991738\%$

ii) $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4} = 4(16.4)$

$i = 29.32\%$

iii) $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4} = 4 \times 15.5$

$i = 67.37\%$

6] i) $160000 = X \, a_{\overline{10}|} @ 8\%$

$$X = \frac{160000}{6.7101} = 23844.65201$$

ii) $PV_4 = 23844.65201 \, a_{\overline{6}|} @ 8\%$

$= 110231.4422$

$110231.4422 = Y \, a_{\overline{6}|} @ 10\%$

$Y = 25309.72428$

iii) $PV_7 = 25309.72428 \, a_{\overline{3}|7}$ @ 10%.

$= 62442.25331$

$62442.75331 = 2 \, a_{\overline{3}|7}$ @ 9%.

$2 = 24865.78179$

8] i) PV of outflow $= 2.7 + 0.2 a_{\overline{3}|7}$

PV of inflow $= \bar{a}_{\overline{10}|7}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$NPV = 0 = \bar{a}_{\overline{10}|7}(0.25v + 0.25v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2 a_{\overline{3}|7}$

$\therefore i = 9.3\%$

ii) $NPV = 0.1v^3 + 0.85 \, \bar{a}_{\overline{10}|7} v^3 - 2.7 - 0.2 a_{\overline{3}|7}$ @ 10%.

$= 1.0089$

9] i) $(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{33+6} \times \frac{41.05-4.5}{38.5} \times \frac{45}{41.05} \times \frac{47}{45+6-2.5}$

$\therefore i = 7.0952\%$.

$30.5(1+i)^3 + 6(1+i)^{2.95} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$

When $i = 7\%$, 46.75

$i = 7.5\%$, 47.037

Using interpolation

$i = 7.20\%$

ii) Linked Rate of Return

2011:

$30.5(1+i)+6(1+i)^{0.75} = 38.5$

$i = 6.266\%$

2012:

$38.5(1+i)+4.5(1+i)^{0.5} = 45$

$i = 4.92\%$

2013:

$45(1+i)+2.5(1+i)^{0.5} = 47$

$i = \cancel{0}10.276\%$

$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10976$

$i = 7.13\%$

11] i) $TWRR = (1+i)^2 = \frac{500}{460} \times \frac{550.50}{500+40} \times \frac{600}{550} \times \frac{X}{600}$

$1.44 = \frac{500}{400} \times \frac{500}{540} \times \frac{X}{550}$

$X = 786.9317$

ii) HWRR

$400(1+i)^2+40(1+i)^{9/12}-50(1+i) = 786.9312$

$i = 30.909\%$

Page No.
Date

12] ii) a) $-800(1+i)^{t} - 50(1+i)^{t-1} + 100\bar{s}_{\overline{t-2}|} = 0$

$\therefore t = 17.767$

b) $100\bar{s}_{\overline{4.233}|}$ @ 6%

$= \cancel{480} \quad \cancel{480} \quad = 480$

iii) $-800(1+i)^{t} - 50(1+i)^{t-1} + 107.971\, s_{\overline{t-2}|}$ @ 7% $= 0$

$-800(1+i)^{18} - 50(1+i)^{17} + 107.971\, s_{\overline{16}|}$ @ 7% $= 97714$

$= 462.29$ (Acc. amount after 22 years)

13] i) $988000 = X(12)\, a^{(12)}_{\overline{25}|}$ @ 7%

$X = 6848$

ii) $X\, a^{(12)}_{\overline{37/3}|} = 6848\left(a^{(12)}_{\overline{37/3}|} @ 7\%\right)$

$= 64.8600$

iii) PV after 10^{th} ~~Se~~ September payment $= X\, a^{(12)}_{\overline{83/6}|}$ @ 7%

PV after 10^{th} October's payment $= X\, a^{(12)}_{\overline{13.75}|}$

Capital repaid $= X\, a^{(12)}_{\overline{13.75}|} - X\, a^{(12)}_{\overline{83/6}|}$

$= 26.80$

iii) Capital repayment in these 12 installments

$X\left(a^{(12)}_{\overline{23/3}|} - a^{(12)}_{\overline{14/3}|}\right)$

$= 516\text{-}9$

$\therefore$ Total interest $= 12X - 51690$

$= 30556$