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Probability & Statistic 1 Assignment - [Unit 1 & 2]

Q.1 i.] 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

Mean $= \bar{x} = \frac{\Sigma x}{n} = \frac{134}{10}$

$= 13.4$

Median $= \left(\frac{n+1}{2}\right)^{th}$ term

$= \frac{11+15}{2}$

$= 13$

Mode $= 18$

Standard deviation, $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

x_i	$(x_i - \bar{x})^2$
4	88.36
7	40.96
9	19.36
9	19.36
11	5.76
15	2.56
18	21.16
18	21.16
18	21.16
25	134.56

$\Sigma(x_i - \bar{x})^2 \quad 374.4$

$\sigma^2 = \frac{374.4}{10} = 37.44$

$\sigma = \sqrt{37.44} = 6.12$

Last term of Q_1 is the $\left(\frac{n}{4}\right)^{th}$ term = 2.5^{th} term

$$\text{So } Q_1 = \frac{7+9}{2} = 8$$

Last term of Q_3 is the $\left(\frac{3n}{4}\right)^{th}$ term = 7.5^{th} term

$$\text{So, } Q_3 = \frac{18+18}{2} = 18$$

Inter Quartile Range = $Q_3 - Q_1$
$= 18 - 8$
$= 10$

2]

x_i	f_i	cf
0	5	5
1	11	16
2	26	42
3	15	57
4	3	60

$n = 60$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{n} = \frac{120}{60}$$

$= 2$

Median = $\left(\frac{n+1}{2}\right)^{th}$ term = 30.5^{th} term

30.5^{th} term lies in the x_i, 2

Thus, Median = 2

Mode = 2

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x_i	f_i	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
0	5	4	20
1	11	1	11
2	26	0	0
3	15	1	15
4	3	4	12

$\Sigma f_i(x_i-\bar{x})^2 = 58$

$\sigma^2 = \frac{58}{60} = 0.97$

$\sigma = \sqrt{0.97}$

$= 0.98$

Last term of Q_1 is $\left(\frac{n}{4}\right)^{th}$ term $= 15^{th}$ term

So, $Q_1 = 1$

Last term of Q_3 is $\left(\frac{3n}{4}\right)^{th} = 45^{th}$ term

45^{th} term, i.e. $Q_3 = 3$

Inter Quartile Range $= Q_3 - Q_1$

$= 3-1$

$= 2$

Q.2 $\lambda = 0.5$

i] Poisson distribution $=$ Poi(0.5)

P(no claim arriving) $= P(X=0)$

$P(X=0) = \frac{\lambda^x e^{-x}}{x!} = \frac{0.5^0 e^{-0.5}}{0!}$

$= 0.60653066$

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2.] This is a binomial distribution
Let one or more than one claim arises be success (s)
& no claim arising be failure (f)

$P(s) = P(X>0) = 1 - P(X=0)$
$= 1 - 0.60653066$
$= 0.39346934$
$P(f) = P(X=0) = 0.60653066$
$\therefore X = Bin(3, 0.39346934)$
$P(X=1) = {}^3C_1 (0.39346934)^1 \cdot (0.60653066)^2$
$= 0.434247843$

3.]

Q.3 1.] $\alpha = 2$ & $\lambda = 1/2$

$E(X) = \dfrac{\alpha}{\lambda}$
$= 2 / 1/2$
$= 4$

$V(X) = \dfrac{\alpha}{\lambda^2} = \dfrac{2}{(1/2)^2}$
$= 8$
$\sigma = \sqrt{8}$
$= 2.83$

2.] Coefficient of skewness $= \dfrac{2}{\sqrt{\alpha}} = \dfrac{2}{\sqrt{2}}$
$= 1.414$

$\therefore$ The data will be positively skewed.
So, shape → rightly skewed.

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3.] $f_x(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$

$= \frac{1/2^{2}}{(2-1)!} x^{2-1} e^{-x/2}$

$= \frac{1}{4} x e^{-x/2}$

$F_x(x) = \frac{1}{4} \int_0^x x e^{-x/2} \, dx$

$= \frac{1}{4} \left[x e^{-x/2} (-2) - \int e^{-x/2} dx \right]_0^x$

$= \frac{1}{4} \left[-2x e^{-x/2} + 2 e^{-x/2} \right]_0^x$

$= \frac{1}{4} \left[2e^{-x/2} - 2x e^{-x/2} - 2 \right]$

$= \frac{1}{4} \cdot 2 \left[e^{-x/2} - x e^{-x/2} - 1 \right]$

$= \frac{1}{2} \left[e^{-x/2} - x e^{-x/2} - 1 \right]$

Q. 4 P(choosing 2 members from the group) $= {}^{18}C_2 = P(A)$

P(atleast one female) = 1 - P(Both male)

$= 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$

P(F) $= 0.705882353$

P(2 qualified females) $= \frac{{}^{7}C_2}{{}^{18}C_2}$

$P(F \cap Q) = 0.137254902$

$P(Q/F) = \frac{P(F \cap Q)}{P(F)} = \frac{0.137254902}{0.705882353}$

$= 0.1945$

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Q.5 $P(L_1) = 0.4$
$P(L_2) = 0.5$
$P(L_3) = 0.1$ } Delivering P

$P(L|L_1) = 0.2$
$P(L/L_2) = 0.4$
$P(L/L_3) = 0.1$ } Arriving late

$$P(L_2/L) = \frac{P(L_2)P(L/L_2)}{P(L_1)(P(L/L_1) + P(L_2)P(L|L_2) + P(L_3) \cdot P(L|L_3)}$$

$$= \frac{0.5 \times 0.4}{0.4 \times 0.2 + 0.5 \times 0.4 + 0.1 \times 0.1}$$

$$= 0.69$$

Q.6 $X \sim U(1, 2)$ & $Y = \frac{3}{6-X}$

$$f_X(x) = \frac{1}{\beta - \alpha} = \frac{1}{2-1}$$

$$= 1$$

$$Y = \frac{3}{6-X}$$

$$Y(6-X) = 3$$

$$X = 6 - \frac{3}{Y}$$

$$g^{-1}(y) = 6 - \frac{3}{y}$$

$$\frac{d g^{-1}(y)}{dy} = \cancel{3 \cdot \frac{-1}{y^2}} \quad -3 \cdot \frac{-1}{y^2}$$

$$= \frac{3}{y^2}$$

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$f_x\, g^{-1}(y) = 1$ (As R.V. X can only take value between 0 & 1)

$$f_y(y) = f_x\, g^{-1}(y) \left| \frac{d\, g^{-1}(y)}{dy} \right|$$

$$= 1 \cdot \frac{3}{y^2}$$

$$= 3y^{-2}$$

Q.7 1] $F(3) = P(X \leq 3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

2] $E(x) = x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 + x_5 p_5$

$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$= 3.25$

3] $V(x) = x_1^2 p_1 + x_2^2 p_2 + x_3^2 p_3 + x_4^2 p_4 + x_5^2 p_5$

$= 1^2(0.05) + 2^2(0.15) + 3^2(0.35) + 4^2(0.4) + 5^2(0.05)$

$= 11.45$

4.]

Q. 8 Let claim amt exceeding 1000 be success. $P(s)$

So, $P(s) = 0.2$

$P(f) = 0.8$

It is a binomial distribution

$P(X < 3) = P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1\, 0.2^1 \times 0.8^{14} + {}^{15}C_2\, 0.2^2 \times 0.8^{13}$

$= 0.398$

Q.9 let winning ~~to~~ prize be success $P(s)$

so $P(s) = 0.01$

$P(f) = 0.99$

It is a binomial distribution

so $P(x=3) = {}^7C_3\, 0.01^3\, 0.99^4$

$= 0.000033621$

Q.11 $P(b) = P(g)$

$P(b) + P(g) = 1$

$2P(b) = 1$

$P(b) = 1/2 = P(g)$

let E be the event of having 3 boys

$P(E) = 1/2 \cdot 1/2 \cdot 1/2$

$= 1/8$

Q.13 $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75}$$

$$= \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100-80}{45}$$

$= 0.444$