PROBABILITY AND STATISTICS - 2

ASSIGNMENT

Q1. $X \sim Poi(\theta)$

$$\frac{X - \theta}{\sqrt{\theta}} \overset{\cdot}{\sim} N(0, 1) \quad \text{— [given]}$$

we know,

$$\frac{\hat{\lambda} - \lambda}{\sqrt{\lambda / n}} \sim N(0, 1)$$

$$\therefore \hat{\lambda} = \frac{\Sigma x_i}{n} = 200.$$

$$95\% \ CI = \hat{\lambda} \pm Z_{2.5\%} \times \sqrt{\frac{\hat{\lambda}}{n}}$$

$$= 200 \pm 1.96 \sqrt{200 \div 1}$$

$$= 200 \pm 27.718$$

$$\therefore 95\% \ CI = (172.282, 227.718)$$

2. $\bar{X} = \Sigma_1^n x_i$

$$S^2 = \frac{1}{n-1} \Sigma_i^n (x_i - \bar{x})^2$$

(i) $E(\bar{x}) = E\left(\frac{\Sigma x_i}{n}\right)$

$= \frac{\Sigma E(x_i)}{n}$ — [since x_i are ~~indepent~~ independent and identically distributed.]

$= \frac{\Sigma \mu}{n} = \frac{n\mu}{n}$

$\therefore E(\bar{x}) = \mu$

$\cancel{E}$ $V(\bar{x}) = V\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n^2} V(\Sigma x_i)$

$= \frac{\Sigma V(x_i)}{n^2}$ — [Since x_is are iids]

$= \frac{\cancel{n}\sigma^2}{n^{\cancel{2}}}$

$\therefore V(\bar{x}) = \frac{\sigma^2}{n}$

(iii) $E(s^2) = \sigma^2$.

$$\therefore E(s^2) = E\left[\frac{1}{n-1}\Sigma(x_i - \bar{x})^2\right]$$

$$= \frac{1}{n-1} \times E(\Sigma(x_i - \bar{x}))^2$$

$$= \frac{1}{n-1} . \Sigma \times E[(x_i - \bar{x})^2] \quad \text{— [since iid]}$$

$$= \frac{1}{n-1} \Sigma[E(x_i^2 - 2x_i\bar{x} + \bar{x}^2)]$$

$$= \frac{1}{n-1} \Sigma[E(x_i^2) - \cancel{2E(\bar{x}^2)} + E(\bar{x}^2)]$$

$$= \frac{1}{n-1} \Sigma[E(x_i^2) - E(\bar{x}^2)]$$

$$= \frac{1}{n-1} \Sigma(\sigma^2 + \mu^2) - n(\sigma^2/n + \mu^2)$$

$$= \frac{1}{n-1} \; n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}$$

$$= \frac{1}{\cancel{n-1}} \sigma^2 \cancel{(n-1)}$$

$$\therefore E(s^2) = \sigma^2$$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}.$$

(iii) we know,

$$V\left[\frac{(n-1)s^2}{\sigma^2}\right] = 2(n-1)$$

$\therefore$ ~~(n-1)s~~

$$\therefore \frac{(n-1)^2 V(s^2)}{\sigma^4} = 2(n-1)$$

$$V(s^2) = \frac{2\cancel{(n-1)}\sigma^2}{(n-1)^{\cancel{2}}}$$

$$\therefore V(s^2) = \frac{2\sigma^4}{(n-1)}$$

(iv) $n=10$ $\quad \sigma^2 = 100.$

$\therefore E(s^2) = \sigma^2.$

$\therefore E(s^2) = 100$.

~~P(150~~ $P(50 < s^2 < 150)$ — To find.

$\therefore P(50 < s^2 < 150) = P(s^2 < 150) - P(s^2 < 50)$

$$\because \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}.$$

$$\therefore P(50 < s^2 < 150) = P\left(\frac{9s^2}{100} < 13.5\right) - P\left(\frac{9s^2}{100} < 4.5\right)$$

$$= P\left(\chi^2_9 < 13.5\right) - P\left(\chi^2_9 < 4.5\right)$$

$$= 0.8587 - 0.1245$$

$$\therefore P(50 < s^2 < 150) = 0.7342$$

3. $n = 4$.

$\therefore P(X=0) = {}^4C_0 \cdot p^0 (1-p)^4$

$= (1-p)^4$.

$P(X=1) = {}^4C_1 \cdot p^1 (1-p)^3$

$= p(1-p)^3 \cdot {}^4C_1$

$P(X=2) = {}^4C_2 \cdot p^2 (1-p)^2$.

$P(X=3) = {}^4C_3 \cdot p^3 (1-p)^1$

$P(X=4) = {}^4C_4 \cdot p^4 (1-p)^0$

$\therefore L = \prod_{i=1}^{180} f(x_i)$

$= ((1-p)^4)^{86} \cdot [4p(1-p)^3]^{75} \cdot [6p^2(1-p)^2]^{16}$

$\times [4p^3(1-p)]^2 \times (p^4)^1$

$= [(1-p)^{344}][p^{75}(1-p)^{225}][p^{32}(1-p)^{32}]$

$\times [p^6(1-p)^2][p^4] \times$ constant.

$= p^{117} \times (1-p)^{603} \times$ constant.

$\ln L = 117 \ln p + 603 \ln(1-p) + \ln(\text{constant})$.

$$\frac{d \ln L}{dp} = \frac{117}{p} - \frac{603}{(1-p)} = 0.$$

$$\frac{117}{p} = \frac{603}{1-p}$$

$117 - 117p = 603p$

$720p = 117$

$$\therefore \hat{p} = \frac{117}{720} = \underline{0.1625}$$

x	O_i	E_i	$(O_i-E_i)^2/E_i$
0	86	36	69.44
1	75	36	42.25
2	16	36	11.11
3	2	36	32.11
4	1	36	34.02
	180.	180	188.93

~~$P(x=0) = (1-p)^4$~~

$$\therefore \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_3$$

$P(x=0) = (1-p)^4 = 0.4919.$

$P(x=1) = 0.3818.$

$P(x=2) = 0.1111$

$P(x=3) = 0.0143.$

$P(x=4) = 0.0006.$

H0: no. of claims conform to the binomial distn.

H1: no. of claims do not conform to the binomial distribution

x	O_i	E_i	$\Rightarrow E_i$	$\Rightarrow E_i$	$(O_i-E_i)^2/E_i$
0	86	88.542	88.542	88.542	0.0729
1	75	68.724	68.724	68.724	0.5731
2	16	19.998	19.998	22.68.	0.5971
3	2	2.574	2.682		
4	1	0.108			
	180				1.2431

$$\therefore \text{TS} : \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{5-1-1-2.}$$

$$\therefore 1.2431 \sim \chi^2_1$$

At 5% level of significance, $\chi^2_{cal} < \chi^2_{tab}$.

$\therefore$ we do not reject H0.

4. $f(x) = \dfrac{c\beta^3}{(x+\beta)^4}$; $x > 0$, $\beta > 0$

(i) Comparing to pdf of Pareto Distribution.

$$PDF = \frac{\alpha \lambda^{\alpha}}{(\lambda + x)^{\alpha+1}}$$

$\therefore$ β corresponds to λ.

$\therefore$ $c = 3$ — [as c corresponds to α]

$$E(X) = \frac{\lambda}{\alpha - 1} = \frac{\beta}{2}$$

$$V(X) = \frac{\alpha \lambda^2}{(\alpha-1)^2(\alpha-2)} = \frac{3\beta^2}{4}$$

(ii) $E(X) = \cancel{\text{Sm}} \dfrac{\lambda}{\alpha - 1}$

$$\bar{x} = \frac{\beta}{2}$$

$$\therefore \hat{\beta} = 2\bar{x}$$

$$\text{Bias} = E(\hat{\beta}) - \beta$$

$$= E(2\bar{x}) - \beta$$

$$= 2E(\bar{x}) - \beta = 2\,\frac{\Sigma E(x_i)}{n} - \beta$$

$$= \frac{\cancel{2}}{\cancel{n}} \times \frac{\cancel{n}\beta}{\cancel{2}} - \beta = 0$$

$\therefore$ Bias $= 0$.

$\therefore$ Estimator is unbiased.

$$MSE = var(\hat{\beta}) + Bias^2$$

$$= V(\hat{\beta}) = V(2\bar{x})$$

$$= 4(V(\bar{x})) = 4\left[V\left(\frac{\sum x_i}{n}\right)\right]$$

$$= \frac{4}{n^2} \times \sum V(x_i)$$

$$= \frac{\cancel{4}}{n^{\cancel{2}}} \times \cancel{n}\frac{3\beta^2}{\cancel{4}}$$

$$\therefore MSE = \frac{3\beta^2}{n}$$

(iii) $\hat{\beta} = b\bar{x}$

$$E(\hat{\beta}) = E(b\bar{x})$$

$$= bE\left(\frac{\sum x_i}{n}\right)$$

$$= \frac{b}{n}\sum E(x_i)$$

$$= \frac{b\beta}{2}$$

$$V(\hat{\beta}) = V(b\bar{x})$$

$$= b^2 V\left(\frac{\sum x_i}{n}\right)$$

$$= \frac{b^2}{n^2}\sum V(x_i)$$

$$= \frac{b^2}{n^{\cancel{2}}} \times \frac{3\beta^2}{4} \times \cancel{n} = \frac{3}{4}b^2\beta^2$$

$$MSE = Var + Bias^2.$$

$$= \frac{3b^2\beta^2}{4} + \left(\frac{b\beta}{2} - \beta\right)^2.$$

$$= \cancel{\frac{3b^2\beta^2}{4}} + \cancel{\frac{b^2\beta^2}{4}} - \cancel{b\beta^2} + \cancel{\beta^2}$$

$$= \frac{3b^2\beta^2}{4} + \left[\frac{b\beta - 2\beta}{2}\right]^2.$$

$$= \frac{3b^2\beta^2}{4n} + \left[\frac{b^2\beta^2 + 4\beta^2 - 4\beta^2 b}{4}\right]$$

$$\therefore M = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$$

$$\frac{dM}{db} = \frac{\beta^2}{4n}(6b + 2nb - 4n)$$

$$\frac{dM}{db} = 0$$

$$\frac{\beta^2}{4n}(6b + 2nb - 4n) = 0.$$

$$6b + 2nb - 4n = 0$$

$$b(3+n) = 2n$$

$$b = \frac{2n}{3+n} = \frac{2}{\left(\frac{3+n}{n}\right)}$$

$$= \frac{2}{\left(1 + \frac{3}{n}\right)}$$

$$\frac{d^2M}{db^2} = \frac{\beta^2}{4n}(6 + 2n)$$

$$\frac{\beta^2}{4n}(6 + 2n) > 0$$

$\because \frac{d^2m}{db^2}$ is minimum.

Thus, $b = \frac{2}{(1+3/n)}$ minimises the MSE.

(iv) $E(\hat{\beta}_2) = \frac{b\beta}{2}$

$= \frac{\cancel{2}}{(1+3/n)} \times \frac{\beta}{\cancel{2}}$

$= \frac{n\beta}{n+3}$

Bias $= E(\hat{\beta}_2) - \beta$

$= \frac{n\beta - \beta(n+3)}{n+3}$

$= \frac{-3\beta}{n+3}$

$\because$ we have a negative bias, it underestimates the parameter β.

$$MSE = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$$

$$= \frac{\beta^2}{4n}\left[b^2(n+3) + 4n(1-b)\right]$$

$$= \beta^2\left[\frac{4n^2}{n+3} + 4n\frac{(3-n)}{n+3}\right]$$

$$= \beta^2\left[\frac{\cancel{4n^2} + 12n - \cancel{4n^2}}{n+3}\right]$$

$$MSE = \frac{12n\beta^2}{n+3}$$

As $n \to \infty$, MSE does not tend to 0

$\therefore$ it is inconsistent.

5. (i) $X \sim U(a, b)$

$$E(X) = \frac{a+b}{2}$$

$$\therefore \bar{x} = \frac{a+b}{2}$$

$$\cancel{\therefore a = 2E(X) - b}$$

$$\therefore a = 2\bar{x} - b \quad — (1)$$

$$V(X) = \frac{1}{12}(b-a)^2$$

$$\therefore s^2 = \frac{1}{12}(b-a)^2$$

$$12s^2 = (b-a)^2 \quad — (2)$$

$$12s^2 = (2\bar{x} - a - a)^2$$

$$\therefore \sqrt{12s^2} = 2\bar{x} - 2a$$

$$\cancel{2}\sqrt{3}\, s = \cancel{2}(\bar{x} - a)$$

$$\cancel{\therefore s = \frac{\bar{x} - a}{\sqrt{3}}}$$

$$\therefore \hat{a} = \bar{x} - \sqrt{3}\, s$$

(ii) $b = 2\bar{x} - a$

From (1)

From (2)

$$12s^2 = (b-a)^2$$

$$2\sqrt{3}\, s = (b - 2\bar{x} + b)$$

$$\cancel{2}\sqrt{3}\, s = \cancel{2}(b - \bar{x})$$

$$\therefore \hat{b} = \bar{x} + \sqrt{3}\, s$$

(iii) Let the sample be 8, 10, 24, 100, 250.

$\therefore \bar{x} = 78.4$.

$S = 103.086$

$\hat{a} = \bar{x} - \sqrt{3}\, S$

$= 78.4 - \sqrt{3}\,(103.086)$

$\therefore \hat{a} = -100.15$

$\hat{b} = \bar{x} + \sqrt{3}\, S$

$= 78.4 + \sqrt{3}\,(103.086)$

$\therefore \hat{b} = 256.95$

One weakness that we can observe here, is the value of $\hat{a}$ is very far from the sample but that of $\hat{b}$ is relatively closer.

(iv) $X \sim U(0, b)$

$L = \prod_{i=1}^{n} f(x_i) = \left(\frac{1}{b}\right)^n$

$\ln L = n \ln(1/b) = -n \ln b$

$\frac{d \ln L}{db} = -n/b = 0$

$\therefore d \ln L / db$ gives $b = \infty$ which is not possible.

and $\frac{d^2 l}{db^2} = n/b^2 > 0$ which is minimum

$\therefore \hat{b} = \max(x_i)$

6. $H0: p = 0.1$

$H1: p > 0.1$

(i) $P(\text{Type } 1 \text{ error}) = P(\text{Rejecting } H0 / H0 \text{ is true})$.

Let X be the no. of hits in step 1 missiles

and Y be the no. of hits in step 2 missiles.

$\therefore P(\text{Type } 1 \text{ error}) = P(X \geq 3 / p = 0.1) \cancel{+ P(}$

$+ P(Y \geq 5 / p = 0.1) \times P(X = 0 / p = 0.1)$

$+ P(X = 1 / p = 0.1) \times P(Y \geq 1 / p = 0.1)$

$+ P(X = 2 / p = 0.1) \times P(Y \geq 3 / p = 0.1)$

(iv) $n = 120$ missiles.

$$\frac{X/n - P}{\sqrt{\frac{P(1-P)}{n}}} \sim N(0, 1)$$

$n = 120$ and $\cancel{p}X = 40$.

$\therefore$ $\cancel{40/120 + Q.}$ $\hat{p} = \frac{40}{120} = 0.3333$

$Z_{10\%} = 1.2816$.

~~$\because$ its a one-sided interval, we find only the lower bound confidence interval.~~

$\therefore CI = 0.3333 - \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \times 1.2816$.

$= \underline{0.27815}$

(v) $H_0: p = 0.278$

$H_1: p > 0.278$.

$\therefore \dfrac{x - np}{\sqrt{npq}} \sim N(0,1)$

$\therefore$ using continuity correction,

$$\frac{39.5 - 33.36}{\sqrt{24.086}} = 1.25108$$

$\therefore$ we do not have sufficient evidence to reject H_0 at 10% level of significance as $Z_{cal} < Z_{tab}$.

(vii) $H_0: \mu_1 = \mu_2$.

$H_1: \mu_1 \neq \mu_2$.

$n_1 = 12$ $\quad$ $n_2 = 15$ $\quad$ $S^2p = \frac{1}{25}[11(2916) + 14(4900)]$

$\bar{x}_1 = 65$ $\quad$ $\bar{x}_2 = 70$ $\quad$ $= \underline{\underline{4027.04}}$

$\sigma_1 = 54$ $\quad$ $\sigma_2 = 70$

$$TS = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{1/n_1 + 1/n_2}} \sim t_{n_1 + n_2 - 2}$$

$$= \frac{(65 - 70) - 0}{4027.04\,(0.15)} = -0.2034$$

~~$t_{cal} > t$.~~ $t_{tab} > t_{cal}$.

CI at 95%. $= (2.2634, 1.8566)$

$\therefore$ we ~~do not~~ have ~~s/~~ insufficient information to reject H_0.

7.

Public	24	45	29	33	20	40	26.5	25	27.5
Private	30	54.5	30	40	28.5	36	30.5	30.5	35.5

(i) $n = 9$.

$\bar{x}_1 = 30$. $\qquad$ $\bar{x}_2 = 35.055$

$S_1^2 = 64.3125$. $\qquad$ $S_1^2 = 67.403$.

(ii) $\because$ sample comes from the same distribution, we apply a 2-sided test.

$H_0 : \cancel{H_1}\ \mu_1 = \mu_2$. $\qquad$ $S_p^2 = \frac{1}{16}[514.5 + 606.627]$

$H_1 : \mu_1 \neq \mu_2$. $\qquad$ $= 70.0704$

$\therefore \cancel{\frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}}} \sim \cancel{N}$

$$T.S = \frac{\cancel{A}(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{(70.0704)(0.2222)} \sim t_{n_1+n_2-2}$$

$$= \frac{-5.055}{15.569} \sim t_{16, 2.5\%}$$

$$= 0.3246$$

CI at 95% LOS $= 2.12 \pm 0.3246$.

$= (1.7954, 2.4446)$

$t_{cal} < t_{tab}$.

we reject H_0.

(iii) $H_0 : \mu_{pr} = \mu_{pb}$

$H_1 : \mu_{pr} > \mu_{pu}$

$$TS = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_{12})}{Sp\sqrt{1/n_1 + 1/n_2}}$$

$$\cancel{TS} = \frac{35.055 - 30}{8.1152\sqrt{2/9}}$$

$\therefore TS = 1.32151$

Tabulated value $= 1.746$.

$T_{cal} < T_{tab}$.

$\therefore$ we reject H_0.

8. (i) An unbiased estimator is ~~an estimator~~ when the expected value of the ~~p~~ estimator is equal to the parameter.

~~(ii)~~ An estimator is said to be unbiased if bias is equal to zero '0' for all values of parameter 'θ'.

(ii) Bias is the measure of difference between the expected value of the estimator and the parameter being estimated.

(iii) $MSE(\hat{\theta}) = Var(\hat{\theta}) +$ ~~Bias($\hat{\theta}$)~~ $Bias^2$.

(iv) The estimator having less MSE is more consistent.

(v) They both will be considered to be consistent when MSE minimizes such that $n \to \infty$ and $MSE \to 0$.

(vi) We can estimate θ by method of Moments. We can also use a more preferred method of Maximum Likelihood Estimate.

9. (i) the variable t_K is defined as

$$\frac{N(0,1)}{\sqrt{t_K / K}} \sim t_K.$$

where,

t_K is the t distribution with 'k' degrees of freedom.

k is the ~~term~~ ~~sample~~ number of samples or simply 'n-1'.

(ii) Mean = 0

Variance = $\frac{K}{K-2}$

(iii) $n = 10$

$\bar{x} = 50$

$s^2 = 48.667$

(a) $\therefore \frac{\bar{x} - \mu}{s \sqrt{n}} \sim t_{n-1}$

$$P\left(t_{9,0.5\%} < \frac{\bar{x} - \mu}{s\sqrt{n}} < t_{9,0.5\%} \right) = 0.99$$

$$TS = \bar{x} \pm t_{9,0.5\%} \sqrt{s^2/n}$$

$$= 50 \pm 3.25 \times \sqrt{\frac{48.667}{10}} = 50 \pm 3.25(2.206)$$

$\therefore$ 99% CI = (42.83, 57.17)

(iv) ~~for~~ From (iii)

$$t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right)$$

$$\therefore \frac{\bar{x} - \mu}{s\sqrt{n}} \sim N\left(0, \sqrt{9/7}\right)$$

$$\therefore \frac{\bar{x} - \mu}{s\sqrt{7n/9}} \sim N(0, 1)$$

$$\therefore TS = \bar{x} - Z_{0.5\%}\sqrt{s^2/n}$$

$$= 50 - 2.58\sqrt{\frac{48.661}{10} \times \frac{9}{7}}$$

$$\therefore 99\% \ CI = (43.54, 56.45)$$

10. $n = 1000$

$\lambda = 3$

$X \sim Poi(n\lambda)$

$\therefore X \sim Poi(3000)$

$H0: \ n\lambda > 3100$

$H1: \ n\lambda \leq 3100.$

(i) $P(\text{Type 1 error}) = P(\text{Reject } H0 / H0 \text{ is true})$

$= P(X > 3100 / X \sim Poi(3000))$

$$P(X > 3100) = P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$$

$= P(Z > 1.8257) = 1 - P(Z < 1.8257)$

$= \cancel{0.0343}$ 0.0343

(ii) Type 2 error = event of accepting hypothesis when it is false.

(iii) Power of test = P(rejecting Ho / Ho is false)

$= P[X > 3100 / X \sim Poi(n\lambda) - N(n\lambda, n\lambda)]$

$= 1 - P\left(Z < \frac{3100 - n\lambda}{\sqrt{n\lambda}}\right)$

The value of power of test will depend on the value of the parameter under hypothesis.

(iv) $\hat{\mu}$ is the estimated value.

$\therefore \hat{\mu} \sim N(\mu, \hat{\mu}/n)$

$\therefore \frac{\hat{\mu} - \mu}{\sqrt{\hat{\mu}/n}} \sim N(0, 1)$

$P\left(-2.578 < \frac{2900 - \mu}{\sqrt{2900/1000}} < 2.578\right) = 0.99$

$\therefore$ 99% CI = (2.7613, 3.0387)

11. $n = 12$.

12

$\Sigma x = 213200$

$\Sigma x^2 = 4919860000$

$\bar{x} = 17767$

$s = 10144$

new $\Sigma x_1 = 213200 - 11000 - 48000 + 27500 + 31500$

$= 213200$

$\bar{x}_1 = \frac{213200}{12} = 17767.$

$\Sigma x_1^2 = 4919860000 - (11000)^2 - (48000)^2 + (27500)^2 + (31500)^2.$

$= 4243360000$

$s_1^2 = \frac{1}{n-1}\left[\Sigma x_1^2 - n(\bar{x}_1)^2\right]$

$= \frac{1}{11}(4243360000 - 3787995468)$

$\therefore s_1^2 = 41396775.64$

We do not observe any changes in the sample mean but we do observe a decrease in the sample variance, signifying that the new data deviates less from the mean of the population.

12. (i) $X \sim U(0, \theta)$

The Cramer-Rao lower Bound result holds true under very general conditions except when the range of distribution ~~parameters~~, involves the parameters, such as this case. This is caused due to the discontinuity in derivative.

(iii) $\text{Bias}[\hat{\theta}(c)] = \left(\frac{cn}{n+1} - 1\right)\theta$

$$\therefore \text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c) - \theta]$$

$$= E(cY - \theta)$$

$$= c \cdot E(Y) - \theta$$

$$= c \cdot \frac{n\theta}{n+1} - \theta$$

$$= \left(\frac{cn}{n+1} - 1\right)\theta$$

~~MSE = Var + Bias²~~

$$MSE[\hat{\theta}(c)] = E[(\hat{\theta}(c) - \theta)^2]$$

$$= E[(cY - \theta)^2] = E[c^2Y^2 - 2c\theta Y + \theta^2]$$

$$= c^2 E(Y^2) - 2c\theta E(Y) + \theta^2$$

$$= c^2 \cdot \frac{n\theta^2}{n+2} - \frac{2c\theta \cdot n\theta}{n+1} + \theta^2$$

$$\therefore MSE[\hat{\theta}(c)] = c^2 \cdot \frac{n\theta^2}{n+2} - c \cdot \frac{2n\theta^2}{n+1} + \theta^2$$

(iv) For $\hat{\theta}(c)$ to be unbiased, we need,

$$\text{Bias}[\hat{\theta}(c)] = 0.$$

$$\therefore \left(\frac{cn}{n+1} - 1\right)\theta = 0.$$

$$\therefore \frac{c_u n}{n+1} = 1$$

$$\therefore c_u = \frac{n+1}{n}$$

(v) in order to minimize MSE, we need,

$$0 = \frac{d}{dc} MSE\{\hat{\theta}(c)\}$$

$$= \frac{d}{dc}\left[c^2 \cdot \frac{n\theta^2}{n+2} - c \cdot \frac{2n\theta^2}{n+1} + \theta^2\right]$$

$$0 = 2c \cdot \frac{n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}$$

$$\therefore c_m = \frac{\cancel{2n\theta^2}}{n+1} \times \frac{n+2}{\cancel{2n\theta^2}}$$

$$\therefore c_m = \frac{n+2}{n+1} > 0 \quad \therefore \text{It is minimum.}$$

(vi) The estimator with the least MSE is preferred.

$\therefore$ when $n \to \infty$, $\hat{\theta}(c_u) = \frac{n+1}{n} Y \to Y$

and when $n \to \infty$, $\hat{\theta}(c_m) = \frac{n+2}{n+1} Y \to Y$

$\therefore$ the 2 estimators become one and same as $n \to \infty$.