Calculus - Assignment 1

1. $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$

$\lim_{h\to 0} \frac{2(9+h^2-6h)-18}{h}$

$\lim_{h\to 0} \frac{\cancel{18}+2h^2-12h-\cancel{18}}{h}$

$\lim_{h\to 0} \frac{2h(h-6)}{\cancel{h}}$

$= 2(-6) = \boxed{-12}$

2. $g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$

(a) at $x=4$

$g(4) = 2x = 2(2) = 4$

$\lim_{x\to 4} 2x = 4$

$\Rightarrow \lim_{x\to 4} g(x) = g(4)$

$\Rightarrow g(x)$ is continuous at $x=4$

(b) at $x=6$

$g(6) = x-1 = 6-1 = 5$

LHL : $\lim_{x\to 6^-} 2x = 2(6) = 12$

RHL : $\lim_{x\to 6^+} x-1 = 6-1 = 5$

$\Rightarrow$ LHL $\neq$ RHL

$g(x)$ is discontinuous at $x=6$

3. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}{-\cancel{(\sqrt{x}-3)}}$

$= \lim_{x \to 9} -(\sqrt{x}+3) = \underline{\underline{-6}}$

4. $f(x) = \frac{4x+5}{9-3x}$

(a) at $x=-1$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \boxed{\frac{1}{12}}$

$\lim_{x \to -1} \frac{4x+5}{9-3x} = \frac{1}{12}$

$\rightarrow f(-1) = \lim_{x \to -1} f(x)$

$\rightarrow f(x)$ is continuous at $x=-1$

(b) at $x=0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\lim_{x \to 0} \frac{4x+5}{9-3x} = \frac{5}{9}$

$\rightarrow f(0) = \lim_{x \to 0} f(x)$

$\rightarrow f(x)$ is continuous at $x=0$

c $x = 3$

$f(3) = \frac{4(3)+(5)}{9-3(3)} = \frac{17}{0} \Rightarrow$ ~~Does~~ not defined

Since $f(3)$ does not exist, $f(x)$ is discont. at $x=3$

5. $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\Rightarrow \lim_{x \to \infty} \frac{\cancel{e^{4x}}[6 - e^{-6x}]}{\cancel{e^{4x}}[8 - e^{-2x} + 3e^{-5x}]}$

$\Rightarrow \lim_{x \to \infty} \frac{\left[6 - \frac{1}{e^{6x}}\right]}{\left[8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}\right]}$

$= \frac{6 - \frac{1}{e^{\infty}}}{8 - \frac{1}{e^{\infty}} + \frac{3}{e^{\infty}}}$

$= \frac{6}{8} = \boxed{\frac{3}{4}}$

Ans $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} = \boxed{\frac{3}{4}}$

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6. $g(y) = \begin{cases} y^2+5 & ; y < -2 \\ 1-3y & ; y \geq -2 \end{cases}$

(a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} 1-3y = 1-3(6) = \underline{\underline{-17}}$

(b) $\lim_{y \to -2} g(y)$

LHL: $\lim_{y \to -2^-} y^2+5$

$= (-2)^2+5 = \underline{\underline{9}}$

RHL: $\lim_{y \to -2^+} 1-3y = 1-3(-2) = \underline{\underline{7}}$

7. $f(t) = (4t^2-t)(t^3-8t^2+12)$

$= (4t^2-t)\frac{d}{dt}(t^3-8t^2+12) + (t^3-8t^2+12)\frac{d}{dt}(4t^2-t)$

$= (4t^2-t)[3t^2-16t] + (t^3-8t^2+12)[8t-1]$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$

$= \underline{20t^4 - 132t^3 + 24t^2 + 96t - 12}$

8. $h(w) = \frac{3w + w^4}{2w^2+1}$

$= \frac{(2w^2+1)\frac{d}{dw}(3w+w^4) - (3w+w^4)\frac{d}{dw}(2w^2+1)}{(2w^2+1)^2}$

$= \frac{(2w^2+1)[3+4w^3] - (3w+w^4)(4w)}{(2w^2+1)^2}$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$$

$$= \frac{4w^5 + \cancel{10}\, 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

9. $f(x) = 2e^x - 8^x$

$$f'(x) = \frac{d}{dx}(2e^x - 8^x)$$

$$= 2e^x - 8^x \log 8$$

10. $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[e^z \frac{d}{dz}[\ln(z)] + \ln(z)\frac{d}{dz}(e^z)\right]$$

$$\boxed{\frac{dy}{dz} = 5z^4 - \frac{e^z}{z} + e^z \ln(z)}$$

11 a) $f(x) = (6x^2 + 7x)^4$

$$f'(x) = 4(6x^2+7x)^3 \frac{d}{dx}(6x^2+7x)$$

$$= 4(6x^2+7x)^3[12x+7]$$

b). $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = e^{(4t+t^7)} \frac{d}{dt}[4t+t^7]$$

$$= e^{(4t+t^7)}[4+7t^6]$$

12 a) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} * \frac{d}{dx}(7-x^3)$$

$$\frac{dz}{dx} = \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)[-6x] + (3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-(3x^4+42x)}{(7-x^3)^2}$$

b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$Q(v) = 2(6+2v-v^2)^{-4}$

$Q'(v) = -8(6+2v-v^2)^{-5}(2-2v)$

$$Q'(v) = \frac{16v-16}{(6+2v-v^2)^5}$$

$$Q''(v) = \frac{(6+2v-v^2)^5(16) - 5(16v-16)(6+2v-v^2)^4(2v-2v)}{(6+2v-v^2)^5}$$

$$= \frac{16(6+2v-v^2) - 5(2-2v)(16v-16)}{(6+2v-v^2)}$$

$$= \frac{96+32v-16v^2 - 5(32v+\cancel{32v}-32v^2-32)}{6+2v-v^2}$$

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$$= \frac{96+32v-16v^2-320v+160v^2+160}{(6+2v-v^2)}$$

$$= \cancel{5144v^2 \mp 32v 5 256}$$

$$= \frac{144v^2-288v+256}{6+2v-v^2}$$

c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{2t}{1+t^2}$$

$$f''(t) = \frac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \boxed{\frac{2-2t^2}{(1+t^2)^2}}$$

13. $f(x) = x^3-3x^2-9x+12$

$f'(x) = 3x^2-6x-9$

when $f'(x)=0$

$\Rightarrow$ $3x^2-6x-9=0$

$x^2-2x-3=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$x=-1,\ x=3$

$f''(x) = 6x-6$

$f''(-1) = -12 \Rightarrow f''(-1)<0$

$f''(3) = 12 \Rightarrow f''(3)>0$

$\rightarrow$ $f(x)$ is maximum at $x=-1$

<u>Ans</u> max value $= f(-1) = -1-3+9+12 = \boxed{17}$

$\rightarrow$ $f(x)$ is minimum at $x=3$

<u>Ans</u> $f(3) = 27-27-27+12 = \boxed{-15}$

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Ans Minimum value $= -15$
Maximum value $= 17$

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$
When $f'(x) = 0$
$12x^2 - 36x + 24 = 0$
$x^2 - 3x + 2 = 0$
$x^2 - 2x - x + 2 = 0$
$\Rightarrow x = 1, 2$

$f''(x) = 24x - 36$
$f''(1) = 24 - 36 = -12 \Rightarrow$ Maxima
$f''(2) = 48 - 36 = 12 \Rightarrow$ Minima

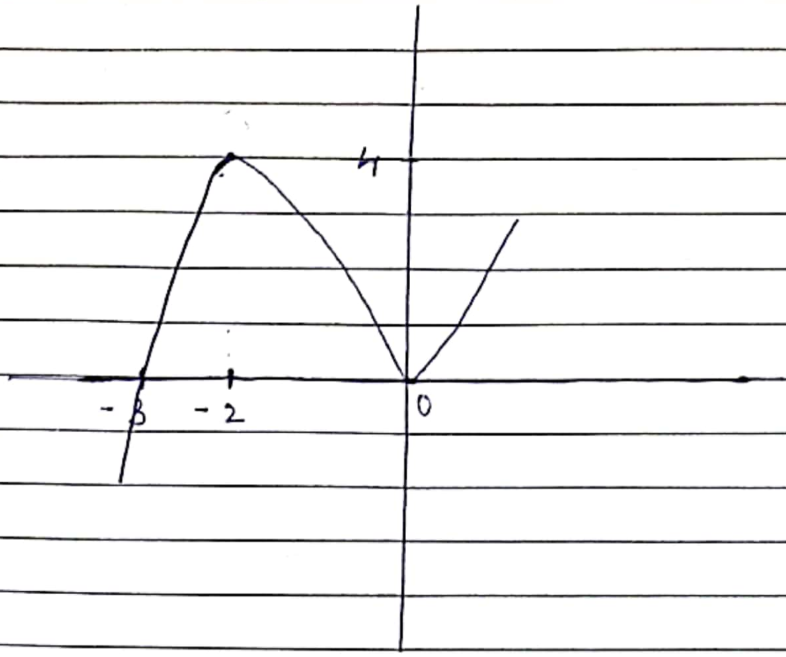
Ans Max value : $f(1) = 4 - 18 + 24 - 7 = \boxed{3}$

Ans Min. value : $f(2) = 4(8) - 18(4) + 24(2) - 7$
$= 32 - 72 + 48 - 7$
$= \boxed{1}$

15. $f(x) = x^2(x+3)$
$f'(x) = 2x(x+3) + x^2$
$= 3x^2 + 6x$
$= 3x(x+2)$
when $f'(x) = 0$
$\Rightarrow x = 0, \; x = -2$

+ve | -ve | +ve
(incre) -2 (dec.) 0 (increasing)

$f''(x) = 6x + 6$
$f''(0) = 6$
$f''(-2) = -6$
$\Rightarrow f(x)$ is max at $x = -2$
$\Rightarrow f(x)$ is min at $x = 0$



16. $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

When $f'(x) = 0$

$\Rightarrow$ $6x - 6 = 0$

$\Rightarrow$ $x = 1$

$f''(x) = 6$

$\Rightarrow$ $f(x)$ is minimum at $x = 1$