Assignment - 1

2.i) $\overline{X} = \sum_i^n x_i \qquad S^2 = \frac{1}{n-1}\sum_i^n (x_i - \overline{X})^2$

8

$E(\overline{X}) = E\left(\frac{\sum x_i}{n}\right)$

9

$\Rightarrow \frac{1}{n} E(\sum x_i)$

10

$\Rightarrow \frac{1}{n} \sum E(x_i)$

11

$\Rightarrow \frac{\sum \overline{X}}{n} = \frac{n}{n} \times \mu = \mu$

Noon

01 $V(\overline{x}) = V\left(\frac{\sum x_i}{n}\right)$

02 $\Rightarrow \frac{1}{n^2} V(\sum x_i)$

03 $\Rightarrow \frac{1}{n^2} \sum V(x_i)$

04 $\Rightarrow \frac{1}{n^2} \sum \sigma^2 \Rightarrow \frac{n\sigma^2}{n^2} \Rightarrow \frac{\sigma^2}{n}$

05 ii) $E(S^2) = E\left(\frac{1}{n-1}\sum_i^n (x_i - \overline{x})^2\right)$

Sunday 03

06 $\Rightarrow \frac{1}{n-1} E[\sum x_i^2) - E(n\overline{x}^2)$

$\Rightarrow \frac{1}{n-1}\left[\sum E(x^2) - nE(\overline{x}^2)\right]$

$\Rightarrow \frac{1}{n-1}\left[\sum(\mu^2+\sigma^2) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right]$

$\Rightarrow \frac{1}{n-1}\left[n\sigma^2 + n\mu^2 - n\mu^2 - \sigma^2\right]$

$\Rightarrow \cancel{\frac{\sigma^2[n-1]}{n-1}} \Rightarrow \sigma^2$

iii) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$.

$V(\chi^2_{n-1}) = V\left(\frac{(n-1)S^2}{\sigma^2}\right)$

$\Rightarrow 2(n-1) = \frac{(n-1)^2}{\sigma^4} \cdot V(S^2)$

$\Rightarrow V(S^2) = \frac{(n-1)^2}{2\cancel{(n-1)} \cdot \sigma^4} = \frac{n-1}{2\sigma^4}$

iv) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$\Rightarrow E\left[\frac{(n-1)S^2}{\sigma^2}\right] = E(\chi^2_{n-1})$

$\Rightarrow \frac{(n-1)E(S^2)}{\sigma^2} = n-1$

$\Rightarrow E(S^2) = \sigma^2$

So, $E(S^2) = 100$

$P(S^2 < 50) = P\left(\frac{(n-1)S^2}{\sigma^2} < \frac{9 \times 50}{100}\right)$

$\Rightarrow P(\chi^2_9 < 4.5)$

$\Rightarrow 0.1245 \Rightarrow 12.45\%$

4.

Probability & Statistics
Assignment - I

$\frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} \sim N(0,1)$

$\hat{\lambda} = \frac{\sum x_i}{n}$

$\hat{\lambda} = 200$

$P(-z_{0.025} < Z < z_{0.025}) = 0.95$

$P(-1.96 < Z < 1.96) = 0.95$

$\Rightarrow P\left(\hat{\lambda} - 1.96\sqrt{\frac{\hat{\lambda}}{n}} < \lambda < \hat{\lambda} + 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.95$

95% CI for $\lambda = (200 - 1.96\sqrt{200},\ 200 - 1.96\sqrt{200})$

$= (172.281,\ 227.718)$

~~6. $H_0: p = 0.1$ $H: p > 0.1$~~

~~$x < 5$ $n = 24$, $x \le 4$~~

Probability & Statistics
Assignment I

1) $$\frac{\hat{\lambda} - \lambda}{\sqrt{\lambda/n}} \sim N(0,1)$$

$$\hat{\lambda} = \frac{\sum X_i}{n}$$

$$\hat{\lambda} = 200$$

$$P(Z_{2.5\%} < Z < Z_{97.5\%}) = 0.95$$

$$P(-1.96 < Z < 1.96) = 0.95$$

$$P\left(-1.96 < \frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} < 1.96\right) = 0.95$$

$$\Rightarrow P\left(\cancel{-1.96} - \hat{\lambda} - 1.96\sqrt{\frac{\hat{\lambda}}{n}} < -\lambda < 1.96\sqrt{\frac{\hat{\lambda}}{n}} - \hat{\lambda}\right) = 0.95$$

~~95% CI~~ for $\lambda = (200 -$

$$\Rightarrow P\left(\hat{\lambda} - 1.96\sqrt{\frac{\hat{\lambda}}{n}} < \lambda < \hat{\lambda} + 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.95$$

95% CI for $\lambda = \left(200 - 1.96\sqrt{200}, 200 + 1.96\sqrt{200}\right)$

$= (172.281, 227.7185)$

3. No. of Claims = X

$X \sim Bin(4, p)$ ~~x=5~~

i) $L = \prod_{i=1}^{180} f(x_i)$

$= (P(X=0))^{86} \times (P(X=1))^{75} \times (P(X=2))^{16} \times (P(X=3))^{2} \times (P(X=4))^{1}$

$\Rightarrow ({}^4C_0 p^0 (1-p)^4)^{86} \cdot ({}^4C_1 p^1 (1-p)^3)^{75} \cdot ({}^4C_2 p^2 (1-p)^2)^{16} \cdot$

$({}^4C_3 p^3 (1-p))^2 \cdot ({}^4C_4 p^4)^1$

$\Rightarrow$ Constant $\cdot p^{117} \cdot (1-p)^{603}$

$\log L = \log(\text{constant}) + 117\log(p) + 603\log(1-p)$

$\frac{d\log L}{dp} = 0 + \frac{117}{p} - \frac{603}{1-p}$

taking, $\frac{d\log L}{dp} = 0$

so, $\frac{117}{p} = \frac{603}{1-p}$

$\Rightarrow 117 - 117p = 603p$

$\Rightarrow 117 = 720p$

$\Rightarrow \hat{p} = \frac{117}{720}$

$\Rightarrow \hat{p} = 0.1625$

ii) Goodness of fit test

H_0: no. of claims to the binomial model
H_1: no. of claims does not conform to the binomial model

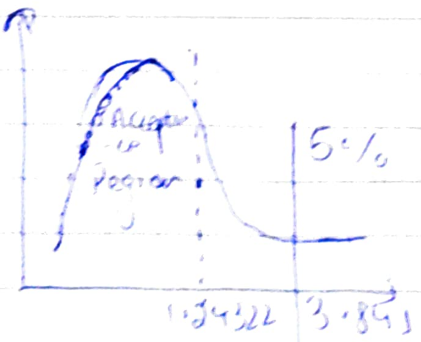
$p = 0.1625$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.542	68.724	19.98	2.574	0.108

$P(X=0) = {}^4C_0 p^0 (1-p)^4 = 0.4919$
$P(X=1) = {}^4C_1 p^1 (1-p)^3 = 0.3818$
$P(X=2) = 0.11$
$P(X=3) = 0.0143$
$P(X=4) = 0.0006$

$$T.S = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{5-1-2-1}$$

$$\Rightarrow 1.24322 \sim \chi^2_1$$



5% level of significance

$\therefore$ Do not reject H_0

4 $f(x) = \dfrac{c\beta^3}{(x+\beta)^4}$; $x>0$, $\beta>0$

(i) Comparing the above $f(x)$ to P.D.F of Pareto distribution (2 parameter version)

For Pareto,

$$PDF = \frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$$

Thus, α corresponds to β

$\therefore C = 3$

$$E(X) = \frac{\lambda}{\alpha-1}$$

$$\Rightarrow \frac{\beta}{3-1} = \frac{\beta}{2}$$

$$V(X) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} = \frac{3\beta^2}{4}$$

ii) $E(X) = \dfrac{\beta}{2}$

$E(X) = \bar{x}$

$\dfrac{\beta}{2} = \bar{x} \Rightarrow \hat{\beta} = 2\bar{x}$

$\hat{\beta} = 2\bar{x}$

ii) Bias $= E(\hat{\beta}) - \beta$

$\Rightarrow E(2\bar{X}) - \beta$

$\Rightarrow 2E(\bar{x}) - \beta$

$\Rightarrow \frac{2}{n} E(\Sigma x_i) - \beta$

$\Rightarrow \frac{2}{n} \times \frac{n\beta}{2} - \beta = 0$

$\therefore$ no bias

Noon

iii) $MSE = V(\hat{\beta}) + (Bias)^2$

$\Rightarrow V(\hat{\beta})$

$\Rightarrow V(2\bar{X})$

$\Rightarrow 2^2 V(\bar{x}) \Rightarrow 4V(\bar{x})$

$\Rightarrow 4V\left(\frac{\Sigma X_i}{n}\right)$

$\Rightarrow \frac{4}{n^2} \Sigma V(X_i)$

$\Rightarrow \frac{4}{n^2} \times \frac{3n\beta^2}{4}$

$\Rightarrow \frac{3\beta^2}{n}$

Sunday 10

As n increases, MSE tends to 0.
Hence, $\hat{\beta}$ is constant.

iii) $\beta = b\bar{X}$

$E(\beta_2) = E(b\bar{X})$

$= bE\left(\frac{\Sigma X_i}{n}\right) \Rightarrow \frac{b}{n} \Sigma E(X_i) \Rightarrow \frac{b}{n} \times \frac{\beta}{2} \times n = \frac{b\beta}{2}$

$$V(\hat{\beta}_2) = V(b\bar{X})$$
$$= b^2 V\left(\frac{\Sigma X_i}{n}\right) = \frac{b^2}{n^2} \Sigma V(X_i)$$

8
$$\Rightarrow \frac{b^2}{n^2} \times \frac{3}{4}\beta^2 \times \cancel{n}$$

9
$$\Rightarrow \frac{3b^2\beta^2}{4n}$$

10
$$MSE = V(\beta_2) + (Bias)^2$$
$$= \frac{3b^2\beta^2}{4n} + \left(\frac{b\beta}{2} - \beta\right)^2$$

11
$$= \frac{3b^2\beta^2}{4n} + \left(\frac{b\beta - 2\beta}{2}\right)^2$$

Noon
$$\Rightarrow \frac{3b^2\beta^2}{4n} + \frac{b^2\beta^2 + 4\beta^2 - 4b\beta^2}{4}$$

01
$$\Rightarrow \frac{3b^2\beta^2 + nb^2\beta^2 + 4n\beta^2 - 4nb\beta^2}{4n}$$

02
$$M = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$$

03
$$\frac{dM}{db} = \frac{\beta^2}{4n}(6b + 2nb - 4n)$$

04

05
$$\frac{dM}{db} = 0$$

06
$$\frac{\beta^2}{4n}(6b + 2nb - 4n) = 0$$

$$\Rightarrow 6b + 2nb - 4n = 0$$
$$\Rightarrow 3b + nb - 2n = 0$$
$$\Rightarrow b(3+n) = 2n$$
$$\Rightarrow b = \frac{2n}{3+n} = \frac{2}{(3+n)/n}$$

$$\Rightarrow b = \frac{2}{\left(1+\frac{3}{n}\right)}$$

$$\frac{d^2M}{db^2} = \frac{\beta^2}{4n}(6+2n)$$

$$\frac{\beta^2}{4n}(6+2n) > 0$$

Hence, $\frac{d^2M}{db^2}$ is minimum

Thus, $b = \frac{2}{(1+3/n)}$ is the minimum MSE.

iv) $E(\hat{\beta}_2) = \frac{b\beta}{2}$

$$= \frac{2}{(1+3/n)} \times \frac{\beta}{2} = \frac{n\beta}{(n+3)}$$

$$\text{Bias} = E(\hat{\beta}_2) - \beta$$

$$= \frac{n\beta}{n+\cancel{\beta}3} - \beta$$

$$\Rightarrow \frac{n\beta - \beta(n+3)}{n+\beta} = \frac{-3\beta}{n+3}$$

Thus we have a negative bias & hence it underestimates the true parameter β.

$$MSE = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$$

$$= \frac{\beta^2}{4n}\left(b^2(n+3) + 4n(1-b)\right) \Rightarrow \frac{\beta^2}{4n}\left[\frac{4n^2}{(n+3)^2} \times (n+3) + \frac{4n(3-n)}{n+3}\right]$$

$$\Rightarrow \frac{12n\beta^2}{n+3}$$

As $n \to \infty$, MSE do not tend to 0. Hence, it is inconsistent

5 i) $X \sim U(a,b)$

$E(X) = \dfrac{a+b}{2}$

$\bar{x} = \dfrac{a+b}{2} \Rightarrow b = 2\bar{x} - a$

$V(X) = \dfrac{(b-a)^2}{12}$

$S^2 = \dfrac{(b-a)^2}{12}$

$12S^2 = (b-a)^2$

$\Rightarrow 2\sqrt{3}S = b - a \Rightarrow 2\sqrt{3}S = 2\bar{x} - a - a$

$\Rightarrow \sqrt{3}S = \bar{x} - a$

$\Rightarrow \hat{a} = \bar{x} - 3\sqrt{3}$

ii) $b = 2\bar{x} - a \Rightarrow \hat{b} = 2\bar{x} - \bar{x} + \sqrt{3}S$

$\hat{b} = \bar{x} + \sqrt{3}S$

iii) Sample of observation : 1, 2, 8, 50, 150

$\bar{x} = 42.2$

$S = 63.57$

$\hat{a} = \bar{x} - \sqrt{3}S = -67.907$ $\qquad$ $\hat{b} = \bar{x} + \sqrt{3}S = 152.3064$

Hence, we can conclude that value of $\hat{b}$ is ~~approx~~ very close to sample value but $\hat{a}$ is very far from the true sample value, so it is the potential weakness of method of moments.

(iv) $X \sim U(0,b)$

$L = \prod_{i=1}^{n} f(x_i) = \left(\dfrac{1}{b}\right)^n$

$\Rightarrow \log L = -n \log b$

$\Rightarrow \frac{d \log L}{db} = \frac{-n}{b}$

$\frac{d \log L}{db} = 0$, gives $b = \infty$ which is not possible

Also,

$\frac{d^2 \log L}{db^2} = \frac{n}{b^2}$, $\frac{n}{b^2} > 0$, which we have a minimum likelihood estimate

Thus, we say, $\hat{b} = \max(x_i)$

6 $H_0 : p = 0.1$
$H_1 : p > 0.1$

(i) P(Type I Error) = P(rejecting H_0 | H_0 is true)

Let X be the no. of hits in first 12 missile & Y be the no. of hits in step 12 missiles

$P(\text{Type I error}) = P(X \geq 3 \mid p=0.1) + P(X=0 \mid p=0.1)\, P(Y \geq 5 \mid p=0.1) + P(X=1 \mid p=0.1)\, P(Y \geq 4 \mid p=0.1) + P(X=2 \mid p=0.1)\, P(Y \geq 3 \mid p=0.1)$

$= (1-0.8891) + 0.2824(1-0.9957) + (0.6590 - 0.2824)(1-0.9744) + (0.8891-0.6590)(1-0.8891)$

$= 0.14727 \approx 15\%$

ii) Probability of regretting null hypothesis when $p = 0.3$

$= P(X \geq 3 \mid p = 0.3) + P(X = 0 \mid p = 0.3) P(X \geq 5 \mid p = 0.3)$
$+ P(X = 1 \mid p = 0.3) P(X \geq 4 \mid p = 0.3) + P(X = 2 \mid p = 0.3)$
$P(Y \geq 3 \mid p = 0.3)$

$= 0.9125 \approx 91\%$

(iii) Probability of type II error
Accepting H_0 when H_0 is fragile

$= 1 - 0.91253$
$= 0.08747$
$\approx 9\%$ (where $p = 0.3$)

(iv) $\dfrac{X/n - p}{\sqrt{\frac{p(1-p)}{n}}}$ $n = 120$, $X = 40$

$\hat{p} = \dfrac{40}{120} = \dfrac{1}{3} = 0.3333$

$Z_{10\%} = 1.2816$

~~Lower~~ $\therefore \hat{p} - 1.2816\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} = 0.3333 - 1.2816\sqrt{\dfrac{0.33(1-0.333)}{120}}$

$= 0.2782$

(v) $p > 0.278$ at 10% LOS
$H_0 : p = 0.278$ vs $H_1 : p > 0.278$
One sample binomial model
$\dfrac{X - np_0}{\sqrt{np_0q_0}} \sim N(0,1)$ with continuity correction

$\dfrac{39.5 - 120(0.278)}{\sqrt{120 \times 0.278 \times 0.722}}$

$\therefore$ Do not reject H_0

(vi) Lower bound of confidence interval implies that there is only a 10% chance of p whereas from the hypothesis test, p could be less than or equal to 0.2780 with probability of more than 10%.

(vii) $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \neq \mu_2$

$$TS = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$$

Given,

$\bar{X}_1 = 65$

$\bar{X}_2 = 70$

$S_1 = 54$

$S_2 = 70$

$n_1 = 12$

$n_2 = 15$

$$S_p^2 = \frac{1}{25}\left(11(2916) + 14(4900)\right) = 4027.04$$

$$\frac{65 - 70}{65.459\sqrt{\frac{1}{12} + \frac{1}{15}}} = -0.2034$$

There is ± 2.060 ($\approx t_{25, 2.5\%}$), so ~~we have insufficient~~ do not reject H_0.

7 i) Public

$\bar{X}_1 = 30$ $\quad\quad$ $\bar{X}_2 = 35.0555$

$S_1^2 = 64.3125$ $\quad\quad$ $S_2^2 = 67.4027$

ii) 2 sided t-test can be applied in case the samples come from populations with equal variances.

We are testing $H_0: \sigma_1^2 = \sigma_2^2$ vs $H_1: \sigma_1^2 \neq \sigma_2^2$

T.S is $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,\, n_2-1}$

8 value of statistic is $\dfrac{64.31}{67.42} = 0.9542$

9

10 $\therefore$ Do not reject H_0.

11 iii) $H_0: \mu_{g} = \mu_{su}$

Noon $H_1: \mu_{g} > \mu_{su}$

01 $T.S = \dfrac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

02

$= \dfrac{35.05555 - 30}{8.1152\sqrt{\frac{2}{9}}}$

03

04 $T.S = 1.3151$

05 $t_{16,\, 5\%} = 1.746$

06 Since, $T.S_{cal}$ < Tabulated value

Do not reject H_0

5 i) Unbiased estimator

- An unbiased estimator is an estimator where expected value is equal to the parameter.

(ii) Biased

A biased estimator is one which delivers an estimate which is consistency diffirent from the parameter to be estimated

iii) The mean square error of an estimator $\hat{\theta}$ is defined by

$$MSE(\hat{\theta}) = var(\theta) + Bias^2$$

(iv) The estimator having less MSE is more consistent

(v) They both would be considered to be consistent when MSE minimizes such that when, $n \to \infty$, $MSE \to 0$.

(vi) i) Method of moments

ii) Maximum likelihood estimate

4) i) Variable t_k is defined as $t_k \Rightarrow \dfrac{N(0,1)}{\sqrt{\chi^2_k / k}}$

where k ~~dow~~ denotes the degree of freedom & the 2 random variables $N(0,1)$ & χ^2_k are independent.

ii) For $k > 2$,

$$E(t_k) = 0$$

$$V(t_k) = \frac{k}{k-2}$$

iii) a) $\dfrac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

$$P\left(t_{9,0.5\%} < \frac{\bar{x} - \mu}{S/\sqrt{n}} < t_{9,99.5\%}\right) = 0.99$$

$$P\left(\bar{x} - t_{9,0.5\%} \frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{9,0.5\%} \frac{S}{\sqrt{n}}\right) = 0.99$$

$t_{7, 0.5\%} = 3.25$

99% confidence interval for μ = (42.83, 57.17)

(b) From (ii),

$t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right)$

$t_{n-1} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$

i.e, $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$

$\frac{\bar{X}-\mu}{S/\sqrt{7n/9}} \sim N(0,1)$

Critical values level of confidence 2.58.

Confidence interval = (45.54, 56.45)

10. $n = 1000$

$\lambda = 3$

$X \sim Poi(N\lambda)$

$X \sim Poi(3000)$

i) P(Type 1 error) = P(reject H_0 | H_0 is true)

$= P(X > 3100 \mid X \sim Poi(3000))$

Using normal approx

$P(X > 3100) = P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$= 3.34\%$

ii) Type II error = event of accepting the hypothesis when it is false

iii) Power of test = P(rejecting H_0 | H_0 is false)

$= P[X > 3100 \mid X \sim Poi(n\lambda) - N(n\lambda, n\lambda)]$

8 $= 1 - P\left(Z < \frac{3100 - n\lambda}{\sqrt{n\lambda}}\right)$

9 The value of power of test will depend on the value of parameter under alternative hypothesis.

10

(iv) $\hat{\mu}$ is estimator of μ. $\hat{\mu}$ followed $N(\mu, \frac{\hat{\mu}}{n})$

11 Hence, $\frac{\hat{\mu} - \mu}{\sqrt{\hat{\mu}/n}} \sim N(0,1)$

Noon

or, $P\left(-2.5758 < \frac{2.9 - \mu}{\sqrt{2.9/1000}} < 2.5758\right) = 0.99$

01

Hence, CI = (2.7613, 3.0387)

02

11. $n = 12$

03 $\sum x = 213,200$

$\sum x^2 = 4,919,860,000$

04 $\bar{x} = 17,767$

$S = 10,144$

05 New Sample Mean

$\sum X' = \sum X - 11000 - 48000 + 27500 + 31500$

06 $= 213200$

$\bar{X}' = \frac{213200}{12}$

$= 17,767$

New Sample Standard deviation

$\sum X^{2'} = 4,919,860,000 - 11000^2 - 48000^2 + 27500^2 + 31500^2$

$= 4,243,360,000$

$$S'^2 = \frac{1}{n-1}\left(\Sigma X^{2'} - n(\bar{X}')\right)$$

$$= \frac{1}{11}\left(4,243,360,000 - 12(17,767)^2\right)$$

$$\Rightarrow 41396775.64$$

$$S' = 6434.03211$$

The sample mean remains the same after the leaving of the 2 permanent employees as the 2 values ~~found~~ removed were outliers & the 2 values included were around the sample mean.

The sample standard deviation reduces as the 2 values now included do not deviate very much from the sample mean causing the variance to reduce & hence the reduction of the sample standard deviation.

12. i) The Cramer-Rao lower bound result holds under very general conditions except where the range of the distribution inwards the parameter, such as the uniform distribution in this case. This is due to a ~~discortion~~ discontinuity, to the derivative in the formula does not make sense.

ii) We have $Y = \max X_i$

Since each X_i dies b/w 0 & θ, the support for Y will be 0 & θ.

For $0 < y < \theta$

$$P[Y < y] = P[\max X_i < y]$$

$$= P\left[\bigcap_{i=1}^{n} (X_i < y)\right] = \prod_{i=1}^{n} \int_0^y \frac{dx}{\theta} = \left(\frac{y}{\theta}\right)^n$$

Thus the probability density function g_y will be,

$$g_x(y) = \frac{d}{dy} P(x < y) = \frac{n}{\theta^n} y^{n-1} \text{ for } 0<y<\theta.$$

Now, for any non-negative real no. k,

$$E(y^k) = \int_0^\theta y^k \frac{n}{\theta^n} \cdot y^{n-1} dy$$

$$= \frac{n}{n+k} \int_0^\theta \frac{n+k}{\theta^n} \cdot y^{n+k-1} dy$$

$$= \frac{n}{n+k} \cdot \frac{\theta^{n+k} - 0}{\theta^n}$$

$$= \frac{n\theta^k}{n+k}$$

iii) Bias of estimator $\hat{\theta}(c)$ is given as:

$$\text{Bias}(\hat{\theta}(c)) = E(\hat{\theta}(c) - \theta)$$

$$= E[cY - \theta]$$

$$= cE(Y) - \theta$$

$$= \frac{cn\theta}{n+1} - \theta$$

$$= \left(\frac{cn}{n+1} - 1\right)\theta$$

MS error of estimator $\hat{\theta}(c)$ is given as

$$MSE[\hat{\theta}(c)] = E[\{\hat{\theta}(c) - \theta\}^2]$$

$$= c^2 E(Y^2) - 2c\theta E(Y) + \theta^2$$

$$= \frac{c^2 n\theta^2}{n+2} - \frac{c2n\theta^2}{n+1} + \theta^2$$

(iv) For $\hat{\theta}(c)$ to be as unbiased estimator of θ, we need $Bias(\hat{\theta}(c_\mu)) = 0$

8 $\left(\frac{c_\mu n}{n+1} - 1\right) \cdot \theta = 0$

9

$\Rightarrow c_\mu = \frac{n+1}{n}$

10

(v) In order to minimise $MSE[\hat{\theta}(c)]$, we need

11 $0 = \frac{d}{dc} MSE[\hat{\theta}(c)] = 2c\frac{n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}$

Noon Thus, $c_m = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta^2} = \frac{n+2}{n+1}$

01

(vi) In order to minimize error in estimation, it is

02 produced to ask for an estimator which has lower mean square error among different competing

03 estimators. So here $\hat{\theta}(c_m)$ will be preferred over $\hat{\theta}(c_\mu)$.

04

As n becomes large, $\hat{\theta}(c_\mu) = \frac{n+1}{n} Y \to Y$.

05

Similarly $\hat{\theta}(c_m) = \frac{n+2}{n+1} Y \to Y$

06

Thus the two estimators becomes one & same as $n \to \infty$.