Probability & Statistics Assignment

1) i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

μ = mean = $\frac{\Sigma x}{n} = \frac{134}{10} = 13.4$

median = 13

mode = 18

ii)

x_i	f_i	$x_i f_i$	CF
0	5	0	5
1	11	11	16
2	26	52	42
3	15	45	57
4	3	12	60
	60	120	

Mean = $\frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{60} = 2$

Median = $\frac{n+1}{2} = \frac{5+1}{2}$ 3rd term = 2

Mode = 2

2) $\lambda = 0.5$

i) $P(X=0) = \frac{e^{-0.5} \cdot \lambda^0}{0!} = \frac{e^{-0.5} \times 1}{1} = 0.60653$

3) i) Mean = $\frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$

Standard deviation = $\sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{(1/2)^2}} = \sqrt{8}$

ii) CDF $= \int_0^{\alpha} \frac{(1/2)^2}{2-1)!} x e^{-1/2x} = \frac{1}{4} \int x e^{-1/2} x dx$

$= \frac{1}{4}\left(x(e^{3/2}) \cdot (-2) - \int e^{-1/2 x} (-2) dx \right.$

$= \frac{1}{4}\left(-2x - 1/2 x - (-2) e^{-1/2 x} \cdot (-2)\right.$

$= \frac{1}{4}\left(-2x e^{-1/2 x} - 4 e^{-1/2 x}\right)_0^{\alpha}$

$= \frac{1}{4}(0 - (-4)) = 1$

4) No. of male = 10

No. of female = 8

Fully qualified = 6 males
= 7 females

Team of ~~stud~~ stud = $10C_6 \times 8C_7$

5) 40% via land = P(1) = 40%

P(2) = 50%

Prob. of package arriving late = 20%, 40%, 20%

6) $X \sim U(1,2)$ and $Y = \frac{3}{6-X}$ $y \sim U\left(\frac{3}{5}, \frac{3}{4}\right)$

7) i) F(3) = 0.05 + 0.15 + 0.35 = 0.55

ii) E(X) = 0.05 + 0.3 + 1.05 + 1.6 + 0.25 = 3.25

iii) $E(X)^2 = 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.35 + 4^2 \times 0.4 + 5^2 \times 0.05$

$= 0.05 + 0.6 + 3.15 + 7.52 + 1.25$

$= 12.52$

$V(X) = 12.52 - 10.56$

$= 1.9575$

8) P(exceeding 1000) = 0.2

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$
$$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$$
$$= (0.8)^{15} + 0.13194395$$
$$= 0.3980$$

9) P(winning) = 0.01 $\quad k = 3$

$$P(n=7) = {}^{7-1}C_{3-1} \cdot (0.01)^3 (0.99)^4$$
$$= 0.000014409$$

11) $P(F) = 0.5 = \frac{1}{2}$

$$P(\text{3 next boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0.125$$

13) $f_X(x) = \frac{1}{6-a} = \frac{1}{120-75} = \frac{1}{45}$

$$P(80 < x < 100) = \int_{80}^{100} \frac{1}{45}\, dx = \frac{1}{45}[x]_{80}^{100} = \frac{1}{45}(100-80)$$
$$= \frac{20}{45} = \frac{4}{9}$$

14) ~~$X \sim$ Gamma~~ $X \sim \text{Gamma}(5, 0.4)$ $\quad \alpha = 5$

$2\lambda X \sim \chi^2_{10}$ $\quad \lambda = 0.4$ $\quad \therefore P(X < 22.87)$

$$P(X < 22.8) = P(2\lambda X < 2\lambda(22.87))$$
$$= P(\chi^2_{10} < 18.322)$$

$P(X < 18) = 0.9450$ $\quad P(X < 18.5) = 0.9529$

0.5 : 0.0079 $\quad$ 1 $\quad$ 0.0158 $\quad$ 0.32 $\quad$ 0.002085

$P(\chi^2_{10} < 18.132) = 0.94770856$