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PROBABILITY AND STATISTICS
ASSIGNMENT - 1

Q1. Mean, median, mode, S.D and IQR

(i) $x_i = 4, 7, 9, 9, 11, 15, 18, 18, 18, 25$

$n = 10$

$$\text{Mean } (\bar{x}) = \frac{\sum x_i}{n}$$

$$= \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \frac{134}{10}$$

$$= 13.4$$

$$\boxed{\bar{x} = 13.4}$$

Median $= \left(\frac{n+1}{2}\right)^{th}$ size of the ~~term~~ class

$= \left(\frac{10+1}{2}\right)^{th}$ size of the ~~term~~ class

$= \frac{11}{2} = 5.5^{th}$ term

Average of 5 and 6^{th} term

$$\frac{11+15}{2} = \frac{26}{2} = 13$$

$$\boxed{\text{Median} = 13}$$

Mode : Most recurring value (frequent value)

$= 18.$

$$\boxed{\text{Mode} = 18}$$

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$\bar{x} = 13.4$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
4	-9.4	88.36
7	-6.4	40.96
9	-4.4	19.36
9	-4.4	19.36
11	-2.4	5.76
15	1.6	2.56
18	4.6	21.16
18	4.6	21.16
18	4.6	21.16
25	11.6	134.56

$\Sigma x_i = 134$ $\qquad \Sigma(x_i - \bar{x})^2 = 374.4$

$$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{\Sigma x_i}}$$

$$= \cancel{\sqrt{\frac{374.4}{134}}} \qquad = \sqrt{\frac{374.4}{10}}$$

$$= \cancel{\sqrt{2.794029851}} \qquad = \sqrt{37.44}$$

$$= \cancel{1.671535178} \qquad = \underline{\underline{6.11882342}}$$

$$\sigma = \underline{\underline{6.11882342}}$$

4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

Q_1 = size of $\left(\frac{N}{4}\right)^{th}$ term

$= \left(\frac{10}{4}\right)^{th}$ term $= 2.5^{th}$ term

$= \frac{7+9}{2} = 8$

Q_3 = size of $3\left(\frac{N}{4}\right)^{th}$ term

$= 7.5^{th}$ term $= \frac{18+18}{2} = 18$

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Inter Quartile Range $= Q_3 - Q_1$
$= 18 - 8$
$= \underline{\underline{10}}$

(ii)

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$	c.f
0	5	0	−2	4	20	5
1	11	11	−1	1	11	16
2	26	52	0	0	0	42
3	15	45	1	1	15	57
4	3	12	2	4	12	60
	60	120		10	58	

Mean $(\bar{x}) = \dfrac{\Sigma x_i f_i}{\Sigma f_i}$

$= \dfrac{12\cancel{0}}{6\cancel{0}} = 2$

$\underline{\underline{\bar{x} = 2}}$

Size of $\left(\dfrac{N+1}{2}\right)^{th}$ term

$= 61/2 = 30.5^{th}$ term

30.5^{th} term is when $x_i = 2$

$\underline{\underline{\text{Median} = 2}}$

Mode : highest frequency

$\underline{\underline{\text{Mode} = 2}}$

$\sigma = \sqrt{\dfrac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}$

$= \sqrt{\dfrac{58}{60}} = \underline{\underline{0.98319208}}$

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Q_1 = size of $\left(\frac{N}{4}\right)^{th}$ term

= 15^{th} term

= 1

Q_3 = size of $3\left(\frac{N}{4}\right)^{th}$ term

= 45^{th} term

= 3

Inter Quartile Range = $Q_3 - Q_1$

= 3 - 1

= 2

Q2. $\lambda = 0.5$

(i) P (no claims arise in a single year)

= $P(X=0)$

$$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{(0.5)^0 e^{-0.5}}{0!}$$

$$= e^{-0.5} \times 1$$

$$= 0.60653066$$

(ii) Using binomial distribution

$${}^nC_r p^r q^{n-r}$$

$${}^nC_r = {}^3C_2$$

$$= \frac{3!}{2!\,1!}$$

$$= \frac{3 \times \cancel{2!}}{\cancel{2!}} = 3$$

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(ii) $q^{n-x} = 0.606530659$

$p^x = 1 - 0.606530659$

$= 0.393469341$

$^nC_x \, p^x q^{n-x} = 3(0.606530659)^2(0.393469341)$

$= 0.434247843$

(iii) Doubt

Q3. $\alpha = 2$ and $\lambda = 1/2$ $\quad 0 < x < \infty$

(i) Mean and standard deviation

$$E(X) = \frac{\alpha}{\lambda}$$

$$= \frac{2}{1/2} = 4$$

$$\sqrt{\sigma(x)} = \sqrt{\frac{\alpha}{\lambda^2}}$$

$$= \sqrt{\frac{2}{1/4}} = \sqrt{8}$$

$$= 2.828427124$$

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(ii) Co-efficient of skewness = $\dfrac{\alpha}{\sqrt{\cancel{2}\alpha}}$

$= \dfrac{2}{\sqrt{2}}$

$= \sqrt{2}$

$= 1.414213562$

Thus the graph will be positively skewed.

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~~Q5.~~

Q4. 10 male 8 female
6 qual. act. 7 qual. act.

P(A) = P(choosing 2 members) = $^{18}C_2$
P(F) = P(atleast one female) = 1 - P(both are males)
P(M) = P(Both are males) = $\frac{^{10}C_2}{^{18}C_2}$

$= \frac{10!}{2!\,8!} \times \frac{2!\,16!}{18!}$

$= \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{2!}\,\cancel{8!}} \times \frac{\cancel{2!}\,\cancel{16!}}{\cancel{18} \times 17 \times \cancel{16!}}$

$= \frac{5}{17}$

$P(F) = 1 - P(M)$

$= 1 - \frac{5}{17}$

$= \frac{12}{17} = 0.70588 2353$

P(E) = P(2 qualified females) = $\frac{^{7}C_2}{^{18}C_2}$

$= \frac{7!}{\cancel{2!}\,5!} \times \frac{\cancel{2!}\,16!}{18!}$

$= \frac{7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!}} \times \frac{\cancel{16!}}{{}_3\cancel{18} \times 17 \times \cancel{16!}}$

$= \frac{7}{51}$

$= 0.137254902$

$P(F \cap E) = 0.137254902$

$P(E|F) = \frac{P(F \cap E)}{P(F)} = \frac{0.137254902}{0.705882353}$

$= \underline{0.19444\ldots}$

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Q5. $P(L_1) = P(\text{Delivering via level 1}) = 40\%$

$P(L_2) = 50\%$ $\quad P(E|L_1) = 20\%$

$P(L_3) = 10\%$ $\quad P(E|L_2) = 40\%$

$P(E|L_3) = 10\%$

$$P(L_2|E) = \frac{P(L_2) \times P(E|L_2)}{P(L_1)P(E|L_1) + P(L_2)P(E|L_2) + P(L_3)P(E|L_3)}$$

$$= \frac{0.5 \times 0.4}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29}$$

$$= 0.689655172$$

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$F_x\ g^{-}(y) = 1$

Q7.

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3)$

$$F(3) = P(X \leq 3)$$
$$= 1 + 2 + 3$$
$$= 0.05 + 0.15 + 0.35$$
$$= \underline{\underline{0.55}}$$

(ii) $E(X)$

$$E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 + x_5 p_5$$
$$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$$
$$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$$
$$= \underline{\underline{3.25}}$$

(iii) $V(X)$

$$V(X) = E(X^2) - [E(X)]^2$$
$$= [(1)^2(0.05) + (2)^2(0.15) + 3^2(0.35) + 4^2(0.4) + 5^2(0.05)] - (3.25)^2 - (10.5625)$$
$$= (0.05 + 0.6 + 3.15 + 6.4 + 1.25) -$$
$$= \underline{\underline{0.8875}}$$

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Q8. Let claim amount exceeding 1000 be $P(E) = 0.2$

claim amt not exceeding 1000 $P(F) = 0.8$

Using binomial situation

$$P(X<3) = P(X \leq 2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14}$$

$$+ {}^{15}C_2 (0.2)^2 (0.8)^{13}$$

$$= 0.035184372 + 0.131941395$$

$$+ 0.230897442$$

$$= \underline{0.398023209}$$

Q9. Let sucess be defined as winning the price

$P(S) = 0.01$

$P(F) = 0.99$

Using binomial distribution.

$$P(X=3) = {}^7C_3 (0.01)^3 \cdot (0.99)^4$$

$$= \frac{7!}{3!\,4!} (0.000001) \cdot (0.96059601)$$

$$= \frac{\cancel{8} \times 7 \times 5 \times \cancel{4}}{\cancel{3}\ \cancel{4}}$$

$$= \underline{0.000033621}$$

Q10. will follow poisson distribution

$$\lambda = \frac{2}{5} = 0.4$$

$$P(X>2) = 1 - P(X<2)$$

$$P(X=0) = \frac{\lambda^x - e^{-\lambda}}{x!}$$

$$= \frac{(0.4)^0 e^{-0.4}}{0!} = e^{-0.4}$$

$$= \underline{0.670320046}$$

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$P(X=1) = \frac{(0.4)^1 e^{-0.4}}{1!}$

$= 0.268128018$

$P(X=2) = \frac{(0.4)^2 e^{-0.4}}{2!}$

$= \frac{\cancel{0.16}^{0.08} e^{-0.4}}{\cancel{2}} = 0.053625604$

$P(X \leq 2) = 0.992073668$

$P(X > 2) = 1 - P(X \leq 2)$

$= 0.007926332$

Q 11. 5 boys and 2 two girls

P(of a boy) = P(of a girl)

$P(B) + P(G) = 1$

$2P(B) = 1$

$P(B) = \frac{1}{2}$

Similarly $P(G) = \frac{1}{2}$

Let A be the event of having 3 boys

$P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$P(A) = \frac{1}{8}$

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Q12. Exponential distribution

$\lambda = 10 \quad \alpha = 1$

$t = 0.1$ days

$P(T < 0.1) = F(0.1)$

$= 1 - e^{-\lambda t}$

$= 1 - e^{-1}$

$= 0.632120559$

Q13. $X \sim U(75, 120)$

$F_X(x) = \frac{1}{120-75} = \frac{1}{45}$

$P(80 < X < 100) = \frac{100-80}{45}$

$= \frac{20}{45}$

$= 0.444444$

Q14. $X \sim \text{gamma}(5, 0.4)$ $\qquad P(X < 22.89)$

$\alpha = 5 \quad \lambda = 0.4$

$2\lambda = 0.8$ $\qquad P(0.8X < 18.312)$

$0.8X \sim \chi^2_{2\times 5 \; \cancel{0.05}}$

$P(\chi^2_{10} < 18.0) \rightarrow 0.9450$

$P(\chi^2_{10} < 18.5) \rightarrow 0.9529$

0.0079

$P(\chi^2_{10} < 18.312)$?

$0.5 \rightarrow 0.0079$

$1 \rightarrow 0.079/0.5$

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$0.312 \rightarrow \frac{0.0079}{0.5} \times 0.312$

$= 0.0049296.$

$P\left(\chi^2_{10} < 18.312\right) = P\left(\chi^2_{10} < 18.0 + 0.0049296\right)$

$= 0.9450 + 0.0049296$

$= \underline{\underline{0.9499296}}$

Q15. $F_X(x) = \frac{K}{(x+5)^4}, \quad x > 0$

(i) $K = ?$

$$\int_0^\infty F_X(x) = 1$$

$$\Rightarrow K \int_0^\infty \frac{1}{(x+5)^4} dx = 1$$

Let $x+5 = t$

$dx = dt$

$$\Rightarrow K \int_5^\infty t^{-4} dt = 1$$

$$\Rightarrow K \left[\frac{t^{-3}}{-3}\right]_5^\infty = 1$$

$$\Rightarrow K \left[-\frac{1}{3t^3}\right]_5^\infty = 1$$

$$\Rightarrow K \left[0 - \left(-\frac{1}{3(125)}\right)\right] = 1$$

$$\frac{K}{375} = 1$$

$$\boxed{K = 375}$$

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192%

(ii) $F(10)$

$$F(10) = P(x < 10)$$

$$= \int_0^{10} \frac{k}{(x+5)^4}$$

$$= 375 \int_0^{10} \frac{1}{(x+5)^4}$$

Let $x+5 = t$

$dx = dt$

$$= 375 \int_{5}^{15} t^{-4} \, dt$$

$$= 375 \left[\frac{-1}{3t^3} \right]_5^{15}$$

$$= 375 \left(-\frac{1}{10125} + \frac{1}{375} \right)$$

$$= 375 \left(\frac{-1 + 27}{10125} \right)$$

$$= \cancel{375} \times \frac{26}{\cancel{10125}\ 27}$$

$$= \underline{\underline{0.962962963}}$$

(iii) $E(x)$

$$E(x) = \int_0^{\infty} x \cdot F_x(x) \, dx$$

$$= 375 \int_0^{\infty} \frac{x}{(x+5)^4} \, dx$$

Let $x+5 = t$

$dx = dt$

$$= 375 \int_5^{\infty} \frac{t-5}{t^4} \, dt$$

$$\cancel{= 375 \left[\int_5^{\infty} \frac{1}{t^3} \, dt - \int_5^{\infty} \frac{5}{t^4} \, dt \right]}$$

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$$= 375 \int_5^{\infty} \left(\frac{1}{t^3} - \frac{5}{t^4} \right) dt$$

$$= 375 \left[-\frac{1}{2t^2} + \frac{5}{3t^3} \right]_5^{\infty}$$

$$= 375 \left[\frac{5}{t^3} - \frac{1}{2t^2} \right]_5^{\infty}$$

$$= 375 \left[0 - 0 - \frac{5}{375} + \frac{1}{50} \right]$$

$$= 375 \left[\frac{1}{50} - \frac{5}{375} \right]$$

$$= \frac{375}{50} - 5$$

$$= \frac{125}{50}$$

$$= 2.5$$

$$\underline{\underline{E(x) = 2.5}}$$