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NUMERICAL METHODS & ALGEBRA

ASSIGNMENT 1

Q1. To find : $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

let $x = \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$ — ①

squaring both the sides

$x^2 = 7+\sqrt{7+\sqrt{7+\ldots}}$ — ②

substituting ① and ②:

$x^2 = 7+x$

$x^2 - x - 7 = 0$

$\therefore x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-1)+\sqrt{(1)^2-4(1)(-7)}}{2(1)}, \dfrac{-(-1)-\sqrt{(-1)^2-4(1)(-7)}}{2(1)}$

$= 3.192582404, -2.192582404$

$\therefore$ Since we know that x has to be positive (greater than 0), the value $x = -2.1925824$ is invalid

$\therefore x = 3.192582404$

$\therefore \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}} = 3.192582404$

Q2. $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$7x^2 + 11x^2 = 6y^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$9x^2 = y^2$$

$$\frac{x^2}{y^2} = \frac{1}{9}$$

$$\frac{x}{y} = \frac{1}{3}$$

Q3. $x + 3y = 4, \; x^2 + y^2 - 4y = 12$

$$x + 3y = 4$$

$$x = 4 - 3y \longrightarrow (1)$$

substituting (1) in the equation:

$$x^2 + y^2 - 4y = 12$$

$$(4-3y)^2 + y^2 - 4y = 12$$

$$16 + 9y^2 - 24y + y^2 - 4y = 12$$

$$10y^2 - 28y + 4 = 0$$

$$5y^2 - 14y + 2 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(5)(+2)}}{2(5)}$$

$$= \frac{14 \pm \sqrt{156}}{10} = \frac{14 \pm 2\sqrt{39}}{10}$$

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$$\cancel{y = \frac{14 + 15.36229}{10}, \quad \frac{14 - 15.36229}{10}}$$

$$\cancel{= 2.936229 \quad \text{or} \quad -0.136229}$$

$$y = \frac{\cancel{2}(7 \pm \sqrt{39})}{\cancel{2}(5)}$$

$$= \frac{7 + \sqrt{39}}{5} \quad \text{or} \quad \frac{7 - \sqrt{39}}{5}$$

$$= 2.6489996 \quad \text{or} \quad 0.1510004$$

when $y = 2.6489996$, $x = -3.9469988$

when $y = 0.1510004$, $x = 3.5469988$

$\therefore$ The ordered pairs that satisfy the system are $(-3.9469988, 2.6489996)$ and $(3.5469988, 0.1510004)$

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Q4. $d = 2r^2 - 16r + 34$

$d \rightarrow$ density of smoke

$r \rightarrow$ speed of the engine

a. when $r = 3.5$:

$d = 2(3.5)^2 - 16(3.5) + 34$

$= 2.5$ million particles per cubic foot

b. For minimum smoke, $\frac{d}{dr}$ should be greater than 0

$\frac{d}{dr} = 4r - 16 > 0 \longrightarrow 4r > 16$

$\therefore r > 4$

$\therefore$ The no. of revolutions for minimum smoke $= 4$ per minute.

$\therefore$ Minimum output $= 2(4)^2 - 16(4) + 34$

$= 2$.

c. when $d = 100$:

$100 = 2r^2 - 16r + 34$

$2r^2 - 16r - 66 = 0$.

$r^2 - 8r - 33 = 0$.

$r^2 - 11r + 3r - 33 = 0$.

$r(r-11) + 3(r-11) = 0$

$(r+3)(r-11) = 0$.

$r = 11$ or $r = -3$ (Invalid)

$\therefore$ Speed of the engine is 100 revolutions per minute.

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Q5. Company 1: 1500 per day plus 25 per km
Company 2: 2100 per day plus 22 per km

Let n be the total no. of km driven in a day

a. Total Cost of renting a car for 1 day from company 1 = $1500 + 25n$

b. Total Cost of renting a car for 1 day from company 2 = $2100 + 22n$

c. For it to be less experience to rent from company 2: Cost of Company 1 > Cost of company 2.
$1500 + 25n > 2100 + 22n$.

d. $1500 + 25n > 2100 + 22n$
$3n > 600$
$n > 200$

∴ It is less experience to rent from company 2 when the no. of km driven in a day is greater than 200.

Q6. $f(x) = x^3 - 7x^2 + 14x - 6 = 0$.

a. Considering $a = 0$, $b = 1$:

$f(a) = f(0) = -6$

$f(b) = f(1) = 2$

Since $f(a)$ & $f(b)$ have opposite signs, this interval is appropriate.

a	b	$f(a)$	$f(b)$	$(a+b/2)$	$f[(a+b/2)]$
0	1	−6	2	0.5	−0.625
0.5	1	−0.625	2	0.75	0.984375
0.5	0.75	−0.625	0.984375	0.625	0.2597656
0.5	0.2597656	−0.625	0.2597656	0.5625	−0.16186523
0.5625	0.625	−0.161865	0.2597656	0.59375	0.05404663
0.5625	0.59375	−0.161865	0.0540466	0.578125	−0.05262375
0.578125	0.59375	−0.0526238	0.0540466	0.5859375	0.0010313987
0.578125	0.5859375	−0.526238	0.0010313987	0.58203125	−0.025716007
0.58203125	0.5859375	−0.025716007	0.0010313987	0.58398437	

$x = 0.58$

Q. Considering $a = 1$, $b = 3.2$

$f(a) = f(1) = 2$

$f(b) = f(3.2) = -0.112$

Since $f(a)$ & $f(b)$ have opposite signs, this interval is appropriate.

a	b	f(a)	f(b)	(a+b/2)	f[(a+b/2)]
1	3.2	~~f(a)~~ 2	-0.112	2.1	1.791
2.1	3.2	1.791	-0.112	2.65	0.552125
2.65	3.2	0.552125	-0.112	2.925	0.0858283
2.925	3.2	0.08582813 ~~1.791~~	-0.112	3.0625	-0.05444339
2.925	3.0625	0.08582813 ~~0.552125~~	-0.05444336	2.99375	0.00632781
2.99375	3.0625	0.006327881 ~~0.08582813~~	-0.05444336	3.028125	-0.02652072
2.99375	3.028125	0.006327881 ~~0.08582813~~	-0.02652072	3.0109375	-0.01069693
2.99375	3.0109375	0.006327881	-0.010696934	3.00234375	-0.00233275
2.99375	3.00234375	0.006327881	-0.00233275	2.9991211	0.001960747
2.9991211	3.00234375	0.001960747	-0.00233275	3.0001953	-1.95236198

$x = 3.00$

c. Considering $a = 3.2$, $b = 4$

$f(a) = f(3.2) = 0.112$

$f(b) = f(4) = 2$

Since $f(a)$ and $f(b)$ have opposite signs, this interval is appropriate.

a	b	f(a)	f(b)	(a+b/2)	f[(a+b/2)]
3.2	4	-0.112	2	3.6	0.336
3.2	3.6	-0.112	0.336	3.4	-0.016
3.4	3.6	-0.016	0.336	3.5	0.125
3.4	3.5	-0.016	0.125	3.45	0.046125
3.4	3.45	-0.016	0.046125	3.425	0.01301563
3.4	3.425	-0.016	0.01301563	3.4125	-0.00199805
3.4125	3.425	-0.00199805	0.01301563	3.41875	0.005381592
3.4125	3.41875	-0.00199805	0.005381592	3.415625	0.001660065

$x = 3.42$

Q7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

Since $f(x)$ has a real zero in $[-1, 0]$, we consider the midpoint -0.5 as x_0

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -0.5 - \frac{f(-0.5)}{f'(-0.5)} = -0.1504524887$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.041816814$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.0406593435$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.04065928832$

Since $f(x)$ has a real zero in $[0, 1]$, we consider the midpoint 0.5 as x_0

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = -0.7050898204$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.3237911142$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.06460313103$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.04068615715$

$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = -0.04065928835$

Q 8. Grades of four exams : 70, 86, 81, 83
Grade on the final exam : x.

a. Converse average $= \frac{70+86+81+83+x}{5} = \frac{320+x}{5}$

b. Since you score a B if the average is greater than or equal to 80 and less than 90:

$$80 \leq \frac{320+x}{5} < 90$$

c. $$80 \leq \frac{320+x}{5} < 90$$

$$400 \leq 320+x < 450$$

$$80 \leq x < 130$$

If we assume that all markes are out of 100:
$80 \leq x < 130$.

~~If we assume~~

∴ The grade on final exam should be greater than or equal to 80 and less than or equal to 100 to obtain a B.

Q9. $x_1 = 8.54$

$x_2 = -11.81$

$x = 0$

$y_1 = 4\% = 0.04$

$y_2 = 20\% = 0.20$

using linear interpolation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$(y - 0.04) = \frac{(0.20 - 0.04)}{(-11.81 - 8.54)}(0 - 8.54)$$

$$y - 0.04 = 0.06714496314.$$

$$y = 0.10714496314$$

$$= 10.714496314\%.$$

$IRR = 10.714496314\%.$

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Q10. Revenue $= R(x) = 32x - 0.21x^2$
Cost $= C(x) = 195 + 12x$.

To remain profitable, the revenues must exceed the cost:

$$R(x) > C(x)$$
$$32x - 0.21x^2 > 195 + 12x.$$
$$20x - 0.21x^2 - 195 > 0.$$
$$0.21x^2 - 20x + 195 < 0.$$

Equating the inequality to 0:

$$0.21x^2 - 20x + 195 = 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(0.21)(195}}{2(0.21)}$$

$$= 84.21142753 \text{ or } 11.02666771.$$

Since $x < 11.02666771$ and $x > 84.211428$ do not satisfy the original inequality:

$11.02666771 < x < 84.2114275 \rightarrow$ satisfies the original inequality.
$$12 \leq x \leq 84.$$

The no. of orders needed for the app to remain profitable should be greater than 12 and less than 84, with the minimum number being 12 orders.