Financial Mathematics
Assignment - 2

Q1. Loan amount = 20000
Repayment = 427.9 /- month
Term = 5

$$\Rightarrow 20000 = (427.9 \times 12)\, a^{(12)}_{\overline{5}|}$$

$$a^{(12)}_{\overline{5}|} = 3.8949$$

Using Trial & Error

At $i = 10\%$; $a^{(12)}_{\overline{5}|} = 3.9615$

$i = 11\%$; $a^{(12)}_{\overline{5}|} = 3.8787$

Interpolating : $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$$\Rightarrow \frac{y - 3.8787}{3.9615 - 3.8787} = \frac{x - 11\%}{10\% - 11\%}$$

$$\Rightarrow \frac{3.8949 - 3.8787}{0.0828} = \frac{x - 11}{-1}$$

$$\Rightarrow -0.0162 = 0.0828x - 0.9108$$

$$\Rightarrow x = 10.8\%$$

APR = 10%

(ii) $i = 10.8\%$

New repayment = 274.49

$$\Rightarrow 20000 = 274.49 \times 12 \times a^{(12)}_{\overline{n}|}$$

$$6.07186 = \frac{1 - v^n}{i^{(12)}}$$

$$6.07186\left[12\left(1.108^{(-12)} - 1\right)\right] = 1 - (1.108)^{-n}$$

$$0.3746 = (1.108)^{-n}$$

$$n \approx 9.$$

ii. New repayment amt = 274.49

Capital o/s after 1 year = $12 \times 427.90 \times a_{\overline{17}|}^{(12)}$ @10.8% on original loan

$= 16775.977$

$\Rightarrow 16775.977 = 12 \times 274.41 \times a_{\overline{n}|}^{(12)}$ @10.8%

$\Rightarrow 0.52456 = 1 - (1.108)^{-n}$

$\Rightarrow 0.47543 = (1.108)^{-n}$

$\Rightarrow n = 7.25$ yrs

iii. Total interest remaining on original loan $= (12 \times 427.90 \times 4) - 16775.977$

$= 3763.223$

Interest paid in 7.25 yrs $= (12 \times 274.49 \times 7.25) - 16775.977$

$= 7104.653$

Extra interest paid $= 7104.653 - 3763.223$

$= 23341.43$

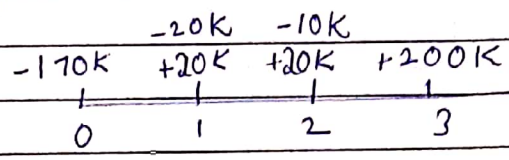
2. i (a) Discounted payback period: It is the number of years that a project take before the cumulative forcasted cashflow equals the initial investment while ~~counts~~ taking into account the time value of money.

(b) Payback period: It the number of years before which the initial investment equals the cash inflows without taking into account the time value of money.

ii Discounted payback period & payback period ignores all cashflows after the said period, while NPV can be problematic in the sense that the ~~assumed~~ estimated discount rate could be wrong, and thereby the calculations surrounding it can be

off.
But NPV is still preferred as it considers all cashflows ~~that~~ which is important to track how profitable a project may or may not be

iii Project A:



→ for IRR ; NPV = 0

⇒ NPV = $-170 + 10v^2 + 200v^3 = 0$

Using trial & error method:

At 7% ; NPV = 1.9939

At 8% : NPV = -2.66

At 7.4% : NPV = 0.1116

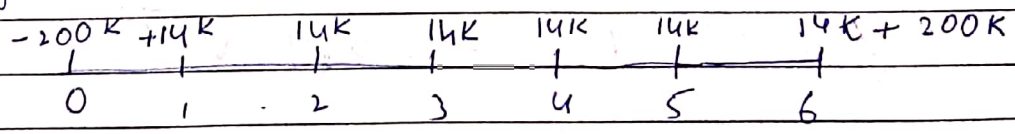
At 7.5% : NPV = -0.354

Interpolating: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

⟹ $i = 7.424\%$

Ans → $IRR_A = \underline{7.4\%}$

Project B:



$NPV = -200 + 14a_{\overline{6}|i} + 200v^6$

Ans $IRR_B = \underline{7\%}$

b. $i = 6\%$

$NPV_A = -170 + 10v^2 + 200v^3$

$= 6823.821$

$NPV_B = -200 + 14 \times a_{\overline{6}|} + 200v^6$

$= 9834.64$

c. IRR of project A & B are 7.4% & 7% respectively. Any interest charged by the bank above these rates would make the respective projects unprofitable as $NPV < 0$ in that case.

Thus maximum rates that a bank could charge ~~are 7.4% or 7% Anything above~~ are 7.4% & 7% respectively above which the projects become unprofitable.

The investor could also look into the discount payback period of the project to analyse which project is able to recover the initial outlay first.

3. Present value of on 1 Nov 1985

Annuity 1 $= 200\,\ddot{a}_{\overline{22}|}\, v^{\left(\frac{1}{4}\right)}$ @ $i = 8\%$

$= 2161.37$

Annuity 2 $= 320 \times a^{(4)}_{\overline{16.25}|} \times (1+i)^{(1 \div 12)}$

$= 2957.87$

Annuity 3 $= 180 \times a^{(12)}_{\overline{18.75}|}$

$= 1780.665$

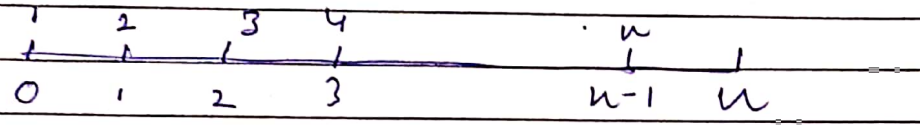
Total value = 2161.36 + 2957.87 + 1780.665
= 6899.9

Let revised annuity amount be x

$\Rightarrow 6899.9 = x\, a^{(2)}_{\overline{2.5}|} (1+i)^{\frac{1}{4}}$

Ans $\Rightarrow$ $x = 656.56$

4. For $(I\ddot{a})_{\overline{n}|}$



$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + 4v^3 + \ldots nv^{n-1}$

$\therefore (1+i)[Ia][v + 2v^2 + 3v^3 + \ldots nv^n]$

$= (1+i)(I\ddot{a})_{\overline{n}|}$

$= \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{\left(\frac{i}{1+i}\right)}$

$$\boxed{(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}}$$

5. (i) TWRR:

For 'property fund'

$(1+i) = \dfrac{16.4}{12.4}$

Ans $i = 32.26\%$

For equity fund:

$(1+i) = \dfrac{15.5}{12.1}$

$i \approx 28.099\%$

ii. (a) For property fund:
Same no. of units bought:

$$\rightarrow 12.4(1+i) + 13.1(1+i)^{\frac{3}{4}} + 14.8(1+i)^{\frac{1}{2}} + 15.8(1+i)^{\frac{1}{4}} = 4 \times 16.4$$

$\rightarrow i \approx 29.316\%$
$i \approx 29.32\%$

(b) Same amount invested

iii (a) For equity fund
Same no. of units bought:

$$\rightarrow 12.1(1+i) + 9.2(1+i)^{\frac{3}{4}} + 10.3(1+i)^{\frac{1}{2}} + 13.1(1+i)^{\frac{1}{4}} = 4 \times 15.5$$

$i = 67.31\%$

(b)

6. Loan = £160,000
Term = 10 yrs
i = 8% p.a.

i Let annual repayment = x

$\Rightarrow 160000 = x\, a_{\overline{10}|} @ 8\%$

$x = \frac{160000}{6.7101}$

Ans $x = £23844.65$

ii Capital o/s after 4 payments

$= 23844.65 \times a_{\overline{6}|}$

$= 110231.4325$

Let revised annual repayment be y

$\rightarrow 110231.4325 = y\, a_{\overline{6}|} @ 10\%$

$y = \frac{110231.4325}{4.3553}$

Ans $y = 25309.722$

iii Capital o/s after 7th repayment

$= 25309.722 \times a_{\overline{3}|}$

$= 62942.75$

Let revised annual repayment be z

$\rightarrow 62942.75 = z\, a_{\overline{3}|} @ 9\%$

$z = 24865.77$

Effective rate of interest :

$(1+i)^{10} = (1.08)^4 (1.1)^3 (1.09)^3$

$(1+i)^{10} = 2.34505$

Ans $i = 8.8968\%$

7. (i) Reasons Govt securities preferred over investing in property :

→ Govt securities are the most secure forms of investments

→ Property returns are lower in short term
→ Expensive to purchase property
→ may find it difficult to rent it out

ii. a) Governments ~~raise~~ issue loans by tender by inviting investors to submit their tender indicating of the amount they are willing to pay. The loan is issued to the highest bidder.

b) Advantage : Govt will know the issue price at outset & are hence able to predict their cost of capital.

Disadvantage : In tender the investor may be willing to pay more than the set price.

c. 3 securities are :

(i) Index linked bond - The returns can change with the ~~ver~~ changing inflation in the market.

(ii) Property - rent payments may vary.

(iii) Shares - Dividend income isn't known.

8. (i) Present value of investment:

$= 2.7 + 0.2a_{\overline{3}|}$ @ i

Expected PV of inflow:

$= 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3)$ @ i

→ Acc to question

$2.7 + 0.2a_{\overline{3}|} = 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3)$

$0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) - 0.2a_{\overline{3}|} = 2.7$

Using trial & error:

At $i = 9\%$, NPV $\Rightarrow 2.75396$

At $i = 9.2\%$, $\Rightarrow 2.7207$

$i = 9.3\%$, $\Rightarrow 2.704$

Ans → $i = 9.3\%$

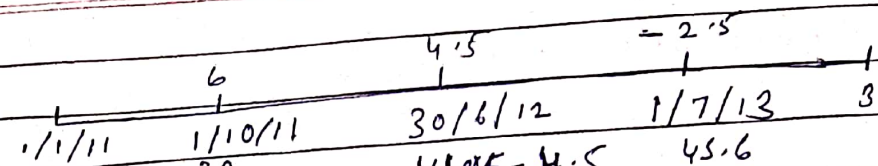
(ii) $NPV = 0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3 - 2.7 - 0.2a_{\overline{3}|}$ @ $i = 10\%$

$= 0.1(0.7513) + 0.85(0.7513)(6.4469) - 2.7 - 0.2(2.4868)$

$= 1.0089$

Since NPV is positive, the project is profitable.

Q9. (i)



TWRR :

$$(1+i)^3 = \frac{33}{30.5} \times \frac{45.6}{39} \times \frac{47}{50.1}$$

$$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05-4.5}{33+6} \times \frac{45.6}{(41.05-4.5+4.5)} \times \frac{47}{(45.6-2.5)}$$

$$= \frac{33}{30.5} \times \frac{36.55}{39} \times \frac{45.6}{41.05} \times \frac{47}{43.1}$$

$$i = 7.09512\%$$

(ii) Linked rate of return:

For year 2011

$$30.5(1+i) + 6(1+i)^{\frac{3}{12}} = 38.5$$

$$i = 6.2566\%$$

For year 2012

$$38.5(1+i) + 4.5(1+i)^{\frac{6}{12}} = 45$$

$$i = 4.911\%$$

For year 2013

$$45(1+i) - 2.5(1+i)^{\frac{6}{12}} = 47$$

$$i = 10.2785\%$$

Linked ROR :

$$(1+i)^3 = (1.062566)(1.04911)(1.102785)$$

Ans $i = 7.12461\%$

(ii) MWRR takes into account the amount & timing of a cashflow ; both these factors are not in the control of a manager incharge of a particular fund.

TWRR does not take into account these factors & hence eliminates their effect.

(i) (contd)

* MWRR

$$\Rightarrow 30.5(1+i) + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$$

Using trial & error

At 10% , = -3.445

At 16% = -0.311

At 17% = 0.2179

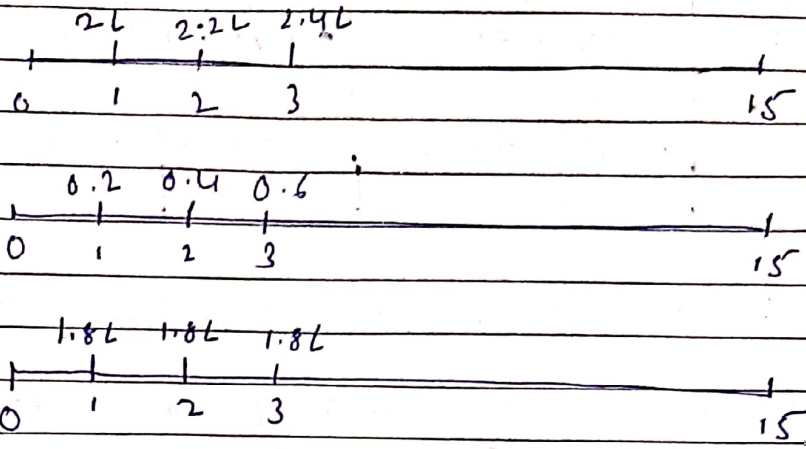
At 16.5% = -0.047

At 16.6% = 0.005876

Interpolating : $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$i = 16.589\%$

Q10. (i)



Let loan amount be x

$$\Rightarrow x = 1.8\, a_{\overline{15}|} + 0.2\, Ia_{\overline{15}|} \text{ @ } i = 12\%$$

$$= 1.8 \times 6.8109 + 0.2 \times 40.7310$$

$$x = 20.40582$$

Ans $x = ₹\ 20,40,582$

~~Cost of house = 0 no+ 80% of~~
~~₹ 204086.98.04~~

Let Cost of House $= y$

$$\Rightarrow 80\% \text{ of } y = x$$

$$\Rightarrow y = \frac{x}{0.8}$$

Ans $y = ₹\ 2550.727.5$

ii) Loan o/s after 8th repayment $= [1.8 + (0.2 \times 8)]\, a_{\overline{7}|} + 0.2\, Ia_{\overline{7}|}$

$$= (3.4 \times 4.5638) + (0.2 \times 16.2080)$$

$$= 18.75852$$

Year	Loan o/s at start	Interest	Install.	Cap. Rep.	Loan o/s at end
9	1875852	225102.24	360000	134897.76	1740954.24
10	1740954.27	208914.51	380000	171085.49	1569868.78

Let revised annual payment = x

$\Rightarrow$ $1569868.78 = x\, a_{\overline{5}|}$ @ $i = 10\%$.

$\Rightarrow$ ~~₹24,14,[illegible]~~

Ans $x = ₹\ 4,14,125.984$

Earlier total interest paid after 10 installments:

= (Total inst − loan o/s after 10 installments

$= \frac{5}{2}\left[800000 + (4)(20000)\right] - 1569868.78$

$= ₹\ 630131.22$

New Bank loan:

Total interest $= (414125.984 \times 5) - 1569868.78$

$= ₹\ 500761.14$

Since Bank 2 is charging lesser interest for the remaining term of the loan, he should transfer the same to Bank 2

ii. i TWRR = 20% = i

$$\rightarrow (1+i)^2 = \frac{500}{460} \times \frac{(550+50)}{500+40} \times \frac{X}{(550+50-50)}$$

$$(1.2)^2 = \frac{5}{2277} X$$

$$X = 655.776$$

iii. MWRR :

$$460(1+i)^2 + 40(1+i)^{1.75} - 50(1+i) = 655.776$$

Using trial & error

At i = 10%

i = 19% , -9.6365

i = 20% , = 1.6575

At i = 19.8% ; = -0.609

i = 19.9% , = 0.5237

Interpolating: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

Ans $i = 19.854\%$

iii. Linked ROR :

For year 2015

$$\rightarrow 460(1+i) + 40(1+i)^{\frac{9}{12}} - 50 = 550$$

$$i = 14.4644\%$$

For year 2016

$$550(1+i) = 655.776$$

$i = 19.232\%$

$\Rightarrow (1+i)^2 = (1.144644)(1.19232)$

Ans $i = 16.82388\%$

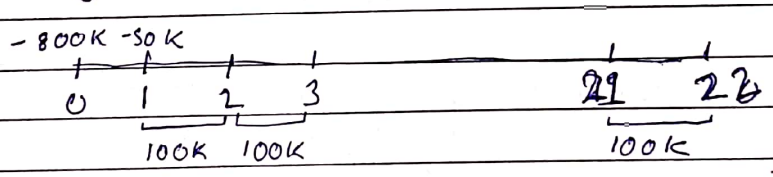
iv. The linked internal rate of return will be equal to the TWRR if the sub intervals are the same in each calculation

v. TWRR is a better measure of fund performance MWRR as the latter does not take into account the amount & timing of the cashflows.

12.

(i) Discounted payback period is the no. of years taken for the cumulative forecasted cashflow to equal the initial investment for a project, taking interest into account

(ii) a.



$i = 7\%$ ⇒ Borrowing rate

PV of investment $= 800 + 50v$

$= 846.7289$

PV of income $= 100\bar{a}_{\overline{n}|} v^2$

$\Rightarrow 846.7289 = 100\bar{a}_{\overline{n}|} v^2$

$846.7289 = 100\left[\frac{1-(1.07)^{-n}}{\ln(1.07)}\right](1.07)^{-2}$

$(1.07)^{-n} = 0.3441$

Ans DPP $= n \approx 15.8485$

b) Lending rate : $i = 6\%$.

Acc profits $= 100\, s_{\overline{7}|}$ @ 6%

Amt $= 746.79$

iii. Since income is earned at end of each year

Amt at end $= 100\, s_{\overline{7}|}$ @ 6%

$= 102.97$

$\Rightarrow 800 + 50(1+i)^{-1} = 102.97\, a_{\overline{7}|}$ @ 7%

$\Rightarrow (1.07)^{-n} = 0.42438$

$\Rightarrow DPP = 12.668$ yrs

13. Let monthly repayment $= x$

Loan amount $= 988000$

$i = 7\%$

Term $= 25$ yrs

(i) $988000 = 12x\, a^{(12)}_{\overline{25}|}$ @ 7%

$82333.33 = x\left[\dfrac{1 - 0.18424}{0.067850}\right]$

$x = 6847.99 \approx 6848$

ii. Loan o/s after 10th march 2029 ie after

$= 6848.00\, a_{\overline{136}|}$ @ $\dfrac{i^{(12)}}{12} = 0.005654$

Ans = 648573.2723

iii. Capital o/s at ~~start of~~ before 10th Oct 2026

$= 6848\, a_{\overline{16}|}$ @ $\frac{i^{(12)}}{12}$

$= 733433.0588$

Interest $= 733433.0588 \times \frac{i^{(12)}}{12}$

$= 4146.903$

Cap. repaid = Inst − Int

$= 2701.096$

iv. Loan repaid for the year 10th Apr − 10th March 2034

$= 6848\, a_{\overline{8}|} - 6848\, a_{\overline{7}|}$

$= 6848\left[a_{\overline{8}|} - a_{\overline{7}|}\right]$

Cap. repaid = 5161 9.69

Interest = ~~5161 9.69 ×~~ $\frac{i^{(12)}}{12}$

$= (12 \times 6848) - 5161 9.69$

$= 30556.31$