Assignment

1) (i) Constant force of mortality is an assumption for ages between x & $x+1$. It ~~says~~ means the force of mortality betw^n age x & $x+1$ is constant.

(ii) Under CFM

$p_x = e^{-\mu_x}$

$\therefore \mu = -\ln(p_x)$

$\mu_{55} = -\ln(1-q_{55})$

$\mu_{55} = 0.0047$

Q2] (i) Select mortalility is the mortality rate of an individual who have recently purchased life/health insurance. They tend to have lower mortality rates than people already insured

(ii) $_{20}p_{[25]:[30]} = {_{20}p_{[25]}} \times {_{20}p_{[30]}}$

$= \dfrac{l_{45}}{l_{[25]}} \times \dfrac{l_{50}}{l_{[30]}}$

$= \dfrac{9801.31}{9951.9913} \times \dfrac{9712.0728}{9923.7497}$

$= 0.9638$

Q3] UDD

$_{1.4}q_{54.5} = 1 - {_{1.4}p_{54.5}}$

$= 1 - {_{0.5}p_{54.5}} \times {_{0.9}p_{55}}$

$= 1 - [(1 - {_{0.5}q_{54.5}})(1 - {_{0.9}q_{55}})]$

$= 1 - \left[\left(1 - \dfrac{0.5 \times q_{54}}{1 - 0.5 q_{54}}\right)\left(1 - 0.9 q_{55}\right)\right]$

$= 1 - \left[\left(1 - \dfrac{0.5 \times 0.00714}{1 - 0.5 \times 0.00714}\right)\left(1 - 0.9 \times 0.00717\right)\right]$

$= 0.01073009$

Q4] $P\ddot{a}_{35:\overline{30|}} = 5000000 A^{\,1}_{35:\overline{30|}} + {}_{30}P A_{35:\overline{30|}}^{\;\;1}$

$\ddot{a}_{35:\overline{30|}} = 14.35$

$A^{\,1}_{35:\overline{30|}} = \cancel{A_{35}}\ A_{35} - v^{30}\ {}_{30}p_{35}\ A_{65}$

$= 0.094 - A_{65} \times 0.40177 = 0.03251$

$A_{35:\overline{30|}}^{\;\;1} = v^{30}\ {}_{30}p_{35}$

$= (1.06)^{-30}\left(\frac{8821.2612}{9894.4299}\right)$

$= 0.15522$

$P = 16771.983$

Q6] $P = 5{,}000{,}000\ A^{\,1}_{45:\overline{20|}}$

$A^{\,1}_{45:\overline{20|}} = A_{45:\overline{10|}} - A_{45:\overline{10|}}^{\;\;1}$

$= 0.46998 - (1.04)^{-20}\left(\frac{8821.2612}{9801.3123}\right)$

$= 0.05922$

$P = 296140.0863$

$\therefore$ Total premium $= 10000 \times P$

$= 296140.08$

(ii) $V = 5000000\ A^{\,1}_{46:\overline{19|}} - 296140.08$

$= 5000000\left[0.34599 - (1.06)^{-19}\left(\frac{8821.2612}{9786.9534}\right)\right] - 296140.08$

$= -255694.26417$

$DSAR = 5055694.26417$

$EDS = 10000\, q_{45} \times DSAR$

$= 74065920.97$

$ADS = 10 \times DSAR$

$= 50556942.6417$

$EDS - ADS = 23508978.33$

~~DSAR~~

Q7] $EPV = 100000\, A'_{40:\overline{20}|} + 200000\, A_{60} v^{20}\, {}_{20}p_{40}$

$[A_{40} - v^{20}\, {}_{20}p_{40} A_{60}]$

$v^{20}\, {}_{20}p_{40} = (1.06)^{-20} \left(\frac{9287.2164}{9856.2863}\right)$

$= 0.29380$

$A_{40} = 0.12313$

$A_{60} = 0.321692$

$EPV = 21917.97945$

$E(PV^2) = 100{,}000^2 ({}^2A'_{40:\overline{20}|}) + 200{,}000^2\, {}^2A_{60} v^{20}\, {}_{20}p_{40}$

${}^2A'_{40:\overline{20}|} = {}^2A_{40} - v^{40}\, p_{40}\, {}^2A_{60}$

$= 0.01414$

~~SD~~ $E(PV^2) = 1798358$

$SD = 36303.73211$

Q8] $A_{65} = 0.5412 \qquad A_{66} = 0.5591$

$A_{65} = q_{65} + (1 - q_{65}) A_{66} v$

$A_{65} = q_{65} (1 - A_{66} v)$

$\frac{0.5412 - 0.5591(1+0.08)^{-1}}{1 - 0.5591(1.08)^{-1}} = q_{65}$

$q_{65} = 0.048754$

$A_{[65]} = 0.75 q_{65} + \frac{(1 - 0.75 q_{65})\, 0.5591}{1.08}$

$A_{[65]} = 0.535321$

$P = 100000 \times 0.53532129$

$= 5352.129$

Q9] i) $P = 120000 \, a_{70}$

$P = 120,000(\ddot{a}_{70} - 1)$

$P = 1267440$

ii) $P(L_0 > 0) - 1$

$P(L_0 \leq 0 | G = G_n) = P(K_{70} > n)$

Q10) $PV = \bar{a}_{\overline{T_x}|}$

$EPV = \bar{a}_x$

ii) $\int_0^\infty e^{-(\mu+\delta)t} \, dt = 16$

$= \dfrac{1}{-(\mu + 0.04)} \left[e^{-(\mu+\delta)t} \right]_0^\infty - 16$

$\mu = \dfrac{1 - 0.64}{16}$

$\mu = 0.0225$

$V = \dfrac{1}{\delta^2} \left({}^2\bar{A}_x - (\bar{A}_x)^2 \right)$

$\bar{A}_x = 1 - \delta \bar{a}_x$

$\bar{A}_x = 1 - 0.64$

$= 0.36$

${}^2\bar{A}_x = \int_0^\infty e^{-2\delta t} e^{-\mu t} \times \mu \, dt$

$= 0.0225 \left[\dfrac{e^{-(2\delta+\mu)t}}{-(2\delta + \mu)} \right]_0^\infty$

$= \dfrac{-0.0225}{(2 \times 0.0225 + 0.04)} (-1) = \dfrac{9}{34}$

$SD = 9.189188$

Q11] (i) Term assurance the premiums are paid annually ~~is~~ or monthly for the particular term where if a policy holder fails to pay premium ~~to~~ the contract will lapse.

(ii) Term assurance

$\ddot{a}_{40:\overline{25|}} = 2{,}500{,}000 \; A^{1}_{40:\overline{25|}}$

$A^{1}_{40:\overline{25|}} = A_{40:\overline{25|}} - A_{40:\overline{25|}}$

$= 0.38907 - (1.04)^{-25}\left(\frac{8821.2612}{9856.2863}\right)$

$= 0.0535448$

$P = \frac{2500000 \times 0.0535448}{15.884}$

$P = 8396.002296$

${}_{10}V = 2{,}500{,}000 \; A^{1}_{50:\overline{15|}} - P\ddot{a}_{50:\overline{15|}}$

$A^{1}_{50:\overline{15|}} = A_{50:\overline{15|}} - A_{50:\overline{15|}}$

$= 0.56719 - (1.04)^{-15}\left(\frac{8821.2612}{9712.0728}\right)$

$= 0.062855$

${}_{10}V = 87334.42964$

Annuity

${}_{10}V = 2{,}500{,}000 \; \ddot{a}_{60}$

$= 3908000$

Total Reserve $= 8500 \times 87334.429 + 9900 \times 3908000$

~~(iii) P~~

Q12 (i) $S_{55:\overline{10|}} = a_{55:\overline{10|}} (1+i)^{10} \, {}_{10}P_{55}$

$= 12.578$

(ii) $\ddot{a}^{(4)}_{[40]:\overline{20|}} = \ddot{a}^{(4)}_{[40]} - v^{20} \, {}_{20}P_{[40]} \, \ddot{a}^{(4)}_{60}$

$= \ddot{a}_{[40]} - \frac{3}{8} - v^{20} \, {}_{20}P_{[40]} \left(\ddot{a}_{60} - \frac{3}{8}\right)$

$= 14.6735$

Q13 $P\ddot{a}_{45:\overline{15|}} = 100{,}000 \, A_{45:\overline{20|}} + 100{,}000 \, A_{45:\overline{20|}}$

$\ddot{a}_{45:\overline{15|}} = 11.886$

$A_{45:\overline{20|}} = 0.48998$

$A_{45:\overline{20|}}^{\ \ 1} = (1.09)^{-20} \, {}_{20}P_{45}$

$= 0.4109519$

$P = 7734.608$

${}_{13}V = 100{,}000 \, A_{58:\overline{7|}} + 100{,}000 \, A_{58:\overline{7|}} - P\ddot{a}_{58:\overline{2|}}$

$A_{58:\overline{7|}} = 0.76516$

$A_{58:\overline{7|}} = (1.09)^{7} \times {}_{7}P_{58}$

$- 0.712$

$\ddot{a}_{56:\overline{2|}} = 1.955$

${}_{13}V = 132603.4289$

$DSAR = - 32603.428$

$EDS = -36657.66$

$ADS = 4 \times DSAR = -130412.71$

$EDS - ADS$

$= 93756.05$

Q14]

$$T_{63} < 2$$

(i) $5000 \ \bar{a}_{\overline{68-2}|} \ v^2, \quad 0 \quad T_{63} > 2$

(ii) $5000 \times 100 \times {}_2p_{63} \times v^2 \ \bar{a}_{65}$

$= 6594347.317$

(iii)