1) $X \sim Poi(\theta)$

P & S - Assignment

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1) \qquad X = 200$$

$$\therefore \theta = X \pm 1.96\left(\sqrt{\theta/n}\right)$$

2) $X_i \sim N(\mu, \sigma^2)$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \qquad Var(\bar{X}) = Var\frac{(\Sigma X_i)}{n} = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$$

$$E(\bar{X}) = E\frac{(\Sigma X_i)}{n} = \frac{n\mu}{n} = \mu$$

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$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \quad E(S^2) = \frac{(n-1)\sigma^2}{n-1} = \sigma^2$$

$$Var(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{n-1}$$

$$(50 < S^2 < 150) \cdot \left(\frac{9 \times 50}{100} < \chi^2_4 < \frac{9 \times 150}{100}\right)$$

4.5, 13.5 (Ans)

3) $n=4$ $\quad Bin(4, P)$ $\quad$ 180 Policy

No. of claims	0	1	2	3	4
	86	75	16	2	1

$$L(P) = ((1-P)^4)^{86}(4P(1-P)^3)^{75}(6P^2(1-P)^2)^{16}(4P^3(1-P))^2(P^4)^1$$

$$= (1-P)^{344} P^{75}(1-P)^{225} P^{32}(1-P)^{32} P^6(1-P)^2 P^4$$

$$= (1-P)^{603} P^{117} \text{ const}$$

$$\ln L(P) = 603 \ln(1-p) + 117 \ln P$$

$\frac{d}{dp} \ln L(p) = \frac{-603}{1-p} + \frac{117}{p} = 0$

$117 - 117p = 603p$

$q = 0.8375 \quad P = 0.1625$

iv) $P(X=0) = (1-p)^4 = 0.492 \quad P(X=2) = 6p^2(1-p)^2 = 0.111$

$P(X=1) = 4p(1-p)^3 = 0.3818 \quad P(X=3) = 4p^3(1-p) = 0.0144$

$P(X=4) = 0.0007$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.55	0.126

By clubbing:-

	0	1	2+
O_i	86	75	19
E_i	88.06	68.72	22.71
$\frac{\Sigma(O_i - E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$\Sigma C_i = 1.2557 \sim \chi_1^2$

$= 1.2557 < 3.841$

$\therefore$ do not reject H_0

5) $U(a,b) \quad E(X) = \frac{a+b}{2} \quad Var(X) = \frac{(b-a)^2}{12}$

$a = \bar{X} - \sqrt{3}S \quad a = 2\bar{X} - b \quad a = b - 2\sqrt{3}S$

$2\bar{x} - b = b - 2\sqrt{3}S$

$b = \bar{x} + \sqrt{3}S \quad a = \bar{X} - \sqrt{3}S$

Sample $= 2, 4, 6, 8, 10 \quad \bar{x} = 6 \quad S = 3.16$

$a = 6 - \sqrt{3}(3.16) \quad b = 6 + \sqrt{3}(3.16) = 11.47$

$= 0.527$

$U(0,b) \quad f(x) = \frac{1}{b} \quad L = \frac{1}{b^n}$

$\log L = -n \log b$

$\frac{d}{db} \log L = \frac{-n}{b} \neq 0$

6) $H_0\, P = 0.1$ $\quad H_1\, P > 0.1$ $\quad$ hits observed

12 missiles fired $\quad H_0$: 3 or more

7) $\bar{X}p\, l_0 = 30$ $\qquad \bar{X}p_{0i} = 35.056$

$S^2_{Pub} = 64.3125$ $\quad S^2_{Poi} = 67.403$ $\quad S_p^2 = 58.123$

$$\frac{(\bar{X}_{Pub} - \bar{X}_{Poi}) - (\mu_{Pub} - \mu_{Poi})}{S_p / \left(\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)} \sim t_{n_1+n_2-2}$$

ii) $H_0: \mu_{Pub} - \mu_{Poi}$ $\quad H_1: \mu_{Pub} \neq \mu_{Poi}$

$$\frac{(30 - 35.056)}{\sqrt{58.123}\sqrt{\frac{2}{9}}} = -1.407 \therefore \text{accept } H_0$$

iii) Private hospital do not result in higher claim

8) i) unbias when $E(\hat{\theta}) = \theta$

ii) bias is when $E(\hat{\theta}) - \theta \neq 0$

iii) MSE is when θ = var + bias^2

9) $t_k = \frac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$ $\qquad \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

ii) $\mu(t_k) = 0$ $\qquad Var(t_k) = \frac{k}{k-2}\, I$

iii) $n = 10$ $\quad \bar{X} = 80$ $\quad s^2 = 48.667$ $\quad CI = 99\%$

a) $t_{0.005, 9} = 3.25$

$$\mu = \bar{X} \pm t_{0.5\%, 9} \frac{S}{\sqrt{n}} = 80 \pm 3.25 \sqrt{\frac{48.667}{10}}$$

$= 57.17, 42.83$

b) $t_{0.005,19} = 2.5750$ $\mu = 50 \pm 2.5788\left(\sqrt{\frac{48.667}{20}}\right)$

$= 58.68, 44.32$

10) $n = 1000$ $X \sim Poi(3)$ $X \sim N(3000, 3000)$

$P(x < 3100)$ then $X \sim Poi(3)$

else X do not follow $Poi(3)$

i) Type I error $= P(H_0 \text{ is reject} / H_0 \text{ is true})$

$= P(X > 3100, \lambda = 3)$ $\therefore X \sim N(3000, 3000)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right) = P(Z > 1.826)$

$= 3.39\%$

ii) Type 2 error $\Rightarrow P(\mu_0 \text{ accept} / \mu_0 \text{ is false})$

i) Power $= P(\text{reject } H_0 / H_0 \text{ false}) = 1 - \left(2 < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$

) $n = 12$ $\Sigma x = 213200$ $\Sigma x^2 = 4919860000$

$\bar{x} = 17767$ $S = 10144$

$x_1 = 11000$ $x_2 = 48000$ $x_1' = 27500$ $x_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 48000$

$A = 154200$

$\Sigma x' = A + x_1' + x_2' = 213200$ $\bar{x} = 17767$

$\Sigma x^2 = 4919860000 = B + 11000^2 + 48000^2$ $B = 2494860000$

$\Sigma' x^2 = B + x_1'^2 + x_2'^2 = 4243360000$

$S = \frac{1}{\sqrt{11}}\sqrt{4243360000 - 12(17767)^2} = 6434.03$