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Subject : ~~No~~ Numerical methods & algebra
Assignment Questions

Q1. radius of circular plate = 12.65 cm
actual length 12.5 cm

(i) Absolute Error
Area of plate = $\pi r^2 = \pi \times 12.65^2$
$= 502.7255\ cm^2$
Area of plate without error = $\pi \times 12.5^2$
$= 490.87385\ cm^2$
Absolute Error = $|502.7255 - 490.87385|$
$= 11.8516\ cm^2$

(ii) Relative Error
12.65^2
$f(x) = x^2$
$f'(x) = 2x$
$a+h = 12 + .65^2$
$a = 12$
$h = 0.65$
$f(a) = 12^2 = 144$
$f'(a) = 2(12) = 24$

$f(a+h) \cong f(a) + hf'(a)$
$= 144 + 0.65 \times 24$
$= 159.6$

$12.65^2 = 160.0225$

∴ ~~Absolute~~ Relative error :
$|159.6 - 160.0225|$
$= 0.4225$

Percentage error

Area with error = 502.7255
Area without error = 490.8738

$$\% \text{ error} = \frac{(502.7255 - 490.8738)}{502.7255} \times 100\%$$

$$= \frac{11.8517}{502.7255} \times 100\%$$

$$= 2.3574\%$$

Q2. Interpolate

x	$f(x)$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

log 5 : Using ~~Regula Falsi~~

(a) Using $x_1 = 4$ & $x_2 = 6$

$$x = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$$

$$= \frac{4(0.7781513) - 6(0.60206)}{0.7781513 - 0.60206}$$

$$= -2.838$$

1(a) Using Bisection Method

$$x = \frac{x_1 + x_2}{2} = \frac{4+6}{2} = 5$$

$$y = \log(x) \Rightarrow \log 5 = 0.69897$$

(b) Using points 4.5 & 5.5; Bisection Method :-

$$x = \frac{x_1 + x_2}{2} = \frac{4.5 + 5.5}{2} = 5$$

$$y = \log(x) = \log 5 = 0.69897$$

Q3. $x^4 - x - 10 = 0$ $\quad a = 1.5, \; b = 2$

(1) $$x = \frac{a+b}{2} = \frac{1.5+2}{2} = \frac{3.5}{2} = 1.75$$

$$y = 1.75^4 - 1.75 - 10 = -2.37109$$

$$\left\{ \begin{array}{llll} \text{at} & a = 1.5 & & y = -6.4375 \\ & b = 2 & & y = 4 \end{array} \right.$$

(2) $x_1 = 1.75$ $\quad y_1 = -2.37109$

$x_2 = 2$ $\quad y_2 = 4$

$$x = \frac{x_1 + x_2}{2} = 1.875$$

$$y = 1.875^4 - 1.875 - 10 = 0.4846$$

(3) $x_1 = 1.75$ $y_1 = -2.37109$
$x_2 = 1.875$ $y_2 = 0.4846$

$$x = \frac{x_1 + x_2}{2} = \frac{1.75 + 1.875}{2} = 1.8125$$

$y = -1.02024$

(4) $x_1 = 1.8125$ $y_1 = -1.02024$
$x_2 = 1.875$ $y_2 = 0.4846$

$$x = \frac{x_1 + x_2}{2} = \frac{1.8125 + 1.875}{2} = 1.84375$$

$y = -0.2877$

(5) $x_1 = 1.84375$ $y_1 = -0.2877$
$x_2 = 1.875$ $y_2 = 0.4846$

$x = 1.859375$ $\left[\text{using } \frac{x_1 + x_2}{2}\right]$

$y = 1.859375^4 - 1.859375 - 10$
$= 0.09337$

$\therefore$ Ans $= 1.859375$

Q4. $x^3 - x - 1$ (Using Newton Raphson method)

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$x_N = ?$

$f(1) = 1^3 - 1 - 1$

$= -1$

$f'(1) = 3(1) - 1 = 2$

$x_{N1} = 1 - \frac{(-1)}{2}$

$= 1 + 1/2 = 1.5$

$y_{N1} = 1.5^3 - 1.5 - 1 = 0.875$

$x_0 = 1.5$

$x_{N2} = f(0.875) = 0.875^3 - 0.875 - 1$

$= -1.2051$

$y_{N2} = -1.2051^3 + 1.2051 - 1$

$= -3.9551$

$x_0 = 1.5$

$f(1.5) = 1.5^3 - 1.5 - 1 = 0.875$

$f'(1.5) = 3(1.5)^2 - 1 = 5.75$

$x_{N2} = 1.5 - 0.875/5.75 = 1.3478$

$y_{N2} = 0.1$

$\therefore$ Ans : 1.3478

Q5. $A^2 = A$

$A^2 - A = 0$

$A(A-1) = 0$

$A = 0 \;/\; A - 1 = 0$

$A = 0 \quad A = 1$

$7A - (1+A)^3$

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$

$a^2 + bc = a$

$ab + bd = b \Rightarrow a + d = 1$

$ac + cd = c$

$bc + d^2 = d$

Q6. Inverse of :

$\begin{bmatrix} -1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = -1(0+6) + 1(-4+0) + 2(4)$

$= -6 - 4 + 8 = -2$

$$M_{11} = \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} = -6 \qquad A_{11} = -6$$

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} = -4 \qquad A_{12} = 4$$

$$M_{13} = \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4 \qquad A_{13} = 4$$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = -1 \qquad A_{21} = 1$$

$$M_{22} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 \qquad A_{22} = 1$$

$$M_{23} = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \qquad A_{23} = 1$$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 6 \end{vmatrix} = -6 \qquad A_{31} = -6$$

$$M_{32} = \begin{vmatrix} -1 & 2 \\ 4 & 6 \end{vmatrix} = -14 \qquad A_{32} = 14$$

$$M_{33} = \begin{vmatrix} -1 & -1 \\ 4 & 0 \end{vmatrix} = 4 \qquad A_{33} = 4$$

$$A^{-1} = -\frac{1}{2} \begin{vmatrix} -6 & 4 & 4 \\ 1 & 1 & 1 \\ -6 & 14 & 4 \end{vmatrix}$$

Q7.

(a)

$\cancel{A = 8i - 9j}$

$\cancel{A^2 = (8i - 9j)^2}$

$\cancel{= (8i - 9j) \cdot (8i - 9j)}$

$\cancel{=}$

$\hat{a} = 8\hat{i} - 9\hat{j}$

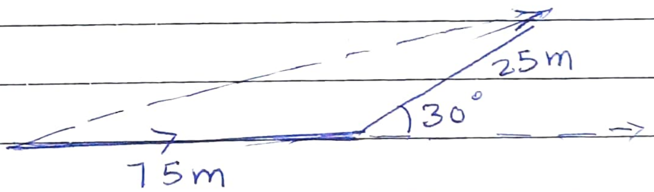
$\hat{b} = 7\hat{i} - 9\hat{j}$

$\bar{a} \cdot \bar{b} = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$

$= 56 - 81$

$= -25$

Q8.



Resultant vector $= \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30}$

$= 97.46$ metre

Q9. sum of all 3 digit no. that leave 2 remainder when divided by 3

series : 101, 104 998

$a = 101$
$d = 3$

$S_n = ?$
$t_n = a + (n-1)d$
$998 = 101 + (n-1)3$
$\frac{897}{3} = (n-1)$

$299 = n - 1$
$n = 300$

$S_{300} = 2a + (n-1)d$
$= 2(101) + 299 \times 3$
$= 1099$

Q10. $t_6 = 32$ $\Rightarrow ar^5 = 32$
$t_8 = 128$ $ar^7 = 128$

Divide t_8 by t_6
$\frac{t_8}{t_6} = \frac{\cancel{a}r^{\cancel{7}\,2}}{\cancel{a}\cancel{r^5}} = \frac{128}{32}$
$r^2 = 4$
$r = \pm 2$ Since $t_6 < t_8$; $r = 2$

$\therefore$ The common ratio is 2

Q11. $(4+3x)^5$

$= 5C_0 (4)^5 (3x)^0 + 5C_1 (4)^{(4)} (3x) +$
$5C_2 (4)^3 (3x)^2 + 5C_3 (4)^2 (3x)^3 +$
$5C_4 (4)^1 (3x)^4 + 5C_5 (4)^0 (3x)^5$

$= 1024 + 1280 \times 3x + 640(3x)^2 + 160(3x)^3 + 20(3x)^4 + (3x)^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q12 $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

If λ is an eigenvalue of A, then λ satisfies

$\det(A - \lambda I) = \det\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right.$

$= \begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda) \begin{vmatrix} 2-\lambda & -3 \\ 2 & -5-\lambda \end{vmatrix}$

$(3-\lambda)[(2-\lambda)(-5-\lambda)+6]$

$= \cancel{3(}\ (3-\lambda)(-10+3\lambda-\lambda^2+6)$

$= (3-\lambda)(\lambda^2+3\lambda-4)$

$= -(\lambda-3)(\lambda+4)(\lambda-1)$

$\therefore$ Eigenvalues of A are $3, 1, -4$.

Eigenvectors:

1. for $\lambda = 3$

$$(A-3I)x = \left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_2 + 2R_1 \rightarrow R_2$

$$= \begin{bmatrix} -1 & -3 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$-R_1 \rightarrow R_1$ & $-1/14\ R_2 \rightarrow R_2$

$$= \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 3R_2 \rightarrow R_1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \therefore x = x_3\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

~~Vec~~ Eigen vector for $\lambda = 1$

$$(A - 3I)\, x = \left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_2 - 2R_1 \to R_2$

$1/2\, R_3 \to R_3$

$R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

Eigen vector for $\lambda = -4$

$$(A - 3I)\, x = \left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_2 - 1/6\, R_1$, $1/7\, R_3$, $1/6\, R_1$, $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = x_1 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$$

Q13 $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x_0) = f(5) = 5^3 - 7(5)^2 + 8(5) - 3$

$= -13$

$f'(x_0) = 3x^2 - 14x + 8$

$f'(5) = 3(5)^2 - 14(5) + 8$

$= 13$

$$x_N = 5 - \frac{(-13)}{13}$$

$= 5 + 1$

$= 6$

$\therefore x_2 = 6$

Q14. $\cancel{(1+}$ $(1 - x + x^2)^4 = [x + (1 + x^2)]^4$

$= {}^4C_0\, x^4 + {}^4C_1\, x^3(1+x^2) + {}^4C_2\, x^2(1+x^2)^2$
$+ {}^4C_3 x(1+x^2)^2 + {}^4C_4\, (1+x^2)^4$

$= x^4 + 4x^3(1+x^2) + 6x^2(1+x^4+2x^2) +$
$4x[1 + x^6 + 3x^2 + 3x^4] + [1 + 4x^2 + 6x^4 + 4x^6 + x^8]$

$= x^8 + 4x^7 + 10x^6 + 16x^5 + 19x^4 + 16x^3 +$
$10x^2 + 4x + 1$

Q15 $e^{-x} = 3\log(x) \Rightarrow$

Using Bisection method

(1) $a = 0.5 \qquad b = 1.5$

$$x = \frac{0.5 + 1.5}{2} = 1$$

$y = 0.555$

(2) $\therefore \quad a = 1 \qquad b = 1.5$

$x = 1.25$

$y = -1.555 \times 10^{-3}$ (-ve)

(3) $a = 1 \quad , \quad b = 1.25$

$$x = \frac{a+b}{2} = \frac{1+1.25}{2} = 1.125$$

$y = 0.063$

(4) $a = 1.125 \qquad b = 1.25$

$$x = \frac{a+b}{2} = \frac{1.125+1.25}{2} = 1.187$$

$y = 0.014$

$\therefore$ Ans : 1.187