Assignment 1

1) Nominal Rate of discount $d^{4} = 8\%$

Effective $(d) = 1 - \left(1 - \frac{8\%}{4}\right)^{4}$

$d = 0.077632$

$= 7.7632\%$

~~i) $\delta = \ln(1+i) = \ln(1-d) = -\ln(1-7.7632) =$~~

i) $\delta = \ln(1+i) = \ln\left(1 + \frac{d}{1-d}\right) = \ln(1 + 0.084166) = 0.08081$

ii) Effective rate of Interest $(i) = \frac{d}{1-d}$

$= \frac{0.07763}{1-0.07763} = 0.084166 = 8.4166\%$

iii) $d^{12} = p\left[1-(1-d)^{1/p}\right] = 12\left[1-(1-0.7763)^{1/12}\right] = 0.080539$

2) $\delta(t) = 0.004t + 0.0002t^{2}$

i) $PV(10) = 1000 \cdot e^{-\int_0^{10}(0.004t + 0.0002t^2)\,dt}$

$= 1000 \cdot e^{-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 1000 \cdot e^{-[0.8667]} = ~~2378.96773~~\ 420.35038$

ii) ~~$e^{\delta n} = e^{\delta \cdot 10} = e^{\int_0^{10}\delta t\,dt}$~~

$PV = 1000 \cdot V(0,10)$

$420.35038 = 1000 \cdot V(0,10)$

$V(0,10) = 0.420350$

$0.420350 = \frac{1}{(1+i)^{10}}$

$(1+i)^{10} = ~~2.38~~\ 2.37897$

$(1+i) = 1.09053$

$i = 0.90533$

$i = 9.05331\%$

3) i) The relationship between i, v and d is an important one. Interest earned on an investment of 1 paid at the beginning of the period is d. Interest earned on an investment of 1 paid at the end of the period is 'i'. Therefore, if we discount 'i' from the end of the period to the beginning of the period with the discounting factor v, so we obtain 'd'. Thus, $d = i.v$

ii) a) $d^{2} = 2\left[1-(1-0.09)^{1/2}\right]$

$= 0.0921216$

$= 9.2122\%$

b) $d^{0.5} = 5\% \rightarrow d^{6} = 0.0125 = 1.25\%$

iii) Simple discount rate $= \dfrac{1}{(1-nd)}$ ~~$= (1-\frac{91}{365}\times 0.05)$~~ $\Big|\ d = \dfrac{1}{1+i}$

$= \dfrac{1}{(1-91/365\times 0.0476)} - 1 = 0.012015 = 1.2015\%$

4) i) The relationship between i, v and d is an important one. Interest earned on an investment of 1 paid at the beginning of the period is 'd'. Interest earned on an investment of 1 paid at the end of the period is 'i'. Therefore, if we discount 'i' from the end of the period to the beginning of the period with the discounting factor v, so we obtain 'd'. Thus, $d = i.v$.

ii) a) $i = 0.08$

$$d = \frac{i}{1+i}$$

$$= \frac{0.08}{1+0.08}$$

$= 0.074074 = 7.4074\%$

a) $d^{(12)} = 12\left[1-(1-0.07407)^{1/12}\right]$

$= 0.07671 = 7.6715\%$

b) $i^{(365)} = i^{(1)} = i = 0.08 = 8\%$

c) $\delta = \ln(1+i) = \ln(1+0.08) = 0.076961 = 7.6961\%$

d) $i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right] = 0.0832 = 8.32\%$

5) i) $d^4 = 6\%$

a) $d = 1 - \left(1 - \frac{6\%}{4}\right)^4$

$= 0.05867 = 5.8663\%$

$i = \frac{d}{1-d} = 0.62319 = 6.2319\%$

$i^2 = 2\left[(1+0.62319)^{1/2} - 1\right]$

$= 0.061378 = 6.1378\%$

b) $d^{12} = 12\left[1-(1-0.05867)^{1/12}\right]$

$= 0.060303 = 6.0303\%$

ii) $i = \frac{220000 - 200000}{200000} = 0.1 = 10\%$

a) $200000 \times \frac{(15+31+30+17)}{365}$

$= 20000 \times \frac{93}{365}$

$= 5095.8904$

Sum paid = Amount + Interest

$= 20000 + 5095.8904$

$= 25095.8904$

6) i) a) Assume, price of toy = ₹ 1000

a) $x - \left(\frac{x \times 30}{100}\right) = x - \frac{3x}{10} = \frac{7x}{10} = \frac{7 \times 1000}{10} = ₹ 700$

Accumulation = present value × Accumulation factor

$1000 = 700 \times A(n)$

$A(n) = 1.4286$

$(1-d) =$

7) $I_n = \frac{26000-20000}{20000} = 0.3$ for 17 months

$0.3 \times \frac{12}{17} = 0.211765$ for 12 months

i) $i = 0.211765$

$i^{(4)} = 4\left[(1+0.211765)^{1/4} - 1\right]$

$= 0.196764 = 19.6764\%$

ii) $D = \frac{0.211765}{1+0.211765} = 0.174757 = 17.47573\%$

$D = 0.174757$

$d^{2} = 2\left[1-(1-0.17476)^{1/2}\right]$

$= 0.18314 = 18.3143\%$

iii) $A_n = ni$

~~An = ni Simple rate of interest~~ $= \frac{26000-20000}{20000}$

$A_1 = 1 \times 0.211765$

$= 0.211765$

iv) when we compare results, 'i' as the ~~biggest~~ highest value then lowered down, we got 'i^4'. ~~which is~~ Afterwards, the value of 'd^2' is less than i^4 and lastly we got the value of 'd' as the lowest.

9) i) Force of interest is an instateneous change in the value of fund because it is continuously compounding.

ii) $i^{(2)} = 0.0775$

a) $i = \left(1 + \frac{0.0775}{2}\right)^2 - 1 = 0.079002$

b) $\delta = \ln(1+i) = \ln(1+0.079002) = 0.0760361$

c) $d = \frac{i}{1+i} = \frac{0.079002}{\cancel{x}1+0.079002} = 0.0732173$

$d^{12} = 12\left[1 - (1-0.073217)^{1/12}\right]$

$= 0.0757957$

d) $i^{1/2} = \frac{1}{2}\left[(1+0.079002)^{1/1/2} - 1\right]$

$= 0.0821222$

10) Assume $0 \le t \le 5$

$\cancel{\delta}\ A(0,t) = e^{-\int_0^t 0.08} = \cancel{e^{-[0.08]^t}}\ e^{-[0.08s]_0^t} = e^{-0.08t}$

Assume $5 \le t \le 10$

$A(0,t) = A\cancel{(0,5)} \cdot \cancel{A(5,10)}$

$= \cancel{e^{[0.08t]}}\ e^{-[0.06t]_0^t} = \cancel{e^{-0.14t}}\ \cancel{e^{-0.4}} \cdot \cancel{e^{-0.3}} = \cancel{e^{-0.7}}\ e^{-0.06t}$

Assume ~~to~~ $t \geq 10$

$A(0,t) = A(0,5) \cdot A(5,10) \cdot A(10,t)$

$= e^{-0.7} \cdot e^{-[0.04s]_t^{10}} = e^{-0.7} \cdot \cancel{e^{-0.4t}}\ e^{-[0.4 - 0.04t]}$

$= e^{-1.1+0.04t}$

Expression for $v(t)$

$v(0,t) = e^{-0.08t}$ $\quad 0 \le t \le 5$

$= e^{-0.06t}$ $\quad \cancel{0}\ 5 \le t \le 10$

$= e^{-1.1+0.04t}$ $\quad t \geq 10$

1) b) i) $(1+i) = e^{\delta}$

$(1+i) = e^{6\%}$

$i = 0.0618365$

$d = \dfrac{0.0618365}{1+0.0618365} = 0.0582355$

~~iii~~

$d^{(12)} = 12\left[1-(1-0.0582)^{1/12}\right]$

$= 0.0598502$

ii) $i = \dfrac{0.06}{1-0.06} = 0.06383$

$i^{(12)} = 12\left[(1+0.06383)^{1/12} - 1\right]$

$= 0.0620352$

c) $d < \delta < i^{(4)} < i$

In terms of equivalent rates, i as a numerical value is the highest then $i^{(4)}$ till limiting to delta (δ). Then, as D as a numerical ~~D is the low~~ value is the lowest, it is first in ascending order. Thus, all the rates are equivalent rates and they all correspond to same effective interest rate.

d) The relationship $d = i.v$ is true. Interest earned on an investment of 1 paid at the beginning of the period is 'd'. Interest earned on an investment of '1' paid at the end of the period is 'i'. Therefore, if we discount 'i' from the end of the period to the beginning of the period with the discount factor v, we obtain 'd'. Thus, $d = i.v$.

12) ~~i~~ First two years $= \left(1+\frac{0.06}{12}\right)^{24} - 1 = 0.12716$

Next, two and half years $= \left(1+\cancel{0.005}\right)^{\cancel{30}} - 1 = \cancel{0.1614}\ 0.15$

Next, one and half years $= d = i = \frac{0.06}{1-0.06} = 0.06383$

$= \left(1+\frac{0.0638}{12}\right)^{18} - 1 = 0.100199$

Accumulated amount $= 7049 \times 1.127 \times 1.15 \times 1.1002$

$= 10152.3372$