Name - Darshan Lodha
Roll NO. - 18

Numerical Methods And Algebra

Assignment 1

1.
Ans Let $x = \sqrt{7+\sqrt{7+\sqrt{7+\dots}}}$ —— ①

$\therefore$ $x = \sqrt{7+x}$ —— ②

Squaring on both sides, we get

$$x^2 = (7+x)$$

$$\therefore x^2 - x - 7 = 0$$

Using formula method,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(-7)}}{2(1)}$$

$x = \frac{1+\sqrt{29}}{2}$ Or $x = \frac{1-\sqrt{29}}{2}$

$\therefore x = 3.192582$ $\therefore x = -2.192582$

But when $x = -2.192582$ is substituted in the RHS of equation 2, we get a positive answer for the resulting square. Thus, in that case LHS. $\neq$. RHS

$\therefore \quad x \neq -2.192582$

$\therefore \quad x = 3.192582$

Ans. The value of $\sqrt{7+\sqrt{7+\sqrt{7+\cdots}}}$ is 3.192582

2

Ans. Given equation:

$$7x^2 - 6y^2 = -11x^2 - 4y^2$$

Dividing throughout by y^2

$$\frac{7x^2}{y^2} - \frac{6y^2}{y^2} = \frac{-11x^2}{y^2} - \frac{4y^2}{y^2}$$

$$\therefore \quad 7\left(\frac{x}{y}\right)^2 - 6 = -11\left(\frac{x}{y}\right)^2 - 4$$

Let $\frac{x}{y}$ be m

$$\therefore \quad 7m^2 - 6 = -11m^2 - 4$$

$$\therefore \quad 18m^2 = 2$$

$$\therefore \quad m^2 = \frac{1}{9}$$

$$\therefore \quad m = \frac{1}{3} \quad \text{or} \quad m = \frac{-1}{3}$$

Resubstituting $m = \frac{x}{y}$

$\therefore \frac{x}{y} = \frac{1}{3}$ Or $\frac{x}{y} = \frac{-1}{3}$

Ans. The value of $\frac{x}{y}$ is $\pm\frac{1}{3}$

3.

Ans Given system of equations:

$x + 3y = 4$ — ①

$x^2 + y^2 - 4y = 12$ — ②

Rearranging equation 1, we get

$x = 4 - 3y$ — ③

From 2 and 3,

$(4-3y)^2 + y^2 - 4y = 12$

$16 - 24y + 9y^2 + y^2 - 4y = 12$

$16 - 28y + 10y^2 = 12$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$$5y^2 - 14y + 2 = 0$$

Using formula method,

$$y = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(5)(2)}}{2(5)}$$

$\therefore y = \frac{7 + \sqrt{39}}{5}$ Or $y = \frac{7 - \sqrt{39}}{5}$

$\therefore y = 2.6489996$ $\underline{y = 0.151}$

$\therefore \underline{y \approx 2.649}$

substituting in equation 3, $x = 4 - 3y$

When $y = 2.649$ When $y = 0.151$

$x = 4 - 3(2.649)$ $\cancel{y}\ x = 4 - 3(0.151)$

$\therefore \underline{x = -3.947}$ $\therefore \underline{x = 3.547}$

Ans. The ordered pairs $\underline{(-3.947, 2.649)}$ and $\underline{(3.547, 0.151)}$ satisfy the given system of equations

4

Ans Given equation

$$d = 2r^2 - 16r + 34$$

where, d = density of smoke in millions of particles per cubic foot

r = speed of engine in hundreds of revolutions per minute.

(a) $r = 3.5$

$$d = 2r^2 - 16r + 34$$

$$\therefore d = 2(3.5)^2 - 16(3.5) + 34$$

$$\therefore d = 2.5 \text{ million particles per cubic foot}$$

i.e. $d = 2500000$ particles per cubic foot

(b) $d = 2r^2 - 16r + 34$ —— ①

Differentiating wrt r.

$$\frac{dd}{dr} = 4r - 16 \quad —— ②$$

Differentiating again wrt r

$$\frac{d^2d}{dr^2} = 4 \quad —— ③$$

Equating equation 2 to zero,

$$0 = 4r - 16$$

$$\therefore 4r = 16$$

$$\therefore r = 4$$

Substituting $r = 4$ in equation 3,

$$\frac{d^2}{dr^2}(d) = 4 \quad > 0$$

By second derivative test,

d is minimum when $r = 4$

$$\therefore \quad d = 2(4)^2 - 16(4) + 34$$

$$\therefore \quad d = 2$$

Ans The number of revolutions per minute for minimum smoke is 400 and the minimum output is 2 million (i.e. 2000000) particles per cubic foot

(c) $d =$ 100~~0~~ million particles per cubic foot

$$\therefore \quad d = 2r^2 - 16r + 34$$

$$\therefore \quad 100 = 2r^2 - 16r + 34$$

$$\therefore \quad 2r^2 - 16r - 66 = 0$$

$$\therefore \quad r^2 - 8r - 33 = 0$$

$$\therefore \quad r^2 - 11r + 3r - 33 = 0$$

$$\therefore \; r(r-11)+3(r-11)=0$$

$$(r-11)(r+3)=0$$

$$\therefore \; r=11 \quad \text{or} \quad r=-3$$

However, speed of the engine cannot be negative

$$\therefore \; r \neq -3$$

$$\therefore \; r = 11$$

Ans. The speed of engine is 1100 revolutions per minute when the density of smoke is determined to be 100 million particles per cubic foot.

5. (a)

Ans. Let C be the total cost of renting a car for 1 day from company 1

According to given information

$$C = 1500 + 25n$$

(n = The total number of km driven in 1 day)

(b) Let C be the total cost of renting a car for 1 day from company 2

According to given information

$$C = 2100 + 22n$$

(n = The total number of km driven in 1 day)

(c) An inequality that can be used to determine for what number of km it is less expensive to rent the car from company 2:

$$2100 + 22n < 1500 + 25n$$

(d) Solving the above inequality,

$$2100 + 22n < 1500 + 25n$$

$$2100 + 22n - 1500 - 22n < 1500 + 25n - 1500 - 22n$$

$$\therefore \quad 600 < 3n$$

$$\therefore \quad 200 < n$$

Thus,
for n greater than 200 km it is less expensive to rent the car from company 2.

6.

Ans Given that: $f(x) = x^3 - 7x^2 + 14x - 6$

(a) $[0, 1]$ (Using bisection method)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
0	1	0.5	−6	2	−0.625
0.5	1	0.75	−0.625	2	0.984375
0.5	0.75	0.625	−0.625	0.984375	0.2597656
0.5	0.625	0.5625	−0.625	0.2597656	−0.1618652
0.5625	0.625	0.59375	−0.1618652	0.2597656	0.0540466
0.5625	0.59375	0.578125	−0.1618652	0.0540466	−0.0526238
0.578125	0.59375	0.5859375	−0.0526238	0.0540466	0.001031399
0.578125	0.5859375	0.58203125	−0.0526238	0.001031399	−0.025716

Ans: 0.58 is the approximate root of $x^3-7x^2+14x-6$ in the interval [0,1]

(b) [1, 3.2] (Using bisection method)

a	b	$t=\frac{a+b}{2}$	f(a)	f(b)	f(t)
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.552125
2.65	3.2	2.925	0.552125	-0.112	0.0858281
2.925	3.2	3.0625	0.0858281	-0.112	-0.054443
2.925	3.0625	2.99375	0.0858281	-0.054443	0.006328
2.99375	3.0625	3.028125	0.006328	-0.054443	-0.026521
2.99375	3.028125	3.0109375	0.006328	-0.026521	-0.0106969
2.99375	3.0109325	3.002344	0.006328	-0.0106969	-0.002333
2.99375	3.002344	2.998047	0.006328	-0.002333	0.001960621
2.998047	3.002344	3.0001955	0.001960621	-0.002333	-0.000195424
2.998047	3.0001955	2.99912125	0.001960621	-0.000195424	0.00088029
2.99912125	3.0001955	2.9996584	0.00088029	-0.00019542	0.000341858

The approximate root of $x^3-7x^2+14x-6$ in the interval [0,1] is 3.00

(c) [3.2, 4] (Using bisection method)

a	b	$t=\frac{a+b}{2}$	f(a)	f(b)	f(t)
3.2	4	3.6	-0.112	2	0.336

3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.046125	0.01301563
3.4	3.425	3.4125	-0.016	0.01301563	-0.0019981
3.4125	3.425	3.41875	-0.0019981	0.01301563	0.00538159

The approximate root to $x^3 - 7x^2 + 14x - 6$ in the interval $[3.2, 4]$ is 3.41

7.

Ans: Given equation:

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

and $$f'(x) = 920x^3 + 54x^2 + 18x - 221$$

Using Newton's method:

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For interval $[-1, 0]$

Using Newton's method and taking $x_0 = 0$

For $n = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0 - \left[\frac{230(0)^4 + 18(0)^3 + 9(0) - 221(0) - 9}{920(0)^3 + 54(0)^2 + 18(0) - 221}\right]$$

$$\therefore x_1 = 0 - \left(\frac{-9}{-221}\right)$$

$$\therefore x_1 = \frac{-9}{221}$$

$$\therefore x_1 = -0.040723982$$

For $x = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \cancel{(-0.040723982) - \left[230(-0.040723982)^3 + 18\right.}$$

$$\therefore x_2 = -0.040659288$$

For $x = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\therefore x_3 = -0.040659288$$

The approximate root of the given equation in the interval ~~-0.040~~ $[-1, 0]$ is -0.0407.

For interval [0, 1]
Using Newton's method and taking $x_0 = 1$
For $n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$\therefore x_1 = 0.964980645$

For $n = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$\therefore x_2 = 0.962411725$

For $n = 2$,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$\therefore x_3 = 0.962398419$

For $n = 3$,

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$\therefore x_4 = 0.962398419$

Ans: The approximate root in interval [0, 1] is <u>0.9624</u>.

8

Ans (a) The grades on the first four exams are 70, 86, 81 and 83

'x' represents the grade on the final exam

Let 'm' represent the course average

$$\therefore m = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\therefore m = \frac{70 + 86 + 81 + 83 + x}{5}$$

$$\therefore m = \frac{320 + x}{5} \quad \text{— ①}$$

(b) The condition to earn a 'B' is given as

$$80 \leq m < 90 \quad \text{— ②}$$

From 1 and 2

$$80 \leq \frac{320 + x}{5} < 90 \quad \text{— ③}$$

(c) Solving the above inequality,

$$80 \times 5 \leq \frac{320 + x}{5} \times 5 < 90 \times 5$$

$$400 \leq 320 + x < 450$$

$$400 - 320 \leq 320 + x - 320 < 450 - 320$$

$$80 \leq x < 130$$

Ans The marks in the final exam should be greater than equal to 80 and less than 130 in order to earn a B.

9.
Ans

Discount rate	NPV
47	8.54
207	−11.81

For IRR, NPV = 0 and the corresponding discount rate is the IRR.
Using linear interpolation,

$$\frac{0 - 8.54}{-11.81 - 8.54} = \frac{y - 47}{207 - 47}$$

$$(0.41965602)(0.16) + 0.04 = y$$

$$\therefore y = 0.107144963$$

$$\therefore y = 10.71449637$$

Ans IRR = 10.71449637.

10.
Ans Given that R(x) and C(x) are the monthly revenue and costs (in lakhs of Rs)
x is the number of orders (thousands)
Let P(x) be the monthy profit
We know,

$$P(x) = R(x) - C(x)$$

$$\therefore P(x) = 32x - 0.21x^2 - 195 - 12x$$

$$\therefore P(x) = 20x - 0.21x^2 - 195$$

Now, for the app to remain profitable,

$$P(x) > 0$$

$$\therefore 20x - 0.21x^2 - 195 > 0$$

$$\therefore -0.21x^2 + 20x - 195 > 0 \quad \text{——} \; (1)$$

Solving the quadratic equation separately,
Using formula method,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-20 \pm \sqrt{(20)^2 - 4(-0.21)(-195)}}{2(-0.21)}$$

$$x = 11.02666771 \quad \text{or} \quad x = 84.21142753$$

Substituting both of these in equation 1,

$$(x - 11.03)(x - 84.21) > 0$$

Case I:

$$(x - 11.03) < 0 \text{ and } (x - 84.21) < 0$$

$$(x - 11.03)(x - 84.21) > 0$$

$$\therefore 11.03 < x < 84.21$$

$$\therefore \underline{12 \leq x \leq 84}$$

Thus,
For the app to remain profitable the number of orders have to be between 12,000 and 84,000.