[redacted]

Assignment - II

(1) $X =$ Claim size on a house insurance

$X \sim \text{Normal}(800, 100^2)$

$Y =$ claim size on a car insurance

$Y \sim \text{Normal}(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3(1200) - 4(800), (3 \times 300^2 + 4 \times 100^2))$

$\sim N(400, 310000)$

$P((X_1 + Y_2 + X_3) - (X_1 + X_2 + X_3 + X_4) > 800)$

$= P\left(Z > \dfrac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 1 - 0.76424$

$= \underline{0.23576}$

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(2) N- number of claims

$N \sim$ Negative Binomial $(3, 0.9)$

X = Claim Amount

$X \sim$ Gamma $(6, 2)$

Y = Total Claim amount

$$M_Y(t) = M_N(\log M_X(t))$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_Y(t) = M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$\left(M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k\right)$$

$$M_Y(t) = \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right)^3$$

$$M_Y(t) = \left(\frac{p\left(1-t/2\right)^{-6}}{1-q\left(1-t/2\right)^{-6}}\right)^3 = \left(\frac{0.9\left(1-t/2\right)^{-6}}{1-(0.1)\left(1-t/2\right)^{-6}}\right)$$

$E(N) = 3/0.9 = 10/3 \quad V(N) = \frac{k(0.1)}{(0.9)^2} = \frac{3}{0.9}\left(1/9\right) = 10/27$

$E(X) = \frac{\alpha}{\lambda} = 6/2 = 3 \quad V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = \frac{3}{2}$

$$V(Y) = E(N)\,V(X) + E(X)^2\,V(N)$$

$$= \frac{10}{3} \times \frac{3}{2} + 9\left(\frac{10}{27}\right)$$

$$= 5 + \frac{10}{3} = \frac{25}{3}$$

$$S.D(Y) = \sqrt{\frac{25}{3}} = 2.886$$

Ques 3. $f_X(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_X(t) = E(e^{tX})$$

$$= \int_{\alpha}^{\infty} e^{tx}\,\lambda e^{-\lambda(x-\alpha)}\,dx$$

$$= \lambda \int_{\alpha}^{\infty} e^{tx}\, e^{-\lambda x}\, e^{\lambda\alpha}\, dx$$

$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)}\, dx$$

$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left[e^{(t-\lambda)x}\right]_0^{\infty} = \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left[e^{-(\lambda-t)x}\right]_0^{\infty}$$

$$M_X(t) = \frac{\lambda \cancel{e^{\lambda\alpha}}}{t-\lambda}\left[0 - \cancel{e^{-\lambda\alpha}} \cdot e^{t\alpha}\right] = \frac{\lambda e^{t\alpha}}{\lambda - t}$$

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Ques 4. $G_V(t) = \left(\frac{p}{1-qt}\right)^k$; $p+q=1$

$\therefore$ given distribution is negative binomial type 2

$$P(V=4) = \frac{\sqrt{k+4}}{\sqrt{5}\,\sqrt{k}}\, p^4 q^4$$

Ques 5. $f(x,y) = ax^2 + axy$ $0 < x < 5$, $10 < y < 20$

$$1 = \int_{10}^{20}\int_0^5 ax^2 + axy \, dx \, dy$$

$$1 = \int_{10}^{20} \left[\frac{ax^3}{3}\right]_0^5 + \left[ay\frac{x^2}{2}\right]_0^5 dy$$

$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2}ay \, dy$$

$$1 = \frac{125a}{3}\left[y\right]_{10}^{20} + \frac{25}{2}a\left[\frac{y^2}{2}\right]_{10}^{20}$$

$$1 = \frac{1250a}{3} + 1875a$$

$$\boxed{a = \frac{3}{6875}}$$

$$f_X(x) = \int_{10}^{20} (x^2 + xy)a \, dy$$

$$= \frac{3}{6875}\left[x^2[y]_{10}^{20} + x\left[\frac{y^2}{2}\right]_{10}^{20}\right]$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right)$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right)$$

$$E(Y/X) = \int y\, f(X/x) = \int_{10}^{20} y \frac{f(x,y)}{f_X(x)}$$

$$= \int_{10}^{20} \frac{y\left(\frac{3}{6875}\right)(x^2+xy)}{\frac{3}{6875}\left(10x^2+150x\right)}\, dy$$

$$= \frac{1}{10}\int_{10}^{20} \frac{y(x+y)}{x+15}\, dy$$

$$= \frac{1}{10(x+15)}\left[\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}\right]$$

$$= \frac{1}{10(x+15)}\left[150x + \frac{7000}{3}\right]$$

$$= \frac{1}{x+15}\left(15x + 700/3\right) = \frac{45x+700}{3(x+15)}$$

$$E(Y/X) = \frac{45x+700}{3(x+15)}$$

$$P\left(Y > \frac{1}{3}x + 10\right) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875}\left(\frac{250+75y}{2}\right) dy$$

$$= \frac{1}{6875(2)}\left[250\left[y\right]_{x/3+10}^{20} + 75\left[\frac{y^2}{2}\right]_{x/3+10}^{20}\right]$$

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$$= \frac{1}{6875(2)}\left[(250)\left(20-\frac{x}{3}-10\right)+(75)\left(700-\frac{x^2}{9}-100-\frac{20x}{3}\right)\right]$$

$$= \frac{1}{6875(2)}\left[5000-\frac{250x}{3}-2500+15000-\frac{25x^2}{3}-7500-\frac{1500x}{3}\right]$$

$$= \frac{1}{6875(2)}\left[10000-\frac{1750x}{3}-\frac{25x^2}{3}\right]$$

Ques 6.

$M(x)t = e^{\lambda(e^t-1)}$ $M(y)t = e^{u(e^t-1)}$

$M(x-y)t = E\left(e^{t(X-Y)}\right)$

$= E\left(e^{Xt-Yt}\right) = \frac{E(e^{Xt})}{E(e^{Yt})} = \frac{e^{\lambda(e^t-1)}}{e^{u(e^t-1)}}$

$= e^{(\lambda-u)(e^t-1)}$

$\therefore X-Y \sim Poi(\lambda-u)$

$\therefore$ Hence prooved

$M_{X+Y}(t) = E\left(e^{t(X+Y)}\right) = E\left[e^{tX+tY}\right]$

$= E(e^{tX}) \cdot E(e^{Yt})$

$= e^{\lambda(e^t-1)} \cdot e^{u(e^t-1)}$

$= e^{(\lambda+u)(e^t-1)}$

$X+Y \sim Poi(\lambda+u)$

$\therefore$ Hence prooved.

Ques 7:

$$Var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$$

$$= \sum_x x^2 \frac{P(X=x, Y=2)}{P(Y=2)} - \left(\sum_x x \frac{P(X=x, Y=2)}{P(Y=2)}\right)^2$$

$$= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.2)}{0.6} - \left[\frac{1(0) + 2(0.3) + 3(0.1) + 4(0.2)}{0.6}\right]^2$$

$$= \frac{53}{6} - \frac{289}{36} = \frac{29}{36}$$

(ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$, $0 < u < 1$, $\frac{u}{2} < v < 2$

$$E(U/V=v) = \int_0^1 u\, f(u/v=v)\, du$$

$$= \int_0^1 u \frac{f(u, V=v)}{f(V=v)}\, du \quad — (1)$$

$$\Rightarrow f_V(V=v) = \int_0^1 \frac{48}{67}(2uv - u^2)\, du$$

$$= \frac{48}{67}\left[\frac{2u^2v}{2} - \frac{u^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left[v - \frac{1}{3}\right]$$

$$= \frac{48}{67} \times \frac{[3v-1]}{3} = \frac{16}{67}(3v-1)$$

$$E\left(U/V=v\right) = \int_0^1 \frac{u\left(\cancel{\frac{48}{67}}^{\,3}\right)\left(2uv-u^2\right)}{\cancel{\frac{16}{67}}\left(3v-1\right)}\, du$$

$$= \frac{3}{3v-1}\int_0^1 u\left(2uv-u^2\right) du$$

$$= \frac{3}{3v-1}\int_0^1 2u^2v - u^3 \, du$$

$$= \frac{3}{3v-1}\left[\frac{2u^3v}{3} - \frac{u^4}{4}\right]_0^1$$

$$= \frac{3}{3v-1}\left[\frac{2v}{3} - \frac{1}{4}\right]$$

$$\boxed{E\left(U/V=v\right) = \frac{8v-3}{4(3v-1)}}$$

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Ques 8. $X \sim \text{Gamma}(3, 2)$

$E(Y/X=x) = 3x+1$ $\quad$ $Var(Y/X=x) = 2x^2+5$

$E(E(Y/X=x)) = E(Y)$ $\quad$ $Var(Var(Y/X=x)) = V(Y)$

$E(Y) = E(3x+1)$ $\quad$ $V(Y) = E(Var(Y/X=x)) + V(E(Y/X=x))$

$= 3E(x)+1$ $\quad$ $= E(2x^2+5) + V(3x+1)$

$= 3\left(\frac{3}{2}\right)+1$ $\quad$ $= 2E(x^2)+5+9V(x)$

$= \frac{11}{2}$ $\quad$ $= 2(3)+5+9\left(\frac{3}{4}\right)$

$= 6+5+6.75$

$= 17.75$

Ques 9. $Z = X+Y$

$E(X) = 3$ $\quad$ $E(Y) = 4$

$\sqrt{V(X)} = 2$ $\quad$ $\sqrt{V(Y)} = 1$

$cov(X, Y) = -0.3$

(i) $cov(X, Z) = cov(X, X+Y)$

$= cov(X, X) + cov(X, Y)$

$= V(X) + (-0.6)$

$= 4 - 0.6$

$= 3.4$

(ii) $Var(Z) = Var(X+Y)$

$= V(X) + V(Y) + 2\,cov(X, Y)$

$= 4 + 1 + 2(-0.6)$

$= 5 - 1.2$

$= 3.8$

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Q10. $f(x,y) = K x^{-\alpha} e^{-y/\beta}$, $1 < x < \infty$

$1 < y < \infty$

$\alpha > 1$, $\beta > 0$

$$1 = K \int_1^\infty x^{-\alpha} dx \int_1^\infty e^{-y/\beta} dy$$

$$1 = K \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^\infty \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^\infty$$

$$1 = K \left[0 - \frac{1}{-\alpha+1} \right] \left[0 + \beta (e^{-1/\beta}) \right]$$

$$1 = K \left[\frac{-1}{\alpha+1} \right] \left[-\beta e^{-1/\beta} \right]$$

$$1 = K \left[\frac{-1+\alpha}{\beta e^{-1/\beta}} \right] = \frac{\alpha - 1}{\beta e^{-1/\beta}} = K$$