Calculus Assignment   Roll No-35

Q1) $\int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$ $= \int_{1/50}^{2} \frac{t\omega^2 dt}{-2\omega^2 e^{2}/\omega}$

Let $t = e^{2}/\omega$
diff wrt ω
$\frac{dt}{dx} = -2e^{2/\omega} \frac{1}{\omega^2}$

$= -\frac{1}{2}\int_{1/50}^{2} dt$

$= -\frac{1}{2}[x]_{1/50}^{2} = -\frac{1}{2}(2-0.02)$

$= -0.99$

Q2) $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

Let $x = (t^4+2t)^{-3}$
diff wrt t
$\frac{dx}{dt} = 3(t^4+2t)^2 (4t^3+2)$

$= \int \frac{2t^3+1}{x(3)(t^4+2t)^2(4t^3+2)}$

multiply and divide by 2

$= \int \frac{\cancel{4t^3+2}}{(x)(6)\cancel{(4t^3+2)}(x)^{2/3}}$

$= \frac{1}{6}\int x^{-5/3} dx = \frac{1}{6} \frac{x^{-5/3+1}}{-5/3+1}$

$= -\frac{1}{4}(t^4+2t)^{-2} + c$

Q3) $f'(x) = x^4 + 3x - 9$ $f(x) = \int f'(x) dx$

$f(x) = \int (x^4 + 3x - 9) dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$[x^3]_{-2}^{1} + [6x]_{1}^{3}$$

$$1 + 8 + 18 - 6 = 21$$

Q5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$

$$\int \frac{3\,\cancel{(8y-1)}\,\cancel{(e^{4y^2-y})}}{\cancel{(e^{4y^2-y})}\,\cancel{(8y-1)}}\,dx$$

$x = e^{4y^2-y}$

diff wrt x

$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$

$$= 3x + c$$

$$= 3(e^{4y^2-y}) + c$$

Q6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$$f'(x) = \int 15\sqrt{x} + 5x^3 + 6\,dx$$

$$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k$$

$$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k\,dx$$

$$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + c$$

$$f(1) = -5/4 = 4 + \frac{1}{4} + 3 + k + c$$

$$k + c = -8.5 \leftarrow (i)$$

$f(4) \Rightarrow 404 = 4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + 4k + c$

$\Rightarrow 4k + c = 164 \quad - (ii)$

$(ii) - (i)$

$$\begin{array}{r} 4k + \cancel{c} = 164 \\ -1k + \cancel{c} = -8.5 \\ \hline 3k = 172.5 \end{array}$$

$k = 57.5$

$c = -66$

$\therefore f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$

Q9) $\frac{dv}{dt} = 9.8 - 0.196v$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$\log(9.8 - 0.196v) = -0.196t + c$

Q12) Taylor series for $f(x) = x^3 - 10x^2 + 6 \quad x = 3$

$f(x) = x^3 - 10x^2 + 6 \qquad f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6$

$$f(x) = f(3) + (x-3)f'(x) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3)$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^3(-\cancel{2})}{\cancel{2}} + \cancel{6}\frac{(x-3)^3}{3\times\cancel{2}} \ldots$$

Q13) Maclaurin series for $(1+x)^{\mu}$

$$f'(x) = \mu(1+x)^{\mu-1}$$
$$f''(x) = (\mu)(\mu-1)(1+x)^{\mu-2}$$
$$f'''(x) = (\mu)(\mu-1)(\mu-2)(1+x)^{\mu-3}$$

$$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots$$

$$(1+x)^{\mu} = 1 + \mu + (\mu-1)(\mu) + \frac{(\mu-1)(\mu-2)}{2} \ldots$$

Q14) $f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2} \qquad f'(0) = 1/2$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \qquad f'(0) = -1/4$$

$$f'''(x) = -\frac{3}{8}(1+x)^{-5/2} \qquad f''(0) = -3/8$$

$$\sqrt{1+x} = \sqrt{1+x} + \frac{x^1}{1!}\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{-3}{8}\right) \ldots$$

Q15) Taylor series

$$f(x) = e^{-6x} \qquad x = -4$$

$$f(x) = e^{-6x} \qquad f(-4) = e^{+24}$$

$$f'(x) = e^{6x}(-6) \qquad f'(4) = e^{24}(6)$$

$$f''(x) = (-6)e^{-6x}(-6) \qquad f''(-4) = e^{24}(6)^2$$

$$f'''(x) = (-6)^3 e^{-6x} \qquad f'''(-4) = e^{24}(6)^3$$

$$e^{-6x} = e^{+24} + (x-4)(e^{24})(6) + \frac{(x-4)^2(e^{24})(6)^2}{2!} + \frac{(x-4)^3(e^{24})(6)^3}{3!}$$

Q16) $f(x) = \ln(3+4x)$ $x = 0$

$f'(x) = \dfrac{4}{3+4x}$ $f'(0) = 4/3$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $f''(0) = (-1)(4)^2(3)^{-2}$

$f'''(x) = (+8)(4x+3)^{-3}(4)^2$ · $f'''(0) = (-1)(-2)(3)^{-2}(4)^3$

$\ln(3+4x) = \ln(3) + \dfrac{x\,4/3}{1!} + \dfrac{x^2(-1)(4)^2(3)^{-2}}{2!} + \dfrac{x^3(4)^3(2)(3)^{-2}}{3!}$