Probability - I

1. (ii)

No. of households	No. of people	$x_i f_i$
0	5	0
1	11	11
2	26	52
3	15	45
4	3	12
		120

Mean $= \frac{\Sigma x_i f_i}{\Sigma x_i} = \frac{120}{60} = 2$

Median $= \frac{N+1}{2} = \frac{60+1}{2} = 30.5 = 2$

Mode $= 2$

Interquartile range $= Q3 - Q1$

$Q1 = 1\left(\frac{N+1}{4}\right) = 15.25$ th $= 1$

$Q3 = 3\left(\frac{N+1}{4}\right) = 3$

$Q = 2$

Standard deviation =

2.

(i) Poisson ($\lambda = 0.5$)

[No. of claims per year]

(i) $P(X=0) = \frac{e^{-0.5} \times 0.5^0}{0!}$

$= e^{-0.5}$

$P(X=0) = 0.60653$

(ii) $3[P(X=0) \cdot P(X=0) \cdot P(X\geq 1)]$

This function would give the probability of one or more claim in one year.

$3[(P(X=0))^2 \cdot P(X\geq 1)]$

$3[(0.60653)^2 \cdot 1 - P(X\leq 0)]$

$3[(0.60653)^2 \cdot (0.39347)]$

$= 0.434351$

(iii)

5. Given that,

$L_1 = 40\% = 0.4$
$L2 = 50\% = 0.5$
$L_3 = 10\% = 0.1$

Now the probability of package getting late delivered through each level can be shown as

$L|L_1 = 20\% = 0.2$
$L|L_2 = 40\% = 0.4$
$L|L3 = 10\% = 0.1$

$$P(L_2|L) = \frac{P(L|L_2) \times P(L_2)}{P(L_1).P(L|L_1) + P(L_2).P(L|L_2) + P(L_3).P(L|L_3)}$$

$$= \frac{40\% \times 50\%}{40\%.20\% + 40\%.50\% + 10\%.10\%}$$

$$= \frac{0.4 \times 0.5}{0.4 \times 0.2 + 0.4 \times 0.5 + 0.1 \times 0.1}$$

$$= \frac{0.2}{0.29} = 0.6896551\overline{7}$$

classmate

Date

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7.

X	$P(X=x)$	$E(x^2)$
1	0.05	0.0025
2	0.15	0.0225
3	0.35	0.4287
4	0.4	0.16
5	0.05	0.0025

(i) $F(3) = P(x=0) + P(X=1) + P(x=2)$

$= 0.05 + 0.15 + 0.35$

$= \underline{\underline{0.55}}$

(ii) $E(x) = \sum_{1}^{5} x\, P(x=x)$

$= 0.05 + 0.15 + 0.35 + 0.4 + 0.05$

$= 3.25$

(iii) $V(x) = E(x^2) - (E[x])^2$

$= 11.45 - [3.25]^2$

$= \underline{\underline{0.8875}}$

(iv) Coefficient of skewness $= \dfrac{3(\text{Mean}-\text{Median})}{S.D}$

$= \dfrac{3(3.25-3)}{\sqrt{0.8875}} = \dfrac{0.75}{\sqrt{0.8875}} = \underline{\underline{0.796117}}$

$S.D = \sqrt{0.8875}$

10. Given that

$\lambda = 2$ (Per 5 days)
So $\lambda = 2/5$

$X \approx Pois(\lambda = 2/5)$

$P(X>2) = 1 - P(X \leq 2)$
$= 1 - 0.99207$
$= 0.00793$

11. Here, the probability of her next three children being boys would be independent of the condition given.

Probability of having a boy is $\frac{1}{2}$

Hence,
$P(\text{Next three boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

12. Here, $X \sim Poisson(10)$
Now waiting time between two claims will be
$X \sim exp$ where $\lambda = 10$

$P(X<0.1) = 1 - e^{-\lambda x}$
$= 1 - e^{-10.0.1}$
$= 1 - e^{-1} = 0.6321$

1.

(i) 4, 7, 9, 9, 11, 13, 18, 18, 18, 25

Here, Mean $= \frac{\Sigma x}{x} = \frac{134}{10}$

$= 13.4$

Median $= \left(\frac{n+1}{2}\right) = 5.5^{th}$ term

$= \cancel{14}\ 13$

(ii) Standard deviation $= \sqrt{\frac{(\Sigma x^2) - \Sigma(x^2)}{n-1}}$

$= 6.45$