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709

SRM - Assignment

classmate
Date
Page

Q1.

Policy year	DY 0	1	2	3
2001	1245	999	610	356
2002	1455	1088	723	
2003	1567	1079		
2004	1491			

inflation from middle to 12 mo:

2002 = 2.10%

2003 = 1.20%

2004 = -0.80%

Ann. prem in 2004 = 5250,000

future inflation (mid-2004) = 2.5%

loss ratio = 75%

(i.) CLM

Inflation adjustment

2001 ⇒ $(1.021) \times (1.012) \times (0.992) = 1.02498$

2002 ⇒ $(1.012) \times (0.992) = 1.003904$

2003 ⇒ 0.992

2004 ⇒ 1

Incremental mid 2004 Prices

AY	DY 0	1	2	3
2001	1276.11	1002.9	605.15	356
2002	1460.68	1079.30	723	
2003	1554.46	1079		
2004	1491			

cumulative claims mid 2004 prices

	DY			
AY	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	
2003	1554.46	2633.46		
2004	1491			

df = 1.73666 1.27566 1.12343

	DY			
AY	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	3665.74
2003	1554.46	2633.46	3359.245	3773.89
2004	1491	2589.36	3302.99	3710.69

incremental table ->
(only 2004)

	DY			
AY	0	1	2	3
2004	1491	1098.36	712.64	407.7
	x (1)	x (1.025)	x (1.025^2)	x $(1.025)^3$
	= 1491	1125.819	748.72	439.05

∴ Total claims o/s = 2813.5875 (in '000)

ii. **B-F**

~~Ult amt~~ =

Initial ult liab = 5,250,000 × 0.75

= 3937500

AY	0	1	2	3
		1.73666	1.27566	
dev. factors (r) =>	–	~~1.28885~~	1.~~275601~~	1.12343
(f) cumulative d.f =>	–	2.48871	1.433048	1.12343
	[0]	[1]	[2]	[3]
$1-\frac{1}{f}$ =>	–	0.10987	0.302186	0.598185

~~EL~~ =

AY	0	1	2	3
2001				3240.13
2002			3262.98	
2003		2633.46		
2004	1491			

	DY
EL => $3937.5 \times \left(\frac{1}{1.43304} - \frac{1}{2.4887}\right) = 1165.51$	1
$3937.5 \times (1/1.12343 - 1/1.43304) = 752.24$	2
$3937.5 \times \left(\frac{1}{} - \frac{1}{1.2343}\right) = 432.612$	3

inf. adjusted = 1194.65

795.57

465.87

∴ Total emerging liab = 2456.09 (in '000)

Q2.

U/W year	Incurred 1	2	3 ← DY
2005	3512	7210	9563
2006	2889	6723	
2007	3876		

Loss Ratio = 87 %

Premium income (EP)

2005	11,041
2006	11,314
2007	12,549

r	-	2.1767	1.3264	1
f		2.8872	1.3264	1
$1-1/f$		0.6536	0.24688	0

Claims paid to date = 20,103,000

AY	2007	2006	2005
E.P	12549	11314	11041
IUL	10917.63	9843.18	9605.67
EL $(IUL \times (1-\frac{1}{f}))$	7135.76	2422.21	0
RL	3876	6723	9563
UL (RL+EL)	11005.76	9145.21	9563

Total UL = 29713.97 ×1000

∴ reserves = 29713970 − 20103000

= $\boxed{9610970}$

Q4. Given:

(i.) no. of claims - cumulative.

	DY		
AY	0	1	2
2005	82	121	156
2006	128	~~[illegible]~~ 163	
2007	98	~~163~~	

(ii) Total claim amounts

	DY		
AY	0	1	2
2005	43512	87210	129563
2006	62889	116723	
2007	73876		

soln:

ACPC method:

ACPC table:

	DY			
AY	0	1	2	ultimate
2005	530.634 (63.89%)	720.744 (86.781%)	830.532 (100%)	→ 830.532
2006	491.320 ([illegible])	716.092 (86.781%)	→	825.1714
2007	753.836 ([illegible])	→	→	1221.4595

Ultimate for claim numbers:

AY	DY 0	1	2	Ultimate
2005	82 (52.566%)	121 (77.564%)	156 (100%) →	156
2006	128 (60.909%)	163 (77.584%)	→	210.149
2007	98 (56.738%)		→	172.7237

Projected ult. loss.

Acc year	ACPC × no. of claims	= Projected
2006	825.1714 × 210.149	= 173,408.9445
2007	1221.4595 × 172.7237	= 210,975.0042
already developed ← (2005)	830.532 × 156	= 129562.99

~~Total = 384023.9487~~

~~outstanding claims = 384023.9487 − [73876 + 116723 + 129563]~~

~~outstanding claims = $\boxed{63,861.9487}$~~

Total = 173408.9445
+ 210925.0042
129562.99

513946.9407

assuming given data is incurred

∴ O/s claims = 513946.9407 − [73876
+116729
+129563]

= $\boxed{193,784.9384}$

Q5.

i. TSC: $\lambda = \dfrac{22}{13500 + 17000/k + 4M + 6M/k}$

$$L = c e^{-4\lambda M} \times \prod_i (\lambda e^{-\lambda x_i}) \cdot e^{-6\lambda M/k} \cdot \prod_j \left(\frac{\lambda}{k} e^{-\lambda y_i/k}\right)$$

$$\log L = c' - 4\lambda M + 10\log\lambda - \lambda \sum x_i - 6\lambda M/k + 12\log\lambda - \lambda/k \sum y_i$$

$$= c' - 4\lambda M + 22\log\lambda - 13500\lambda - 6\lambda M/k - 17000\lambda/k$$

$$\log L' = -4M + \frac{22}{\lambda} - 13500 - 6M/k - 17000/k = 0$$

$$\therefore \hat{\lambda} = \frac{22}{13500 + \frac{17000}{k} + 4M + \frac{6M}{k}} \qquad \text{also } \log L'' = \frac{-22}{\lambda^2} < 0$$

h/P

iii. (a.) $\hat{\lambda} = \dfrac{22}{13500 + \frac{17000}{1.1} + 4\times1600 + \frac{6\times1600}{1.1}} = \boxed{0.000499}$

(b.) $$\int_0^M x \cdot \lambda e^{-\lambda x} dx + M \int_M^\infty \lambda e^{-\lambda x} dx = \left[-xe^{-\lambda x}\right]_0^M + \int_0^M e^{-\lambda x} dx + M\left[e^{-\lambda x}\right]_M^\infty$$

$= \frac{1}{\lambda}(1-e^{-\lambda M})$

$= \frac{1}{0.000499}\left(1-e^{-0.000499 \times 1600}\right)$

$\therefore \cancel{1120}\ 1102.107 = \int_0^R x.\mu e^{-\mu x} dx + R\int_R^\infty \mu e^{-\mu x} dx$

$= \frac{1}{\mu}(1-e^{-\mu R})$

$= \frac{1.05 \times 1.1}{0.0004999}\left(1-e^{-0.000499R/1.05 \times 1.1}\right)$

solve for R

$\therefore R = \underline{\underline{1496.52}}$

Q6.

i. Heterogeneous group, parameters vary by pattern not overall level.

ii. (a) $E(\lambda_i) = 0.5 \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{6} \times 0.9$

$= \underline{\underline{0.3}}$

$V(\lambda_i) = 0.17 - 0.3^2$

$= \underline{\underline{0.08}}$

(b.) $E(S_i|\lambda_i) = [\lambda_i m_1]$ & $Var(S_i|\lambda_i) = \lambda_i m_2$

(c.) $E(S_i) = E[E(S_i|\lambda_i)] = E(\lambda_i|m_1]$

$= 0.3m_1$

$= 0.3 \times 250$

$= \underline{\underline{75}}$

iii.

(d.) For the whole portfolio, variables are iid

$\therefore$ mean $= 100 \times 75$
$= 7500$

var $= 100 \times 23810$
$= 2381000.$

iii. Homogenous

$V(\lambda_i) = 0.5\ (0.2^2 + 0.4^2) - 0.3^2$
$= 0.01$

$E\left(\sum_{i=1}^{n} S_i\right) = n\,E(S_i) = 0.3 \times 100 \times 250$
$= 7500$

$V\left[\sum_{i=1}^{n} S_i\right] = E\left(var\left(\sum_{i=1}^{n} S_i|\lambda\right)\right) + var\left(E\left(\sum_{i=1}^{n} S_i|\lambda\right)\right)$

$= 0.3 \times 100 \times 200 + 0.01 \times 100^2 \times 250^2$
$= 6256000$

iv. mean is same but variance is much higher. This is because parameter uncertainty is at overall level so it affects agg. claim variance much more.

Q7. N : No. of claims

$N \sim Poi\ (50)$

X : Ind. claims

Claim amt => $100X$

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) & , 1 < x < 5 \\ 0 & , ow \end{cases}$$

(80% chance of additional 200)

i. $E(100X)$

$= 100\, E(X)$

$$= 100 \cdot \int_1^5 x \cdot \frac{3}{32}(6x - x^2 - 5)\, dx$$

$$100 \times \frac{3}{32} \int_1^5 6x^2 - x^3 - 5x\ dx$$

$$= \frac{300}{32}\left[\frac{\cancel{6}^{2}x^3}{\cancel{3}} - \frac{x^4}{4} - \frac{5x^2}{2}\right]_1^5$$

$$\frac{300}{\cancel{4}\times 32}\left[2(5)^3 - \frac{(5)^4}{4} - \frac{5(5)^2}{2} - 2 + \frac{1}{4} + \frac{5}{\cancel{4}}\right]$$

$$= \frac{300}{32}\left(250 - 156.25 - 62.5 - \overset{0.5}{\cancel{4}}\right)$$

$=$

$$\frac{300}{32}\left(2x^3 - \frac{x^4}{4} - \frac{5}{2}x^2\right)_1^5 = \frac{300}{32} \times 32$$

$$= \boxed{300}$$

ii. $V(100x)$

$= 100^2 \, V(x)$

$= \cancel{100^2 \cdot E(x^2)} - (\overset{100}{E(x)})^2$

$\therefore E(100x^2)$

$\therefore E(100x^2) = 100^2 \, E(x^2)$

$$= 100^2 \times \frac{3}{32} \int_1^5 x^2 \cdot (6x - x^2 - 5) \, dx$$

$$\frac{100^2 \times 3}{32} \times \int_1^5 6x^3 - x^4 - 5x^2 \, dx$$

$$= \frac{100^2 \times 3}{32} \left[\frac{6x^4}{4} - \frac{x^5}{5} - \frac{5x^3}{3} \right]_1^5$$

$$= \frac{100^2 \times 3}{32} \left[\frac{625}{6} + \frac{11}{30} \right]$$

$= 98000$

$\therefore V(100x) = 98000 - 300^2$

$= \boxed{8000}$

ii. Y = repair amount

$E(Y) = 0.3 \times 200$

$= 60$

$V(Y) = 0.3 \times 200^2 - 60^2$

$= 8400$

let Z = amt + repair

$$E(Z) = E(X+Y)$$
$$= E(X) + E(Y)$$
$$= 300 + 60$$
$$= 360$$

$$V(Z) = V(X+Y)$$
$$= V(X) + V(Y)$$
$$= 8000 + 8400$$
$$= 16400$$

$$E(Z) = E(N) \cdot E(Z)$$
$$= 50 \times 360$$
$$= 18000$$

$$V(Z) = 50 \times 16400 + 50 \times 360^2$$
$$= 7300000.$$