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Numerical Methods & Linear Algebra — Practice Questions : Unit 3

1
Ans $4x + 15 < 23$

$4x < 8$

$\therefore x < \frac{8}{4}$

$\therefore x < 2$

(A) $x < 2$

2
Ans. $x^2 - 3 > 25$

$x^2 > 28$

$\therefore x > \sqrt{28}$ or $x < -\sqrt{28}$

But, we need to solve for $x > 0$,

$\therefore x > \sqrt{28}$

$x \in (\sqrt{28}, \infty)$

(B) $(\sqrt{28}, \infty)$

3.
Ans Given that : $-7 \leq x \leq 6$ and $-5 \leq y \leq 8$

Thus, $xy = (-7)(8) = -56$

and $xy = (6)(8) = 48$

(C) Maximum : 48, Minimum : -56

4.

Ans $5y + 3 \geq -7y + 13$

$5y + 7y \geq 13 - 3$

$\therefore 12y \geq 10$

$\therefore y \geq \frac{10}{12}$

Ans (B) $y \geq \frac{10}{12}$

5.

Ans Given that $|r| < 1$

$\therefore -1 < r < 1$

$\therefore 1 > -r > -1$

$\therefore -1 < -r < 1$ — (Rearranging)

$\therefore 0 < (1-r) < 2$

$\therefore 0 < 5(1-r) < 10$

$\therefore 0 < a < 10$

(c) $0 < a < 10$.

6.

Ans Given equation : $x^2 - 6x + 12 = 0$
Comparing the above equation with $ax^2 + bx + c = 0$,
we get, $a = 1$, $b = -6$, $c = 12$

$b^2 - 4ac = (-6)^2 - 4(1)(12)$

$b^2 - 4ac = -12$

$\because$ $b^2 - 4ac < 0$, the roots are not ~~con~~ real

i.e. complex

(C) Complex

7.

Ans Given equation : $5x^2 - 10x + 20 = 0$

Comparing the above equation with $ax^2 + bx + c = 0$,

we get, $a = 5$, $b = -10$, $c = 20$

$\therefore$ $b^2 - 4ac = (-10)^2 - 4(5)(20)$

$b^2 - 4ac = -300$

$\because$ $b^2 - 4ac < 0$, the roots are complex.

(C) Complex.

8.

Ans A quadratic equation is of the form $ax^2 + bx + c = 0$

where $a \neq 0$

(D) $x^2 - 4 = 0$

9.

Ans. Given polynomial equation $(x-1)(x-2) = x(x+1)$

$\therefore$ $x^2 - 2x - x + 2 = x^2 + x$

$-4x + 2 = 0$

$\therefore$ $4x - 2 = 0$.

(B) Linear equation

10.

Ans. If α and β are the roots of a quadratic equation, then the equation is given as,

$$(x-\alpha)(x-\beta)=0$$

Roots are given as $\alpha = 1$ and $\beta = -3$

$$\therefore (x-1)(x+3)=0$$
$$\therefore x^2+3x-x-3=0$$
$$\therefore x^2+2x-3=0$$

(D) $x^2+2x-3=0$

11

Ans. Given inequality,

$$|2x-5| > |x+1|$$

Considering 4 cases for the same.

	$\lvert 2x-5\rvert$	$\lvert x+1\rvert$
Case I:	$(2x-5)$	$(x+1)$
Case II:	$-(2x-5)$	$(x+1)$
Case III:	$(2x-5)$	$-(x+1)$
Case IV:	$-(2x-5)$	$-(x+1)$

Case I:

$|2x-5| = (2x-5)$ $\quad\quad$ $|x+1| = (x+1)$

$\therefore (2x-5) > 0$ $\quad\quad$ $(x+1) > 0$

$\therefore \quad 2x > 5 \qquad\qquad \therefore \quad x > -1$

$\therefore \quad x > \frac{5}{2}$

$\therefore \quad x > 2.5$

Considering the equation,

$(2x-5) > (x+1)$

$\therefore \quad 2x - x > 1 + 5$

$\therefore \quad \underline{x > 6}$

Case II :

$|2x-5| = -(2x-5) \qquad\qquad |x+1| = (x+1)$

$\therefore \quad 2x - 5 < 0 \qquad\qquad \therefore \quad (x+1) > 0$

$\therefore \quad 2x < 5 \qquad\qquad \therefore \quad x > -1$

$\therefore \quad x < 2.5$

Considering the equation,

$-(2x-5) > (x+1)$

$-2x + 5 > x + 1$

$5 - 1 > 2x + x$

$4 > 3x$

$\frac{4}{3} > x$

$\therefore \quad x < \frac{4}{3}$

$x < 1.3333$

$\therefore \quad \underline{x \in (-1, 1.3333)}$

Case III:

$|2x-5| = (2x-5)$

$(2x-5) > 0$

$2x > 5$

$x > 2.5$

$|x+1| = -(x+1)$

$(x+1) < 0$

$x < -1$

There is no common intersection

Thus, solution is not possible for this case

Case IV:

$|2x-5| = -(2x-5)$

$(2x-5) < 0$

$2x < 5$

$x < 2.5$

$|x+1| = -(x+1)$

$(x+1) < 0$

$x < -1$

Considering the equation,

$$-(2x-5) > -(x+1)$$
$$(2x-5) < (x+1)$$
$$2x - x < 6$$
$$x < 6$$

common intersection is,

$\therefore$ $x < -1$.

12

Ans $f(x) = 2|3x+7|$

$$f(x) = \begin{cases} 2(3x+7) & \text{for } (3x+7) \geq 0 \\ -2(3x+7) & \text{for } (3x+7) < 0 \end{cases}$$

When

$3x + 7 \geq 0$

$3x \geq -7$

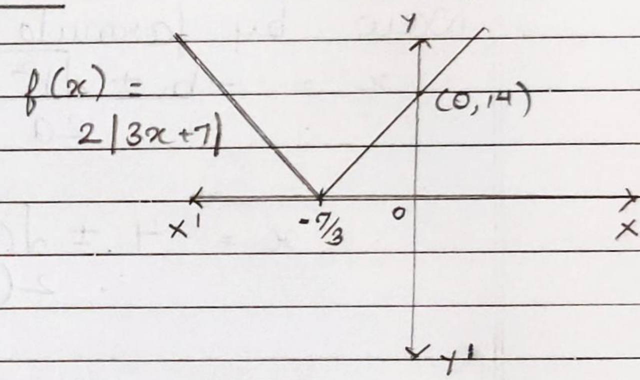
$x \geq -\frac{7}{3}$

$3x + 7 < 0$

$3x < -7$

$x < -\frac{7}{3}$

$$\therefore f(x) = \begin{cases} 6x + 14 & x \geq (-7/3) \\ -6x - 14 & x < (-7/3) \end{cases}$$



13. Ans Given equation:

$4x^2 - 4x - 3 = 0$

By factorization method,

$4x^2 - 4x - 3 = 0$

$4x^2 - 6x + 2x - 3 = 0$

$\therefore 2x(2x^2 - 3) + 1(2x - 3) = 0$

$(2x-3)(2x+1) = 0$

$\therefore x = \frac{3}{2}$ or $x = \frac{-1}{2}$

By formula method,

$4x^2 - 4x - 3 = 0$

comparing the above equation with $ax^2 + bx + c = 0$ we get,

$a = 4$, $b = -4$ and $c = -3$.

Now, by formula method,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)}$

$\therefore x = \frac{3}{2}$ or $x = \frac{-1}{2}$.

By completing squares method,

$4x^2 - 4x - 3 = 0$

$x^2 - x - \frac{3}{4} = 0$

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$$= \left(\frac{1}{2} \times (-1)\right)^2$$

Third term $= \frac{1}{4}$

$$x^2 - x + \frac{1}{4} - \frac{3}{4} - \frac{1}{4} = 0$$

$$\left(x - \frac{1}{2}\right)^2 - (1)^2 = 0$$

$$\left(x - \frac{1}{2} + 1\right)\left(x - \frac{1}{2} - 1\right) = 0$$

$$\left(x + \frac{1}{2}\right)\left(x - \frac{3}{2}\right) = 0$$

$\therefore$ $x = \frac{3}{2}$ Or $x = -\frac{1}{2}$.

14

Ans. Let the two numbers be x and y.

According to given conditions,

$xy = 91$ — ①

$|x - y| = 6$

$\therefore$ $(x - y) = 6$ Or $-(x - y) = 6$

$\therefore \quad x = 6 + y \quad — ②$

$\therefore$ From ① and ②

$$(6+y)y = 91$$
$$6y + y^2 = 91$$
$$\therefore y^2 + 6y - 91 = 0$$
$$\therefore y^2 + 13y - 7y - 91 = 0$$
$$\therefore y(y+13) - 7(y+13) = 0$$
$$\therefore (y+13)(y-7) = 0$$
$$\therefore y = -13 \text{ or } y = 7$$

In ②,

$$x = 6 - 13 \text{ or } x = 6 + 7$$
$$\therefore x = -7 \qquad x = 13$$

$$-x + y = 6$$
$$\therefore y - 6 = x \quad — ③$$

From ① and ③,

$$(y-6)y = 91$$
$$y^2 - 6y - 91 = 0$$
$$y^2 - 13y + 7y - 91 = 0$$
$$y(y-13) + 7(y-13) = 0$$
$$(y-13)(y+7) = 0$$
$$\therefore y = 13 \text{ or } y = -7$$

In ②,

$$x = 13 - 6 \text{ or } x = -7 -$$
$$\therefore x = 7 \text{ or } x = -13$$

Thus, the numbers can either be −7 and −13 or 7 and 13.

15.
Ans. Given quadratic inequality,

$$x^2 - 6x - 16 \leq 0$$
$$\therefore x^2 - 8x + 2x - 16 \leq 0$$
$$\therefore x(x-8) + 2(x-8) \leq 0$$
$$(x-8)(x+2) \leq 0$$

Case I :

FOR EDUCATIONAL USE

$$x^2 - 6x - 16 \leq 0$$

Third term $= \left(\frac{1}{2} \times \text{coefficient of } x\right)^2$

$$= \left(\frac{1}{2} \times (-6)\right)^2$$

$$= 9$$

$\therefore$ $x^2 - 6x + 9 - 16 - 9 \leq 0$

$\therefore$ $(x-3)^2 - (5)^2 \leq 0$

$(-5)^2 \leq (x-3)^2 \leq (5)^2$

$-5 \leq (x-3) \leq 5$

$\underline{-2 \leq x \leq 8}$

16

Ans Given equation:

$$x^4 + 2x^2 - 3 = 0$$

Let $x^2 = m$

$\therefore$ $m^2 + 2m - 3 = 0$

$\therefore$ $m^2 + 3m - m - 3 = 0$

$\therefore$ $m(m+3) - 1(m+3) = 0$

$(m+3)(m-1) = 0$

$\therefore$ $m = -3$ or $m = +1$

$\therefore$ $x^2 = -3$ $x^2 = 1$

Since, x should be real,

$\therefore$ $\underline{x = \pm 1}$

17 (a)

Ans: $\frac{1}{x^2+2x-8} > 0$

This is possible if $x^2+2x-8>0$

$x^2+4x-2x-8>0$

$\therefore \cancel{x^2} - x(x+4)-2(x+4)>0$

$\therefore (x+4)(x-2)>0$

Case I:

$x+4>0$ and $x-2>0$

$x>-4$ $\quad$ $x>2$

$\underline{x>2}$

Case II:

$x+4<0$ and $x-2<0$

$x<-4$ $\quad$ $x<2$

$\underline{x<-4}$

17 (b)

Ans: Given inequality,

$-x^2-8x-12\leq 0$

$x^2+8x+12\geq 0$

$\therefore x^2+6x+2x+12\geq 0$

$x(x+6)+2(x+6)\geq 0$

$(x+6)(x+2)\geq 0$

Case I:

$(x+6)\geq 0$ and $(x+2)\geq 0$

$x\geq -6$ $\quad$ $x\geq -2$

$\underline{x\geq -2}$

Case II:

$(x+6)\leq 0$ and $(x+2)\leq 0$

$x\leq -6$ $\quad$ $x\leq -2$

$\underline{x\leq -6}$

17. (c)

Ans. Given inequality,

$$x^2 + 6x - 7 \geq 0$$
$$x^2 + 7x - x - 7 \geq 0$$
$$x(x+7) - 1(x+7) \geq 0$$
$$(x+7)(x-1) \geq 0$$

Case I:

$(x+7) \geq 0$ and $(x-1) \geq 0$

$x \geq -7$ $\quad$ $x \geq 1$

$\underline{x \geq 1}$

Case II:

$(x+7) \leq 0$ and $(x-1) \leq 0$

$x \leq -7$ $\quad$ $x \leq 1$

$\underline{x \leq -7}$