Mucharla Harsha Vardhan
A, 49

Financial Mathematics Assignment-1

1) Given,

$d^{(4)} = 8\%$

(i) We know that,

$$1-d = e^{-\delta} = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\Rightarrow \quad e^{-\delta} = \left(1 - \frac{0.08}{4}\right)^4$$

$$\Rightarrow \quad \boxed{\delta = 8.08108\%}$$

(ii) We know that,

$$(1+i) = \frac{1}{1-d} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$\Rightarrow \quad (1+i) = \frac{1}{\left(1 - \frac{0.08}{4}\right)^4}$$

$$\Rightarrow \quad \boxed{i = 8.41658\%}$$

②

(iii) We know that,

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = 1-d = \left(1-\frac{d^{(4)}}{4}\right)^{4}$$

$$\Rightarrow \left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{0.08}{4}\right)^{4}$$

$$\Rightarrow \boxed{d^{(12)} = 8.05393\%}$$

2) Given,

$$\delta(t) = 0.004t + 0.0002t^2$$

(i) Here,

$$PV_0 = C\, e^{-\int_0^t \delta(t)\,dt} = 1000\, e^{-\int_0^{10}(0.004t + 0.0002t^2)\,dt}$$

$$= 1000 \cdot e^{-\left[0.004\left[\frac{t^2}{2}\right]_0^{10} + 0.0002\left[\frac{t^3}{3}\right]_0^{10}\right]}$$

$$= 1000 \cdot e^{-\left[0.004\left[\frac{100}{2}-0\right] + 0.0002\left[\frac{1000}{3}-0\right]\right]}$$

$$= 1000 \cdot e$$

$$\cancel{= 1000 \cdot e}$$

$$\cancel{\Rightarrow \boxed{PV_0 = 1305.605172}}$$

$$\Rightarrow \boxed{PV_0 = 765.9283384}$$

(ii) We know that,

$$i_n = \frac{A(0,n) - A(0,n-1)}{A(n-1)}$$

Then, $$i_{10} = \frac{1000 - 765.9283384}{765.9283384} = 30.5605172\%$$

4)

(ii) Given, $i = 8\%$

a) Then, we know that,

$$1+i = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$$

$$\Rightarrow 1 - \frac{d^{(12)}}{12} = (1.08)^{-\frac{1}{12}}$$

$$\Rightarrow \boxed{d^{(12)} = 7.671478\%}$$

(4)

b) We know that,

$$1+i = \left(1+\frac{i^{(365)}}{365}\right)^{365}$$

$$\Rightarrow 1+\frac{i^{(365)}}{365} = (1.08)^{\frac{1}{365}}$$

$$\Rightarrow \boxed{i^{(365)} = 7.696916\%}$$

c) We know that,

$$\delta = \log(1+i) = \log(1.08) = 7.696104\%$$

d) We know that,

$$1+i = \left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$\Rightarrow 1+\frac{i^{(0.5)}}{0.5} = (1.08)^{2}$$

$$\Rightarrow i^{(0.5)} = 8.32\%$$

(5)

5) (i) Given,

$$d^{(4)} = 6\%$$

a) We know that,

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$\Rightarrow \quad 1+\frac{i^{(2)}}{2} = (0.985)^{-\frac{4}{2}}$$

$$\Rightarrow \quad \boxed{i^{(2)} = 6.13775\%}$$

b) We know that,

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4}$$

$$\Rightarrow \quad 1-\frac{d^{(12)}}{12} = (0.985)^{\frac{4}{12}}$$

$$\Rightarrow \quad \boxed{d^{(12)} = 6.03025\%}$$

(ii) Interest rate for the amount borrowed for 1 year

$$i = \frac{220000 - 200000}{2,00,000} = 10\%$$

a) But amount is repaid on 17th July, 2005 i.e 94 days.

Lets assume that interest is reduced proportionally.

Then, effective interest rate for 94 days,

$$\frac{i^{\left(\frac{365}{94}\right)}}{\frac{365}{94}} = (1+i)^{\cancel{365}\ \frac{94}{365}} - 1$$

$$= 2.48494\%$$

$\therefore$ Sum paid by Amit to terminate the contract is,

$$AV_{\left(\frac{94}{365}\right)} = 2,00,000\left(1 + \frac{i^{\left(\frac{365}{94}\right)}}{\frac{365}{94}}\right)$$

$$= 2,00,000\,(1.0248494)$$

$$= 2,04,969.8719$$

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6) (ii)

$\cancel{d^{(12)} = \frac{1}{1+i}}$

$$d^{(12)} = 12\left[1 - (1+i)^{\frac{-1}{12}}\right] ;$$

$$i^{(2)} = 2\left[(1+i)^{\frac{1}{2}} - 1\right] ;$$

$$\delta = \log(1+i)$$

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7) Given,

$AV(0) = 20{,}000$; $AV\left(\frac{17}{12}\right) = 26{,}000$

$$i_{\frac{17}{12}} = \frac{26{,}000 - 20{,}000}{20{,}000} = 30\% = \frac{i^{\left(\frac{12}{17}\right)}}{\frac{12}{17}}$$

(i) We know that,

$$\left(1 + \frac{i^{(4)}}{4}\right)^4 = 1+i = \left(1 + \frac{i^{\left(\frac{12}{17}\right)}}{\frac{12}{17}}\right)^{\frac{12}{17}}$$

$$\Rightarrow \quad 1 + \frac{i^{(4)}}{4} = (1.3)^{\frac{3}{17}}$$

$$\Rightarrow \quad \boxed{i^{(4)} = 18.95525\%}$$

(ii) We know that,

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \frac{1}{1+i} = \frac{1}{\left(1 + \frac{i^{\left(\frac{12}{17}\right)}}{\frac{12}{17}}\right)^{\frac{12}{17}}}$$

⑨

$$\Rightarrow 1 - \frac{d^{(2)}}{2} = (1.3)^{\frac{-6}{17}}$$

$$\Rightarrow \boxed{d^{(2)} = 17.68824\%}$$

(iii) We know that, at $t=1$, Simple Interest = Compound interest = $1+i$.

And,

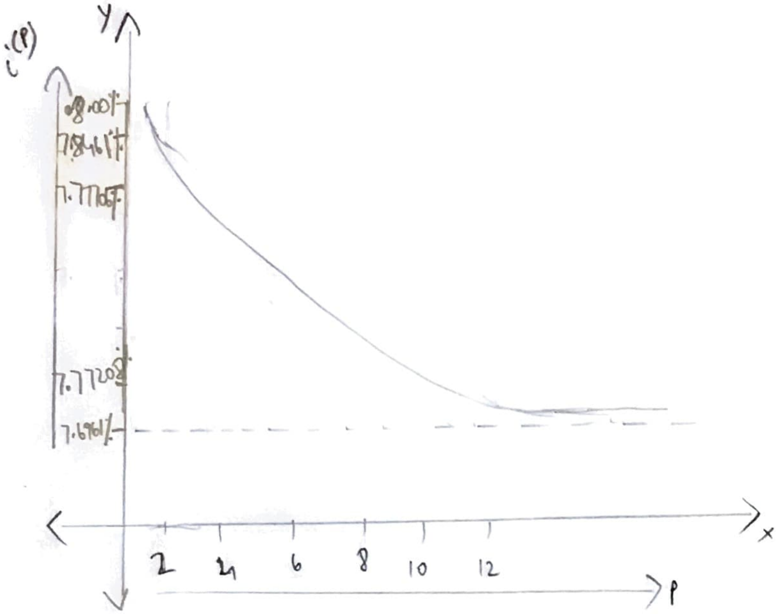
$$1+i = \left(1 + \frac{i^{(\frac{12}{17})}}{\frac{12}{17}}\right)^{\frac{12}{17}} = (1.3)^{\frac{12}{17}}$$

$$\Rightarrow \boxed{i = 20.3457\%}$$

(iv) From (i), (ii) and (iii), we observe,

$$i^{(\frac{12}{17})} > i > i^{(4)} > d^{(2)}$$

8)



for $i = 8\%$;

$i^{(2)} = 7.8461\%$

$i^{(4)} = 7.7706\%$

$i^{(12)} = 7.7208\%$

$\delta = 7.6961\%$

Q) (i) Force of Interest is defined as a nominal rate of interest compounded continuously (or infinitely) for a given time period. It is denoted by 'δ'.

(ii) Given

$$i^{(2)} = 7.75\%$$

Then,

$$i = \left(1+\frac{i^{(2)}}{2}\right)^{2} - 1 = (1.03875)^{2} - 1 = 7.90016\% \quad ;$$

$$\delta = \log(1+i) = \log(1.0790016) = 7.60367\% \quad ;$$

$$d^{(12)} = \cancel{\left(1+\frac{i^{(2)}}{2}\right)} \; 12\left[1-(1+i)^{-\frac{1}{12}}\right] = 12\left[1-(1.0790016)^{-\frac{1}{12}}\right]$$

$$= 7.57958\% \quad ;$$

$$i^{(\frac{1}{2})} = \frac{1}{2}\left[(1+i)^{2}-1\right] = \frac{1}{2}\left[(1.0790016)^{2}-1\right] = 8.21228\%$$

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10) Given,

$$\delta(t) = \begin{cases} 0.08 & ; 0 \le t \le 5 \\ 0.06 & ; 5 \le t \le 10 \\ 0.04 & ; t \ge 10 \end{cases}$$

$$v(0,5) = e^{-\int_0^5 \delta(t)\,dt} = e^{-\int_0^5 0.08\,dt} = e^{-0.08[t]_0^5} = e^{-0.08\times 5} = 0.67032$$

$$v(5,10) = e^{-\int_5^{10} \delta(t)\,dt} = e^{-\int_5^{10} 0.06\,dt} = e^{-0.06[t]_5^{10}} = e^{-0.06\times 5} = 0.74082$$

$$v(10,t) = e^{-\int_{10}^{t} \delta(t)\,dt} = e^{-\int_{10}^{t} 0.04\,dt} = e^{-0.04[t]_{10}^{t}} = e^{-0.04t+0.4}$$

So,

$$v(t) = \begin{cases} 0.67032 & ; 0 \le t \le 5 \\ 0.74082 & ; 5 \le t \le 10 \\ e^{-0.04t+0.4} & ; t \ge 10 \end{cases}$$

And,

$$PV_0 = v(0,5) \cdot v(5,10) \cdot v(10,t)$$

$$= (0.67032)(0.74082)(e^{-0.04t+0.4})$$

$$PV_0 = 0.74082 \cdot e^{-0.04t}$$

11) a) We know that,

$$1-d = \left(1 - \frac{d^{\left(\frac{12}{9}\right)}}{\frac{12}{9}}\right)^{\frac{12}{9}}$$

Given,

$$d^{\left(\frac{12}{9}\right)} = 18\% \text{ p.a} \quad , \quad C = 50,000 .$$

$$\therefore PV_0 = C \times \left(1 - \frac{d^{\left(\frac{12}{19}\right)}}{\frac{12}{19}}\right)$$

$$= 50,000 \times \left(1 - \frac{0.18}{\frac{12}{19}}\right)$$

$$= 35750$$

∴ Initial amount lent to borrower = ₹35,750/-

b)(i) Given, $\delta = 6\%$. We know that,

$$\delta = -\log(1-d) = -\log\left(\left(1 - \frac{d^{(12)}}{12}\right)^{12}\right)$$

$$\Rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta} = e^{-0.06}$$

$$\Rightarrow 1 - \frac{d^{(12)}}{12} = e^{-\frac{0.06}{12}}$$

$$\Rightarrow d^{(12)} = 5.98503\%$$

(ii) Given, $d^{(4)} = 6\%$. We know that,

$$\left(1-\frac{d^{(4)}}{4}\right)^{4} = \frac{1}{1+i} = \frac{1}{\left(1+\frac{i^{(12)}}{12}\right)^{12}}$$

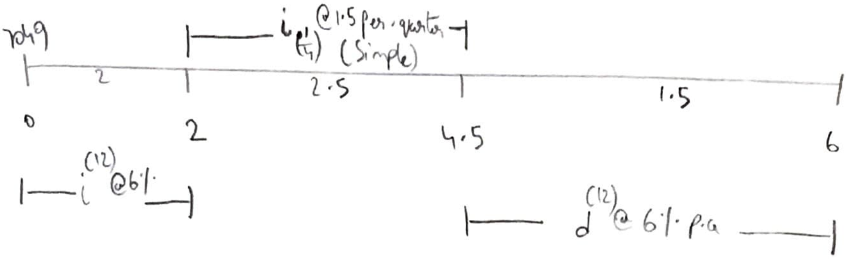
$$\Rightarrow \left(1-\frac{0.06}{4}\right)^{-\frac{4}{12}} = 1+\frac{i^{(12)}}{12}$$

$$\Rightarrow i^{(12)} = 6.0607\%$$

c) The order is,

$$i \cancel{\delta} > i^{(4)} > \delta > d$$

12) Given, $C = 7049$



$$AV = C\cdot\left(1+\frac{i^{(12)}}{12}\right)^{12\times 2}\cdot\left(1+10\times i_{(4)}'\right)\cdot\left(1-\frac{d^{(12)}}{12}\right)^{-(12+6)}$$

$$= 7049\left(1+\frac{0.06}{12}\right)^{24}\cdot\left(1+10\times 0.0015\right)\cdot\left(1-\frac{0.06}{12}\right)^{-18}$$

$$\Rightarrow AV = 9999.89361$$

13) $A(t) = 2$

i) For, $i = 10\%$ p.a,

$$(1+0.1)^n = 2$$

$$\Rightarrow n\log(1.1) = \log 2$$

$$\Rightarrow n = \frac{\log 2}{\log(1.1)} = 7.27254 \text{ years}$$

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(ii) At, $d^{(12)} = 10\%$ p.a.

$$\left(1 - \frac{d^{(12)}}{12}\right)^{-12n} = 2$$

$$\Rightarrow \left(1 - \frac{0.1}{12}\right)^{-12n} = 2$$

$$\Rightarrow -12n \log(0.991667) = \log 2$$

$$\Rightarrow n = \frac{\log 2}{-12 \cdot \log(0.991667)} = 6.902828 \text{ years}$$

3) (ii) To find, $d^{(2)}$

a) When, $\frac{d^{(4)}}{4} = 2.25\%$;

$$\left(1-\frac{d^{(2)}}{2}\right) = \left(1-\frac{d^{(4)}}{4}\right)^2 = (1-0.025)^2$$

$$\Rightarrow d^{(2)} = 9.875\%$$

b) When, $d^{(0.5)} = 5\%$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(0.5)}}{0.5}\right)^{0.5} = (0.9)^{\frac{1}{2}}$$

$$d^{(2)} = 2\left[1-(0.9)^{\frac{1}{4}}\right] = 5.19925\%$$

(iii) Given,

$i = 5\%$. Let $i' =$ Simple interest per day

We know that,

$$(1+i) = (1 + 365\, i')$$

$$\Rightarrow (1 + 365\, i') = 1.05$$

$$\Rightarrow i' = 0.0001369$$

Then Accumulated value by simple interest for 91 days is

$$(1 + 91\, i') = 1.012466$$

So, the rate of discount for which the bill is discounted is given by,

$$(1.012466)^{-1} = (1 - 91 d') \qquad \left(\because 1-d = \frac{1}{1+i} ;\ AV = \frac{1}{PV}\right)$$

$$d' = 0.000135302 \qquad (d' = \text{Simple discount for a day})$$

Then, $d = 365 d' = 4.93853\%$